DOI: 10.7641/CTA.2013.12077

具有时变采样周期网络控制系统的严格耗散控制

李 媛[1,2], 张庆灵[1†], 邱占芝[3], 蔡 敏[3]
(1. 东北大学 系统科学研究所, 辽宁 沈阳 110819;
2. 沈阳工业大学 理学院, 辽宁 沈阳 110870; 3. 大连交通大学 理学院, 辽宁 大连 116028)

摘要: 研究具有时变采样周期和数据丢包的网络控制系统的渐近稳定和严格耗散控制问题. 针对采样周期时变且在标称周期上下波动, 数据丢包数有界, 利用参数不确定的方法, 网络控制系统建模为一类带有参数不确定的离散时滞系统. 构造一个改进的李雅普诺夫–卡拉索夫斯基函数, 基于线性矩阵不等式方法及Jensen不等式方法, 给出系统严格(Q, S, R)–耗散的充分条件, 得到控制器的设计方法. 数值实例表明所提出的方法具有较小的保守性, 同时也减少了计算量.

关键词: 网络控制系统; 时变采样周期; 数据丢包; 严格耗散控制; 参数不确定性
中图分类号: TP273 **文献标识码**: A

Strictly dissipative control for networked control systems with time-varying sampling periods

LI Yuan[1,2], ZHANG Qing-ling[1†], QIU Zhan-zhi[3], CAI Min[3]
(1. Institute of Systems Science, Northeastern University, Shenyang Liaoning 110819, China;
2. School of Science, Shenyang University of Technology, Shenyang Liaoning 110870, China;
3. School of Science, Dalian Jiaotong University, Dalian Liaoning 116028, China)

Abstract: This paper studies the asymptotic stability and strictly dissipative control problems for networked control systems (NCSs) with time-varying sampling periods and packet dropouts. The sampling period is time-varying and fluctuates across the nominal period. The packet dropouts vary in a bounded interval. The NCSs are modeled as a class of discrete-time system with parametrical uncertainties by the parameter uncertainty method. An improved Lyapunov-Krasovskii function is constructed. On the basis of LMIs formulation and the discrete Jensen inequality, we derive some new sufficient conditions for strict (Q, S, R)-dissipativity, and present the controller design methods. The numerical example shows that the design method is less conservative and with reduced computational complexity in comparison with conventional methods.

Key words: networked control systems (NCSs); time-varying sampling periods; packet dropouts; strictly dissipative control; parameter uncertainty

1 Introduction

Networked control systems (NCSs) are feedback control systems whose feedback paths are implemented by a real-time network. In many modern distributed control systems, remotely located sensors, actuators and controllers are often connected over a shared communication network. Compared with the control system by traditional point to point direct link, there are several advantages for NCSs, for example, less cost of installation, higher flexibility and reliability, easy maintenance and so on[1–2]. However, communication networks are usually unreliable, and may be subject to undesirable packet dropouts, time-varying sampling periods and network-induced delays, which may significantly degrade the system performance. Therefore, the negative effects caused by communication networks should be taken into account in designing NCSs to obtain desired control performance.

The interest in the stability of NCSs has grown in recent years due to its theoretical and practical significance[3–7]. In most of the existing results concerning stability of NCSs, the sampling period is constant. In practical engineering, the actual sampling period often varies due to dynamic bandwidth allocation and scheduling decisions, this variation can degrade the control performance and even make the systems unstable. In [8], the NCS is modeled as a class of discrete time-delay system by the parameter uncertainty method and D-stability of the NCS is studied. But repeated eigenvalues of the coefficient matrix are not considered in [8].

Dissipativeness is an important concept in control theory, and it has been widely applied in stability analysis for linear systems and nonlinear systems. It extends from pas-

Received 13 April 2012; revised 4 May 2013.
†Corresponding author. E-mail: qlzhang@mail.neu.edu.cn; Tel.: +86 024–83689929.
This work was supported by the National Natural Science Foundation of China (Nos. 61074029, 61104093, 61273008).

sivity analysis and H_∞ performance. The problem of dissipative control has been attracting the attention of many researchers[9–13]. The dissipativity analysis in discrete-time systems is studied in [9–10]. The robust dissipative control problem for time-delay systems is reported in [11–12]. Robust dissipative control for internet-based switching systems is studied in [13]. But there is virtually no results on strict dissipativity for the NCSs with time-varying sampling periods, which motivates the present dissipativity investigation of the NCSs.

In this paper, the asymptotical stability and strictly dissipative control problems are investigated for the NCSs with time-varying sampling periods and packet dropouts. Sufficient conditions for strict dissipativity are derived via the LMI formulation, and the design of strictly dissipative controller is further given. The paper is organized as follows: In Section 2, the NCSs model with packet dropouts and time-varying sampling periods is made, which is equivalent to a class of discrete time-delay system with parametrical uncertainties by mathematical transformation and matrix theory. The strictly dissipative control problem for the foregoing model is considered in Section 3, and Section 4 is an illustrative example, and Section 5 gives some conclusion remarks.

2 Problem formulation

The structure of the considered NCSs is shown in Fig.1, where the plant is described by the following linear system model:

$$\begin{cases} \dot{x}(t) = A_1x(t) + B_1u(t) + C_1w(t), \\ z(t) = A_2x(t) + B_2u(t) + C_2w(t), \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the system state vector, $u(t) \in \mathbb{R}^m$ is the control input vector, $w(t) \in \mathbb{R}^p$ is the disturbance input, $z(t) \in \mathbb{R}^q$ is the controlled output, A_1, A_2, B_1, B_2, C_1, C_2 are known constant matrices of appropriate dimensions, respectively. Suppose the sensor, the controller and the actuator are clock-driven.

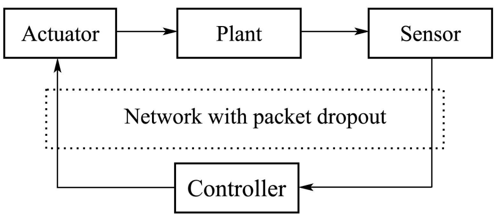

Fig. 1 Diagram of the NCSs with packet dropouts

Assume that T_k is the length of the kth sampling period, t_k is the latest sampling instant, $T_k = t_{k+1} - t_k = \bar{T} + \Delta T_k$ ($\bar{T}$ is the nominal value of the sampling period), and assume that the time-varying sampling interval is bounded, and such that $\Delta T_k = T_k - \bar{T} \in [\Delta T_{\min}, \Delta T_{\max}]$.

Suppose $u(k-L_k)(L_k \geqslant 1)$ is the available control input at the instant t_k, where $L_k - 1$ is the number of consecutive packet dropout, $L_{\max}$ and $L_{\min}$ are the upper-bound and lower-bound of L_k, respectively. So the discrete time expression of the system (1) is as follows:

$$\begin{cases} x(k+1) = \varPhi(T_k)x(k) + \varGamma_1(T_k)u(k-L_k) + \\ \qquad\qquad \varGamma_2(T_k)w(k), \\ z(k) = A_2x(k) + B_2u(k-L_k) + C_2w(k), \end{cases} \tag{2}$$

where

$$\begin{cases} \varPhi(T_k) = \mathrm{e}^{A_1T_k}, \\ \varGamma_1(T_k) = \int_0^{T_k} \mathrm{e}^{A_1t}\mathrm{d}tB_1, \\ \varGamma_2(T_k) = \int_0^{T_k} \mathrm{e}^{A_1t}\mathrm{d}tC_1. \end{cases} \tag{3}$$

Considering the matrix A_1 can be diagonalizable and can be turned into Jordan standard, the NCSs (2) could be modeled as a class of discrete time-delay system with parametrical uncertainties.

1) The matrix A_1 can be diagonalizable.

Let $\lambda_1, \cdots, \lambda_n$ be the n different eigenvalues of the matrix A_1 , there exists an invertible matrix $\varLambda$, where column vectors of $\varLambda$ are eigenvectors corresponding to λ_i of A_1, such that

$$A_1 = \varLambda\mathrm{diag}\{\lambda_1, \cdots, \lambda_n\}\varLambda^{-1}. \tag{4}$$

Combining the formula (4), one gets

$$\begin{cases} \varPhi(T_k) = \\ \varLambda\mathrm{diag}\{\mathrm{e}^{\lambda_1T_k}, \cdots, \mathrm{e}^{\lambda_nT_k}\}\varLambda^{-1} = D_1F_1(T_k)E_1, \\ \varGamma_1(T_k) = \\ \varLambda\mathrm{diag}\{\int_0^{T_k}\mathrm{e}^{\lambda_1t}\mathrm{d}t, \cdots, \int_0^{T_k}\mathrm{e}^{\lambda_nt}\mathrm{d}t\}\varLambda^{-1}B_1 = \\ D_{10} + D_{12}F_2(T_k)E_{12}, \\ \varGamma_2(T_k) = \\ \varLambda\mathrm{diag}\{\int_0^{T_k}\mathrm{e}^{\lambda_1t}\mathrm{d}t, \cdots, \int_0^{T_k}\mathrm{e}^{\lambda_nt}\mathrm{d}t\}\varLambda^{-1}C_1 = \\ D_{20} + D_{22}F_2(T_k)E_{22}. \end{cases} \tag{5}$$

Where scalars $\alpha_1, \cdots, \alpha_n$ meet the following conditions: ① $\lambda_i > 0,\ \ \alpha_i > \Delta T_{\max}$; ② $\lambda_i < 0,\ \ \alpha_i < \Delta T_{\min},\ i = 1, \cdots, n$ and

$$\begin{aligned} & D_1 = \varLambda\mathrm{diag}\{\mathrm{e}^{\lambda_1(\bar{T}+\alpha_1)}, \cdots, \mathrm{e}^{\lambda_n(\bar{T}+\alpha_n)}\}, \\ & D_{12} = D_{22} = \\ & \varLambda\mathrm{diag}\{\frac{1}{\lambda_1}\mathrm{e}^{\lambda_1(\bar{T}+\alpha_1)}, \cdots, \frac{1}{\lambda_n}\mathrm{e}^{\lambda_n(\bar{T}+\alpha_n)}\}, \\ & D_{10} = \varLambda\mathrm{diag}\{-\frac{1}{\lambda_1}, \cdots, -\frac{1}{\lambda_n}\}\varLambda^{-1}B_1, \\ & D_{20} = \varLambda\mathrm{diag}\{-\frac{1}{\lambda_1}, \cdots, -\frac{1}{\lambda_n}\}\varLambda^{-1}C_1, \\ & F_1(T_k) = F_2(T_k) = \\ & \mathrm{diag}\{\mathrm{e}^{\lambda_1(\Delta T_k-\alpha_1)}, \cdots, \mathrm{e}^{\lambda_n(\Delta T_k-\alpha_n)}\}, \\ & E_1 = \varLambda^{-1},\ E_{12} = \varLambda^{-1}B_1,\ E_{22} = \varLambda^{-1}C_1. \end{aligned}$$

2) The matrix A_1 can be turned into Jordan standard.

Let $\lambda^\star$ be the r repeated eigenvalue of the matrix A_1 and the other eigenvalues be different, there exists an invertible matrix $\varLambda$, where column vectors of $\varLambda$ are eigenvectors corresponding to λ_i of A_1, such that

$$A_1 = \varLambda\mathrm{diag}\{J_1, J_2\}\varLambda^{-1}, \tag{6}$$

where

$$J_1 = \mathrm{diag}\{\lambda_1, \cdots, \lambda_{n-r}\},$$

$$J_2=\begin{bmatrix}1 & 0 & \cdots & 0\\ \lambda^\star & 1 & \cdots & 0\\ \vdots & \ddots & \ddots & \vdots\\ \frac{1}{(r-1)!}\lambda^\star & \cdots & \lambda^\star & 1\end{bmatrix}.$$

Combining the formula (6), one gets

$$\begin{cases}\Phi(T_k)=\\ \Lambda\mathrm{diag}\{\mathrm{e}^{J_1T_k},\mathrm{e}^{J_2T_k}\}\Lambda^{-1}=D_1F_1(T_k)E_1,\\ \Gamma_1(T_k)=\\ \Lambda\int_0^{T_k}\mathrm{diag}\{\mathrm{e}^{\lambda_1t},\cdots,\mathrm{e}^{\lambda_{n-r}t},\mathrm{e}^{J_2t}\}\mathrm{d}t\Lambda^{-1}B_1=\\ D_{10}+D_{12}F_2(T_k)E_{12},\\ \Gamma_2(T_k)=\\ \Lambda\int_0^{T_k}\mathrm{diag}\{\mathrm{e}^{\lambda_1t},\cdots,\mathrm{e}^{\lambda_{n-r}t},\mathrm{e}^{J_2t}\}\mathrm{d}t\Lambda^{-1}C_1=\\ D_{20}+D_{22}F_2(T_k)E_{22}.\end{cases}\tag{7}$$

Where scalars $\alpha_1,\cdots,\alpha_{n-r}$ and matrices M_1,M_2 meet the following conditions: ① $\lambda_i>0,\ \alpha_i>\Delta T_{\max},\ \lambda_i<0,\ \alpha_i<\Delta T_{\min},\ i=1,\cdots,n-r$; ② M_1,M_2 are diagonal reversible matrices, $\|M_1^{-1}\hat{J}_2\|<1,\ \|M_2^{-1}\tilde{J}_2\|<1$, and

$$D_1=\Lambda\mathrm{diag}\{\mathrm{e}^{\lambda_1(\bar{T}+\alpha_1)},\cdots,D_{n-r},\ M_1\},$$
$$D_{12}=D_{22}=$$
$$\Lambda\mathrm{diag}\{\frac{1}{\lambda_1}\mathrm{e}^{\lambda_1(\bar{T}+\alpha_1)},\cdots,\frac{1}{\lambda_{n-r}}D_{n-r},\ M_2\},$$
$$D_{10}=\Lambda\mathrm{diag}\{-\frac{1}{\lambda_1},\cdots,-\frac{1}{\lambda_{n-r}},\check{J}_2\}\Lambda^{-1}B_1,$$
$$D_{20}=\Lambda\mathrm{diag}\{-\frac{1}{\lambda_1},\cdots,-\frac{1}{\lambda_{n-r}},\check{J}_2\}\Lambda^{-1}C_1,$$
$$F_1(T_k)=\mathrm{diag}\{\mathrm{e}^{\lambda_1(\Delta T_k-\alpha_1)},\cdots,F_{n-r},\ M_1^{-1}\hat{J}_2\},$$
$$F_2(T_k)=\mathrm{diag}\{\mathrm{e}^{\lambda_1(\Delta T_k-\alpha_1)},\cdots,F_{n-r},\ M_2^{-1}\tilde{J}_2\},$$
$$E_1=\Lambda^{-1},\ E_{12}=\Lambda^{-1}B_1,\ E_{22}=\Lambda^{-1}C_1,$$

$$\hat{J}_2=\begin{bmatrix}\mathrm{e}^{\lambda^\star T_k} & 0 & \cdots & 0\\ T_k\mathrm{e}^{\lambda^\star T_k} & \mathrm{e}^{\lambda^\star T_k} & \cdots & 0\\ \vdots & \ddots & \ddots & \vdots\\ \frac{(T_k)^{r-1}}{(r-1)!}\mathrm{e}^{\lambda^\star T_k} & \cdots & T_k\mathrm{e}^{\lambda^\star T_k} & \mathrm{e}^{\lambda^\star T_k}\end{bmatrix},$$

$$\check{J}_2=\begin{bmatrix}-\frac{1}{\lambda^\star} & 1 & \cdots & 1\\ \frac{1}{(\lambda^\star)^2} & -\frac{1}{\lambda^\star} & \cdots & 1\\ \vdots & \ddots & \ddots & \vdots\\ (-1)^r\frac{1}{(\lambda^\star)^r} & \cdots & \frac{1}{(\lambda^\star)^2} & -\frac{1}{\lambda^\star}\end{bmatrix},$$

$$\tilde{J}_2=\begin{bmatrix}\frac{1}{\lambda^\star}\mathrm{e}^{\lambda^\star T_k} & 0 & \cdots & 0\\ \tilde{J}_{22} & \frac{1}{\lambda^\star}\mathrm{e}^{\lambda^\star T_k} & \cdots & 0\\ \vdots & \ddots & \ddots & \vdots\\ \tilde{J}_{2r} & \cdots & \tilde{J}_{22} & \frac{1}{\lambda^\star}\mathrm{e}^{\lambda^\star T_k}\end{bmatrix},$$

$$\tilde{J}_{22}=\frac{1}{(\lambda^\star)^2}(\lambda^\star T_k-1)\mathrm{e}^{\lambda^\star T_k},$$
$$\tilde{J}_{2r}=\frac{1}{(\lambda^\star)^r}\sum_{j=1}^{r}\frac{(-1)^{j+1}(\lambda^\star T_k)^{r-j}}{(r-j)!}\mathrm{e}^{\lambda^\star T_k},$$
$$D_{n-r}=\mathrm{e}^{\lambda_{n-r}(\bar{T}+\alpha_{n-r})},$$
$$F_{n-r}=\mathrm{e}^{\lambda_{n-r}(\Delta T_k-\alpha_{n-r})}.$$

Definition 1 The quadratic energy supply function E associated with the system (2) is defined by

$$E(w,z,T)=\langle z,Qz\rangle_T+2\langle z,Sw\rangle_T+\langle w,Rw\rangle_T,\tag{8}$$

where Q,S,R are real matrices of appropriate dimensions with Q,R symmetric.

Definition 2[13–15] The system (2) with energy supply E is said to be (Q,S,R)–dissipative, if for any $T\geqslant 0$ and some real function$\eta(\cdot)$ with $\eta(0)=0$,

$$E(w,z,T)+\eta(x_0)\geqslant 0.$$

Furthermore, if for any scalar $\alpha>0$,

$$E(w,z,T)+\eta(x_0)\geqslant\alpha\langle w,w\rangle_T,\tag{9}$$

the system (2) is said to be strictly (Q,S,R)–dissipative.

Remark 1 The above performance of strict (Q,S,R)–dissipativity includes H_∞ performance and passivity as special cases:

1) When $Q=-I,\ S=0$ and $R=\gamma^2I$, (9) reduces to a H_∞ norm constraint[16–17].

2) When $Q=0,\ S=I$ and $R=0$, (9) reduces to a strict passive problem[18–19].

Without loss of generality, we make the following assumption.

Assumption 1 1) $Q\leqslant 0$; 2) $R+C_2^{\mathrm{T}}S+S^{\mathrm{T}}C_2+C_2^{\mathrm{T}}QC_2>0$.

A static controller is considered here with the form

$$u(k)=Kx(k),\tag{10}$$

where K is the controller gain to be determined. Substituting the formula (10) into (2) and combining (5)−(7), we can obtain the following closed-loop system:

$$\begin{cases}x(k+1)=\tilde{A}_1x(k)+\tilde{B}_1x(k-L_k)+\tilde{C}_1w(k),\\ z(k)=A_2x(k)+\tilde{B}_2x(k-L_k)+C_2w(k),\end{cases}\tag{11}$$

where

$$\tilde{A}_1=D_1F_1(T_k)E_1,$$
$$\tilde{B}_1=(D_{10}+D_{12}F_2(T_k)E_{12})K,$$
$$\tilde{C}_1=D_{20}+D_{22}F_2(T_k)E_{22},\ \tilde{B}_2=B_2K.$$

The following lemmas will be used in this paper.

Lemma 1[20] For any positive semi-definite symmetric matrix $W\in\mathbb{R}^{n\times n}$, if the two positive integers r and r_0 satisfy $r\geqslant r_0\geqslant 1$, the following inequality holds:

$$(\sum_{i=r_0}^{r}x(i))^{\mathrm{T}}W(\sum_{i=r_0}^{r}x(i))\leqslant$$
$$(r-r_0+1)\sum_{i=r_0}^{r}x^{\mathrm{T}}(i)Wx(i).$$

Lemma 2[21] Let $\Sigma_1, \Sigma_2, \Sigma_3, \Sigma$ with $\Sigma = \Sigma^{\mathrm{T}} > 0$ be real constant matrices with appropriate dimensions and $\Delta(k)$ be a real matrix function satisfying $\Delta^{\mathrm{T}}(k)\Delta(k) \leqslant I$, then for any scalar $\rho > 0$, the following matrix inequality holds:

$$(\Sigma_3 + \Sigma_1\Delta(k)\Sigma_2)\Sigma^{-1}(\Sigma_3 + \Sigma_1\Delta(k)\Sigma_2)^{-1} \leqslant \\ \rho^{-1}\Sigma_1\Sigma_1^{\mathrm{T}} + \rho\Sigma_3(\Sigma - \rho\Sigma_2^{\mathrm{T}}\Sigma_2)^{-1}\Sigma_3^{\mathrm{T}}.$$

Lemma 3[22] For any matrices $U \in \mathbb{R}^{n\times n}$ and $V \in \mathbb{R}^{n\times n}$, if the matrix V satisfies $V > 0$, then we have

$$V^{-1} \geqslant U + U^{\mathrm{T}} - U^{\mathrm{T}}VU.$$

The purpose of this paper is to design a state-feedback stabilizing controller (10) such that the resulting closed-loop system (11) is asymptotically stable and strictly (Q, S, R)–dissipative. Then, the strictly dissipative control problem for the NCSs addressed in this paper is expressed as follows.

3 Main results

Theorem 1 For given scalars $L_{\max} > 0$, $L_{\min} > 0$ and matrices Q, S, R with Q, R symmetric, the system (11) is asymptotically stable and strictly (Q, S, R)–dissipative, if there exist symmetric positive-definite matrices $P, W, Q_1, Q_2, Z_1, Z_2, Z_3$ satisfying the following matrix inequality

$$\begin{bmatrix} \Lambda_1 & \Lambda_2 \\ \Lambda_2^{\mathrm{T}} & \Lambda_3 \end{bmatrix} < 0, \tag{12}$$

where

$$\begin{cases} \Lambda_1 = \\ \begin{bmatrix} \Lambda_{11} & 0 & Z_1 & Z_2 & -\Lambda_2^{\mathrm{T}}S & (Q_-^{\frac{1}{2}}A_2)^{\mathrm{T}} \\ \star & \Lambda_{22} & Z_3 & Z_3 & -\tilde{B}_2^{\mathrm{T}}S & (Q_-^{\frac{1}{2}}\tilde{B}_2)^{\mathrm{T}} \\ \star & \star & \Lambda_{33} & 0 & 0 & 0 \\ \star & \star & \star & \Lambda_{44} & 0 & 0 \\ \star & \star & \star & \star & \Lambda_{55} & (Q_-^{\frac{1}{2}}C_2)^{\mathrm{T}} \\ \star & \star & \star & \star & \star & -I \end{bmatrix}, \\ \Lambda_{11} = -P + (\tilde{L}+1)W + Q_1 + Q_2 - Z_1 - Z_2, \\ \Lambda_{22} = -W - 2Z_3, \\ \Lambda_{33} = -Q_1 - Z_1 - Z_3, \\ \Lambda_{44} = -Q_2 - Z_2 - Z_3, \\ \tilde{L} = L_{\max} - L_{\min}\ , \\ \Lambda_{55} = -C_2^{\mathrm{T}}S - S^{\mathrm{T}}C_2 - (R - \alpha I), \\ \Lambda_2^{\mathrm{T}} = \\ \begin{bmatrix} \tilde{A}_1 & \tilde{B}_1 & 0 & 0 & \tilde{C}_1 & 0 \\ L_{\min}(\tilde{A}_1 - I) & L_{\min}\tilde{B}_1 & 0 & 0 & L_{\min}\tilde{C}_1 & 0 \\ \tilde{L}(\tilde{A}_1 - I) & \tilde{L}\tilde{B}_1 & 0 & 0 & \tilde{L}\tilde{C}_1 & 0 \\ L_{\max}(\tilde{A}_1 - I) & L_{\max}\tilde{B}_1 & 0 & 0 & L_{\max}\tilde{C}_1 & 0 \end{bmatrix}, \\ \Lambda_3 = \mathrm{diag}\{-P^{-1}, -Z_1^{-1}, -Z_2^{-1}, -Z_3^{-1}\}. \end{cases} \tag{13}$$

Proof Construct the following Lyapunov-Krasovskii function:

$$\begin{cases} V(k) = \\ V_1(k) + V_2(k) + V_3(k) + V_4(k) + V_5(k), \\ V_1(k) = x^{\mathrm{T}}(k)Px(k), \\ V_2(k) = \sum\limits_{l=k-L_k}^{k-1} x^{\mathrm{T}}(l)Wx(l), \\ V_3(k) = \sum\limits_{l=k-L_{\min}}^{k-1} x^{\mathrm{T}}(l)Q_1x(l) + \\ \qquad \sum\limits_{l=k-L_{\max}}^{k-1} x^{\mathrm{T}}(l)Q_2x(l), \\ V_4(k) = \sum\limits_{l=-L_{\max}+1}^{-L_{\min}} \sum\limits_{j=k+l}^{k-1} x^{\mathrm{T}}(j)Wx(j), \\ V_5(k) = L_{\min}\sum\limits_{l=-L_{\min}}^{-1} \sum\limits_{j=k+l}^{k-1} y^{\mathrm{T}}(j)Z_1y(j) + \\ \qquad L_{\max}\sum\limits_{l=-L_{\max}}^{-1} \sum\limits_{j=k+l}^{k-1} y^{\mathrm{T}}(j)Z_2y(j) + \\ \qquad \tilde{L}\sum\limits_{l=-L_{\max}}^{-L_{\min}-1} \sum\limits_{j=k+l}^{k-1} y^{\mathrm{T}}(j)Z_3y(j), \end{cases} \tag{14}$$

where $y(j) = x(j+1) - x(j)$. So

$$\Delta V_1(k) = \\ (\tilde{A}_1x(k) + \tilde{B}_1x(k-L_k) + \tilde{C}_1w(k))^{\mathrm{T}}P(\tilde{A}_1x(k) + \\ \tilde{B}_1x(k-L_k) + \tilde{C}_1w(k)) - x^{\mathrm{T}}(k)Px(k), \tag{15}$$

$$\Delta V_2(k) = \\ x^{\mathrm{T}}(k)Wx(k) - x^{\mathrm{T}}(k-L_k)Wx(k-L_k) + \\ \sum_{l=k+1-L_{\min}}^{k-1} x^{\mathrm{T}}(l)Wx(l) - \sum_{l=k+1-L_k}^{k-1} x^{\mathrm{T}}(l)Wx(l) + \\ \sum_{l=k+1-L_{k+1}}^{k-L_{\min}} x^{\mathrm{T}}(l)Wx(l), \tag{16}$$

$$\Delta V_3(k) = \\ x^{\mathrm{T}}(k)Q_1x(k) - x^{\mathrm{T}}(k-L_{\min})Q_1x(k-L_{\min}) + \\ x^{\mathrm{T}}(k)Q_2x(k) - x^{\mathrm{T}}(k-L_{\max})Q_2x(k-L_{\max}), \tag{17}$$

$$\Delta V_4(k) = \\ \tilde{L}x^{\mathrm{T}}(k)Wx(k) - \sum_{l=k+1-L_{\max}}^{k-L_{\min}} x^{\mathrm{T}}(l)Wx(l), \tag{18}$$

$$\Delta V_5(k) = \\ L_{\min}^2 y^{\mathrm{T}}(k)Z_1y(k) - L_{\min}\sum_{l=k-L_{\min}}^{k-1} y^{\mathrm{T}}(l)Z_1y(l) + \\ L_{\max}^2 y^{\mathrm{T}}(k)Z_2y(k) - L_{\max}\sum_{l=k-L_{\max}}^{k-1} y^{\mathrm{T}}(l)Z_2y(l) + \\ \tilde{L}^2 y^{\mathrm{T}}(k)Z_3y(k) - \tilde{L}\sum_{l=k-L_{\max}}^{k-L_{\min}-1} y^{\mathrm{T}}(l)Z_3y(l). \tag{19}$$

To notice that $L_k \geqslant L_{\min}, L_{k+1} \leqslant L_{\max}$, so

$$\Delta V_2(k) \leqslant \\ x^{\mathrm{T}}(k)Wx(k) - x^{\mathrm{T}}(k-L_k)Wx(k-L_k) + \\ \sum_{l=k+1-L_{\max}}^{k-L_{\min}} x^{\mathrm{T}}(l)Wx(l). \tag{20}$$

Using Lemma 1, we have that

$$-L_{\min}\sum_{l=k-L_{\min}}^{k-1} y^{\mathrm{T}}(l)Z_1y(l) \leqslant -(\sum_{l=k-L_{\min}}^{k-1} y(l))^{\mathrm{T}}Z_1(\sum_{l=k-L_{\min}}^{k-1} y(l)) = -(x(k)-x(k-L_{\min}))^{\mathrm{T}}Z_1(x(k)-x(k-L_{\min})), \tag{21}$$

$$-L_{\max}\sum_{l=k-L_{\max}}^{k-1} y^{\mathrm{T}}(l)Z_2y(l) \leqslant -(\sum_{l=k-L_{\max}}^{k-1} y(l))^{\mathrm{T}}Z_2(\sum_{l=k-L_{\max}}^{k-1} y(l)) = -(x(k)-x(k-L_{\max}))^{\mathrm{T}}Z_2(x(k)-x(k-L_{\max})), \tag{22}$$

$$-\tilde{L}\sum_{l=k-L_{\max}}^{k-L_{\min}-1} y^{\mathrm{T}}(l)Z_3y(l) = -\tilde{L}\sum_{l=k-L_{\max}}^{k-L_k-1} y^{\mathrm{T}}(l)Z_3y(l) - \tilde{L}\sum_{l=k-L_k}^{k-L_{\min}-1} y^{\mathrm{T}}(l)Z_3y(l) \leqslant -(\textstyle\sum_{l=k-L_{\max}}^{k-L_k-1} y(l))^{\mathrm{T}}Z_3(\sum_{l=k-L_{\max}}^{k-L_k-1} y(l)) - (\sum_{l=k-L_k}^{k-L_{\min}-1} y(l))^{\mathrm{T}}Z_3(\sum_{l=k-L_k}^{k-L_{\min}-1} y(l)) = -(x(k-L_k)-x(k-L_{\max}))^{\mathrm{T}}Z_3(x(k-L_k)-x(k-L_{\max}))-(x(k-L_{\min})-x(k-L_k))^{\mathrm{T}}Z_3(x(k-L_{\min})-x(k-L_k)). \tag{23}$$

Combining (15)−(23), we have that when $w(k)=0$,

$$\Delta V(k) \leqslant x^{\mathrm{T}}(k+1)Px(k+1) + x^{\mathrm{T}}(k)((\tilde{L}+1)W + Q_1+Q_2-P)x(k)-x^{\mathrm{T}}(k-L_k)Wx(k-L_k)+ (x(k+1)-x(k))^{\mathrm{T}}(L_{\min}^2Z_1+L_{\max}^2Z_2+ \tilde{L}^2Z_3)(x(k+1)-x(k)) - x^{\mathrm{T}}(k-L_{\min})\cdot Q_1x(k-L_{\min})-x^{\mathrm{T}}(k-L_{\max})Q_2x(k-L_{\max})- (x(k)-x(k-L_{\min}))^{\mathrm{T}}Z_1(x(k)-x(k-L_{\min}))- (x(k)-x(k-L_{\max}))^{\mathrm{T}}Z_2(x(k)-x(k-L_{\max})) - (x(k-L_k)-x(k-L_{\max}))^{\mathrm{T}}Z_3(x(k-L_k)- x(k-L_{\max})) - (x(k-L_{\min})-x(k-L_k))^{\mathrm{T}}\cdot Z_3(x(k-L_{\min})-x(k-L_k))=\xi^{\mathrm{T}}(k)\Phi'\xi(k), \tag{24}$$

where

$$\xi(k)=\begin{bmatrix} x(k)\\ x(k-L_k)\\ x(k-L_{\min})\\ x(k-L_{\max})\end{bmatrix},\ \Phi'=\begin{bmatrix}\Phi'_{11} & \Phi'_{12} & Z_1 & Z_2\\ \star & \Phi'_{22} & Z_3 & Z_3\\ \star & \star & \Phi'_{33} & 0\\ \star & \star & \star & \Phi'_{44}\end{bmatrix},$$

$$\Phi'_{11} = \tilde{A}_1^{\mathrm{T}}P\tilde{A}_1 - P + (\tilde{L}+1)W + Q_1 + Q_2 - Z_1-Z_2+(\tilde{A}_1-I)^{\mathrm{T}}(L_{\min}^2Z_1+L_{\max}^2Z_2+ \tilde{L}^2Z_3)(\tilde{A}_1-I),$$

$$\Phi'_{12} = (\tilde{A}_1-I)^{\mathrm{T}}(L_{\min}^2Z_1+L_{\max}^2Z_2+ \tilde{L}^2Z_3)\tilde{B}_1+\tilde{A}_1^{\mathrm{T}}P\tilde{B}_1,$$

$$\Phi'_{22} = \tilde{B}_1^{\mathrm{T}}(L_{\min}^2Z_1+L_{\max}^2Z_2+\tilde{L}^2Z_3)\tilde{B}_1+ \tilde{B}_1^{\mathrm{T}}P\tilde{B}_1 - W - 2Z_3,$$

$$\Phi'_{33} = -Q_1-Z_1-Z_3,\ \Phi'_{44} = -Q_2-Z_2-Z_3.$$

By Schur complement and (24), if (12) holds, we have that

$$\Delta V(k) \leqslant -\lambda_{\min}(-\Phi')\xi^{\mathrm{T}}(k)\xi(k) \leqslant -\alpha\xi^{\mathrm{T}}(k)\xi(k) \leqslant -\alpha x^{\mathrm{T}}(k)x(k), \tag{25}$$

where $\alpha = \inf\{\lambda_{\min}(-\Phi')\}$. This proves that the system (11) is asymptotically stable for $L_{\min}\leqslant L_k \leqslant L_{\max}$.

Combining (12) and (24), one gets

$$\Delta V(k) - z^{\mathrm{T}}(k)Qz(k) - 2z^{\mathrm{T}}(k)Sw(k) - w^{\mathrm{T}}(k)(R-\alpha I)w(k) \leqslant \begin{bmatrix}\xi^{\mathrm{T}}(k) & w^{\mathrm{T}}(k)\end{bmatrix}\tilde{\Phi}'\begin{bmatrix}\xi^{\mathrm{T}}(k) & w^{\mathrm{T}}(k)\end{bmatrix}^{\mathrm{T}} \leqslant 0, \tag{26}$$

where

$$\tilde{\Phi}' = \begin{bmatrix}\tilde{\Phi}'_{11} & \tilde{\Phi}'_{12} & Z_1 & Z_2 & \tilde{\Phi}'_{15}\\ \star & \tilde{\Phi}'_{22} & Z_3 & Z_3 & \tilde{\Phi}'_{25}\\ \star & \star & \tilde{\Phi}'_{33} & 0 & 0\\ \star & \star & \star & \tilde{\Phi}'_{44} & 0\\ \star & \star & \star & \star & \tilde{\Phi}'_{55}\end{bmatrix},$$

$$\tilde{\Phi}'_{11} = \tilde{A}_1^{\mathrm{T}}P\tilde{A}_1 - P + (\tilde{L}+1)W + Q_1+Q_2 - Z_1 - Z_2 + (\tilde{A}_1-I)^{\mathrm{T}}(L_{\min}^2Z_1+L_{\max}^2Z_2+ \tilde{L}^2Z_3)(\tilde{A}_1-I) - A_2^{\mathrm{T}}QA_2,$$

$$\tilde{\Phi}'_{12} = (\tilde{A}_1-I)^{\mathrm{T}}(L_{\min}^2Z_1+L_{\max}^2Z_2+\tilde{L}^2Z_3)\tilde{B}_1+ \tilde{A}_1^{\mathrm{T}}P\tilde{B}_1 - \tilde{A}_2^{\mathrm{T}}Q\tilde{B}_2,$$

$$\tilde{\Phi}'_{15} = (\tilde{A}_1-I)^{\mathrm{T}}(L_{\min}^2Z_1+L_{\max}^2Z_2+\tilde{L}^2Z_3)\tilde{C}_1+ \tilde{A}_1^{\mathrm{T}}P\tilde{C}_1 - A_2^{\mathrm{T}}QC_2 - A_2^{\mathrm{T}}S,$$

$$\tilde{\Phi}'_{22} = \tilde{B}_1^{\mathrm{T}}(L_{\min}^2Z_1+L_{\max}^2Z_2+\tilde{L}^2Z_3)\tilde{B}_1+ \tilde{B}_1^{\mathrm{T}}P\tilde{B}_1 - W - 2Z_3 - \tilde{B}_2^{\mathrm{T}}Q\tilde{B}_2,$$

$$\tilde{\Phi}'_{25} = \tilde{B}_1^{\mathrm{T}}(L_{\min}^2Z_1+L_{\max}^2Z_2+\tilde{L}^2Z_3)\tilde{C}_1+ \tilde{B}_1^{\mathrm{T}}P\tilde{C}_1 - \tilde{B}_2^{\mathrm{T}}QC_2 - \tilde{B}_2^{\mathrm{T}}S,$$

$$\tilde{\Phi}'_{33} = -Q_1-Z_1-Z_3,\ \tilde{\Phi}'_{44} = -Q_2-Z_2-Z_3,$$

$$\tilde{\Phi}'_{55} = \tilde{C}_1^{\mathrm{T}}(P+L_{\min}^2Z_1+L_{\max}^2Z_2+\tilde{L}^2Z_3)\tilde{C}_1 - C_2^{\mathrm{T}}QC_2 - C_2^{\mathrm{T}}S - S^{\mathrm{T}}C_2 - (R-\alpha I).$$

Taking (26) the sum from 0 to T, since $V(x(T+1))\geqslant 0$, we have that

$$E(w,z,T) = \langle z,Qz\rangle_T + 2\langle z,Sw\rangle_T + \langle w,Rw\rangle_T \geqslant \alpha\langle w,w\rangle_T + \sum_{k=0}^{\mathrm{T}}[V(k+1)-V(k)] = \alpha\langle w,w\rangle_T + V(T+1) - V(0) \geqslant \alpha\langle w,w\rangle_T - V(0).$$

Therefore, from the Definition 2, we can get the system (11) is strictly (Q,S,R)–dissipative. This completes the proof.

Remark 2 In this paper, it is in contrast with [13] for an improved Lyapunov-Krasovskii functional, the added term

$\tilde{L} \sum_{l=-L_{\max}}^{-L_{\min}-1} \sum_{j=k+l}^{k-1} y^{\mathrm{T}}(j) Z_3 y(j)$ of the Lyapunov-Krasovskii functional in this paper can reduce conservative. Another different method to deal with the cross term problem by using the discrete Jensen inequality, neither system transformation nor free-weighting matrix is required, which decreases calculational amount.

Theorem 2 For given scalars $L_{\max} > 0$, $L_{\min} > 0$, $\rho > 0$ and matrices Q, S, R with Q, R symmetric, the system (11) is asymptotically stable and strictly (Q, S, R)–dissipative, if there exist symmetric positive-definite matrices $X, \tilde{W}, \tilde{Q}_1, \tilde{Q}_2, \tilde{Z}_1, \tilde{Z}_2, \tilde{Z}_3$ and matrix$Y$ satisfying the following LMI:

$$\begin{bmatrix} \tilde{\Lambda}_1 & \sqrt{\rho}\,\tilde{\Sigma}_3 & \tilde{\Sigma}_1 & 0 \\ \star & \tilde{\Lambda}_3 & 0 & \rho\Sigma_2^{\mathrm{T}} \\ \star & \star & -\rho I & 0 \\ \star & \star & \star & -\rho I \end{bmatrix} < 0, \tag{27}$$

where

$$\begin{cases} \tilde{\Lambda}_1 = \\ \begin{bmatrix} \tilde{\Lambda}_{11} & 0 & \tilde{Z}_1 & \tilde{Z}_2 & -(A_2X)^{\mathrm{T}}S & (Q_-^{\frac{1}{2}}A_2X)^{\mathrm{T}} \\ \star & \tilde{\Lambda}_{22} & \tilde{Z}_3 & \tilde{Z}_3 & -(B_2Y)^{\mathrm{T}}S & (Q_-^{\frac{1}{2}}B_2Y)^{\mathrm{T}} \\ \star & \star & \tilde{\Lambda}_{33} & 0 & 0 & 0 \\ \star & \star & \star & \tilde{\Lambda}_{44} & 0 & 0 \\ \star & \star & \star & \star & \tilde{\Lambda}_{55} & (Q_-^{\frac{1}{2}}C_2)^{\mathrm{T}} \\ \star & \star & \star & \star & \star & -I \end{bmatrix}, \\ \tilde{\Lambda}_{11} = -X + (\tilde{L}+1)\tilde{W} + \tilde{Q}_1 + \tilde{Q}_2 - \tilde{Z}_1 - \tilde{Z}_2, \\ \tilde{\Lambda}_{22} = -\tilde{W} - 2\tilde{Z}_3,\ \tilde{\Lambda}_{33} = -\tilde{Q}_1 - \tilde{Z}_1 - \tilde{Z}_3, \\ \tilde{\Lambda}_{44} = -\tilde{Q}_2 - \tilde{Z}_2 - \tilde{Z}_3,\ \tilde{L} = L_{\max} - L_{\min}, \\ \tilde{\Lambda}_{55} = -C_2^{\mathrm{T}}S - S^{\mathrm{T}}C_2 - (R - \alpha I), \\ \tilde{\Lambda}_3 = \mathrm{diag}\{-X, -X - X^{\mathrm{T}} + \tilde{Z}_1, \\ \qquad -X - X^{\mathrm{T}} + \tilde{Z}_2, -X - X^{\mathrm{T}} + \tilde{Z}_3\}, \\ \tilde{\Sigma}_1 = \begin{bmatrix} E_1X & 0 & 0 & 0 & 0 & 0 \\ 0 & E_{12}Y & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & E_{22} & 0 \end{bmatrix}^{\mathrm{T}}, \\ \Sigma_2^{\mathrm{T}} = \begin{bmatrix} D_1^{\mathrm{T}} & L_{\min}D_1^{\mathrm{T}} & \tilde{L}D_1^{\mathrm{T}} & L_{\max}D_1^{\mathrm{T}} \\ D_{12}^{\mathrm{T}} & L_{\min}D_{12}^{\mathrm{T}} & \tilde{L}D_{12}^{\mathrm{T}} & L_{\max}D_{12}^{\mathrm{T}} \\ D_{22}^{\mathrm{T}} & L_{\min}D_{22}^{\mathrm{T}} & \tilde{L}D_{22}^{\mathrm{T}} & L_{\max}D_{22}^{\mathrm{T}} \end{bmatrix}, \\ \tilde{\Sigma}_3^{\mathrm{T}} = \\ \begin{bmatrix} 0 & D_{10}Y & 0 & 0 & D_{20}^{\mathrm{T}} & 0 \\ -L_{\min}X & L_{\min}(D_{10}Y) & 0 & 0 & L_{\min}D_{20} & 0 \\ -\tilde{L}X & \tilde{L}(D_{10}Y) & 0 & 0 & \tilde{L}D_{20} & 0 \\ -L_{\max}X & L_{\max}(D_{10}Y) & 0 & 0 & L_{\max}D_{20} & 0 \end{bmatrix}. \end{cases} \tag{28}$$

Furthermore, the desired controller gain is given by $K = YX^{-1}$.

Proof Replacing $\tilde{A}_1$, $\tilde{C}_1$ and $\tilde{B}_2$ in (12) with $D_1F_1(T_k)E_1, (D_{10} + D_{12}F_2(T_k)E_{12})K, D_{20} + D_{22}F_2(T_k)E_{22}$ and B_2K, respectively, we find that (12) is equivalent to the following inequality:

$$(\Sigma_3 + \Sigma_1\Delta(T_k)\Sigma_2)(-\Lambda_3)^{-1}(\Sigma_3 + \Sigma_1\Delta(T_k)\Sigma_2)^{\mathrm{T}} + \Lambda_1 < 0, \tag{29}$$

where

$$\Sigma_1 = \begin{bmatrix} E_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & E_{12}K & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & E_{22} & 0 \end{bmatrix}^{\mathrm{T}},$$

$$\Sigma_3 = \begin{bmatrix} 0 & -L_{\min}I & -\tilde{L}I & -L_{\max}I \\ (D_{10}K)^{\mathrm{T}} & \Sigma_{31} & \Sigma_{32} & \Sigma_{33} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ D_{20}^{\mathrm{T}} & L_{\min}D_{20}^{\mathrm{T}} & \tilde{L}D_{20}^{\mathrm{T}} & L_{\max}D_{20}^{\mathrm{T}} \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$\Sigma_{31} = L_{\min}(D_{10}K)^{\mathrm{T}},\ \Sigma_{32} = \tilde{L}(D_{10}K)^{\mathrm{T}},$

$\Sigma_{33} = L_{\max}(D_{10}K)^{\mathrm{T}},$

$\Delta(T_k) = \mathrm{diag}\{F_1^{\mathrm{T}}(T_k), F_2^{\mathrm{T}}(T_k), F_2^{\mathrm{T}}(T_k)\}.$

Since $F_1^{\mathrm{T}}(T_k)F_1(T_k) < I,\ F_2^{\mathrm{T}}(T_k)F_2(T_k) < I$, we have $\Delta^{\mathrm{T}}(T_k)\Delta(T_k) < I$. For any scalar $\rho > 0$, using Lemma 2 and Schur complement, if the following inequality is feasible, (29) is also feasible.

$$\Lambda_1 + \rho^{-1}\Sigma_1\Sigma_1^{\mathrm{T}} + \rho\Sigma_3(-\Lambda_3 - \rho\Sigma_2^{\mathrm{T}}\Sigma_2)^{-1}\Sigma_3^{\mathrm{T}} < 0. \tag{30}$$

Using Schur complement again, if the following inequality holds, the inequality (30) holds.

$$\begin{bmatrix} \Lambda_1 & \sqrt{\rho}\,\Sigma_3 & \Sigma_1 & 0 \\ \star & \Lambda_3 & 0 & \rho\Sigma_2^{\mathrm{T}} \\ \star & \star & -\rho I & 0 \\ \star & \star & \star & -\rho I \end{bmatrix} < 0. \tag{31}$$

Let

$T = \mathrm{diag}\{T_1, T_2, T_3, T_2\},$

$T_1 = \mathrm{diag}\{X, X, X, X, I, I\},$

$T_2 = \mathrm{diag}\{I, I, I, I\},\ T_3 = \mathrm{diag}\{I, I, I, I, I, I\}.$

Pre- and post- multiply (31) by T^{T} and T, and define

$P^{-1} = X,\ X^{\mathrm{T}}WX = \tilde{W},\ X^{\mathrm{T}}Q_1X = \tilde{Q}_1,$

$X^{\mathrm{T}}Q_2X = \tilde{Q}_2,\ X^{\mathrm{T}}Z_1X = \tilde{Z}_1,\ X^{\mathrm{T}}Z_2X = \tilde{Z}_2,$

$X^{\mathrm{T}}Z_3X = \tilde{Z}_3,\ KX = Y,$

we can obtain

$$\begin{bmatrix} \tilde{\Lambda}_1 & \sqrt{\rho}\,\tilde{\Sigma}_3 & \tilde{\Sigma}_1 & 0 \\ \star & \Lambda_3 & 0 & \rho\Sigma_2^{\mathrm{T}} \\ \star & \star & -\rho I & 0 \\ \star & \star & \star & -\rho I \end{bmatrix} < 0. \tag{32}$$

By Lemma 3, we can get

$$\begin{cases} -Z_1^{-1} \leqslant -X - X^{\mathrm{T}} + X^{\mathrm{T}}Z_1X, \\ -Z_2^{-1} \leqslant -X - X^{\mathrm{T}} + X^{\mathrm{T}}Z_2X, \\ -Z_3^{-1} \leqslant -X - X^{\mathrm{T}} + X^{\mathrm{T}}Z_3X. \end{cases} \tag{33}$$

According to the inequalities (33), if (27) is feasible, (32) is also feasible. The proof is completed.

In order to illustrate the generality of our analysis and synthesis approach, we offer the following results.

Corollary 1 For given scalars $L_{\max} > 0$, $L_{\min} > 0$, $\rho > 0$, the system (11) is asymptotically stable with disturbance attenuationγ, if there exist symmetric positive-definite matrices$X, \tilde{W}, \tilde{Q}_1, \tilde{Q}_2, \tilde{Z}_1, \tilde{Z}_2, \tilde{Z}_3$ and matrix satisfying the following LMI:

$$\begin{bmatrix} \tilde{\varLambda}_1 & \sqrt{\rho}\,\tilde{\varSigma}_3 & \tilde{\varSigma}_1 & 0 \\ \star & \tilde{\varLambda}_3 & 0 & \rho\varSigma_2^{\mathrm{T}} \\ \star & \star & -\rho I & 0 \\ \star & \star & \star & -\rho I \end{bmatrix} < 0, \tag{34}$$

where

$$\tilde{\varLambda}_1 = \begin{bmatrix} \tilde{\varLambda}_{11} & 0 & \tilde{Z}_1 & \tilde{Z}_2 & 0 & (A_2X)^{\mathrm{T}} \\ \star & \tilde{\varLambda}_{22} & \tilde{Z}_3 & \tilde{Z}_3 & 0 & (B_2Y)^{\mathrm{T}} \\ \star & \star & \tilde{\varLambda}_{33} & 0 & 0 & 0 \\ \star & \star & \star & \tilde{\varLambda}_{44} & 0 & 0 \\ \star & \star & \star & \star & -\gamma^2 I & C_2^{\mathrm{T}} \\ \star & \star & \star & \star & \star & -I \end{bmatrix}. \tag{35}$$

Furthermore, the desired controller gain is given by $K = YX^{-1}$.

Corollary 2 For given scalars $L_{\max} > 0,\ L_{\min} > 0,\ \rho > 0$, the system (11) is asymptotically stable and strictly positive, if there exist symmetric positive-definite matrices $X, \tilde{W}, \tilde{Q}_1, \tilde{Q}_2, \tilde{Z}_1, \tilde{Z}_2, \tilde{Z}_3$ and matrix Y satisfying the following LMI:

$$\begin{bmatrix} \tilde{\varLambda}_1 & \sqrt{\rho}\,\tilde{\varSigma}_3 & \tilde{\varSigma}_1 & 0 \\ \star & \tilde{\varLambda}_3 & 0 & \rho\varSigma_2^{\mathrm{T}} \\ \star & \star & -\rho I & 0 \\ \star & \star & \star & -\rho I \end{bmatrix} < 0, \tag{36}$$

where

$$\tilde{\varLambda}_1 = \begin{bmatrix} \tilde{\varLambda}_{11} & 0 & \tilde{Z}_1 & \tilde{Z}_2 & -(A_2X)^{\mathrm{T}} & 0 \\ \star & \tilde{\varLambda}_{22} & \tilde{Z}_3 & \tilde{Z}_3 & -(B_2Y)^{\mathrm{T}} & 0 \\ \star & \star & \tilde{\varLambda}_{33} & 0 & 0 & 0 \\ \star & \star & \star & \tilde{\varLambda}_{44} & 0 & 0 \\ \star & \star & \star & \star & -C_2^{\mathrm{T}} - C_2 & 0 \\ \star & \star & \star & \star & \star & -I \end{bmatrix}. \tag{37}$$

Furthermore, the desired controller gain is given by $K = YX^{-1}$.

Remark 3 According to Theorem 2, Corollary 1 can be deduced by setting $Q = -I,\ S = 0,\ (R - \alpha I) = \gamma^2 I$. Corollary 2 can be deduced by setting $Q = 0,\ S = I,\ (R - \alpha I) = 0$.

4 Numerical example

Considering an unstable system as follows

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} -0.36 & -1.12 \\ -0.40 & 1.11 \end{bmatrix} x(t) + \\ \qquad \begin{bmatrix} 0.41 \\ -1.22 \end{bmatrix} u(t) + \begin{bmatrix} 1.31 \\ 0.62 \end{bmatrix} w(t), \\ z(t) = [-0.31 \quad -0.02]x(t) - \\ \qquad 0.68u(t) + 0.32w(t). \end{cases} \tag{38}$$

Suppose $\bar{T} = 0.4\,\mathrm{s}$, $\Delta T_{\max} = 0.06\,\mathrm{s}$, $\Delta T_{\min} = -0.02\,\mathrm{s}$, we discretize the system (38) and can obtain

$$E_1 = \begin{bmatrix} 0.7887 & -0.7734 \\ 0.3651 & 1.0425 \end{bmatrix},\ E_{12} = \begin{bmatrix} 1.2669 \\ -1.1222 \end{bmatrix},$$

$$E_{22} = \begin{bmatrix} 0.5537 \\ 1.1246 \end{bmatrix},\ D_1 = \begin{bmatrix} 0.9585 & 0.4016 \\ -0.3357 & 0.4096 \end{bmatrix},$$

$$D_{10} = \begin{bmatrix} -37.6343 \\ 12.4628 \end{bmatrix},\ D_{20} = \begin{bmatrix} -15.6963 \\ 6.2149 \end{bmatrix},$$

$$D_{12} = D_{22} = \begin{bmatrix} 29.7498 & -0.2673 \\ -10.4183 & -0.2726 \end{bmatrix},$$

$$F_1(T_k) = F_2(T_k) = \mathrm{diag}\{\mathrm{e}^{\lambda_1(\Delta T_k - \alpha_1)},\ \mathrm{e}^{\lambda_2(\Delta T_k - \alpha_2)}\}.$$

where $\lambda_1 = 0.0322,\ \lambda_2 = -1.5022,\ \alpha_1 = 0.08,\ \alpha_2 = -0.03$, then $F_1(T_k),\ F_2(T_k)$ satisfy $F_1^{\mathrm{T}}(T_k)F_1(T_k) \leqslant I,\ F_2^{\mathrm{T}}(T_k)F_2(T_k) \leqslant I$.

For simplicity, suppose $L_{\max} = 3,\ L_{\min} = 1$. Applying Theorem 2, we obtain the stabilizing networked controller gain $K = [-0.0309 \quad -0.0059]$. To simulate, we take the initial state as $x(0) = [1 \quad -1]^{\mathrm{T}}$, the system state response and controlled output with above parameters can be seen in Fig.2.

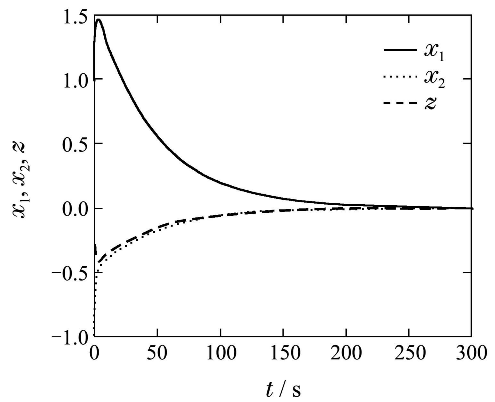

Fig. 2 State response and controlled output of the NCSs

5 Conclusions

The asymptotic stability and strictly dissipative control problems for the NCSs with packet dropouts and time-varying sampling periods have been discussed in this paper. The time-varying sampling period fluctuates across the nominal period. The number of packet dropouts is assumed to have both an upper-bound and a lower-bound. An improved Lyapunov-Krasovskii functional has been constructed. The discrete Jensen inequality is used to deal with the cross term. Based on the LMIs formulation, some new sufficient conditions for strict (Q, S, R)-dissipativity have been derived, and the controller design methods have been presented. A numerical example has illustrated the effectiveness of the proposed approach.

References:

[1] MONTESTRUQUE L A, ANTSAKLIS P J. On the model-based control of networked systems [J]. *Automatica*, 2003, 39(10): 1837 – 1843.

[2] MONTESTRUQUE L A, ANTSAKLIS P J. Stability of model-based networked control systems with time-varying transmission times [J]. *IEEE Transactions on Automatic Control*, 2004, 49(9): 1562 – 1573.

[3] ZHANG W, BRANICKY M S, PHILIPD S M. Stability analysis of networked control systems [J]. *IEEE Control Systems Magazine*, 2001, 21(1): 84 – 99.

[4] YUE D, PENG C, HAN Q L. *Analysis and Synthesis of Network Control Systems* [M]. Beijing: Science Press, 2006.

[5] LI J N, ZHANG Q L, CAI M. Modeling and robust stability of networked control systems with packet reordering and long delay [J]. *International Journal of Control*, 2009, 82(10): 1773 – 1785.

[6] DONKERS M C F, HEEMELS W P M H, BERNARDINI D, et al. Stability analysis of stochastic networked control systems [J]. *Automatica*, 2012, 48(5): 917 – 925.

[7] WANG J F, YANG H Z. Exponential stability of a class of networked control systems with time delays and packet dropouts [J]. *Applied Mathematics and Computation*, 2012, 218(17): 8887 – 8894.

[8] WANG Yan, JI Zhicheng, XIE Linbo, et al. Co-design for networked control systems based on vary-period sample approach [J]. *Journal of System Simulation*, 2008, 20(8): 2108 – 2114.
(王艳, 纪志成, 谢林柏, 等. 基于变采样周期方法的网络控制系统协同设计 [J]. 系统仿真学报, 2008, 20(8): 2108 – 2114.)

[9] LIU Fei, SU Hongye, CHU Jian. Robust strictly dissipative control for linear discrete time-delay systems [J]. *Acta Automatica Sinica*, 2002, 28(6): 897 – 903.
(刘飞, 苏宏业, 褚健. 线性离散时滞系统鲁棒严格耗散控制 [J]. 自动化学报, 2002, 28(6): 897 – 903.)

[10] SHAO Hanyong. Robust dissipative control for a class of uncertain linear discrete-time systems with time-delay [J]. *Control Theory & Applications*, 2006, 23(3): 443 – 448.
(邵汉永. 线性离散时滞系统的鲁棒耗散控制 [J]. 控制理论与应用, 2006, 23(3): 443 – 448.)

[11] YANG Li, ZHANG Qingling, YANG Xiaoguang, et al. Robust impulse dissipative control of singular systems with uncertainties [J]. *Control Theory & Applications*, 2007, 24(6): 1038 – 1042.
(杨丽, 张庆灵, 杨晓光, 等. 一类不确定广义时滞系统的鲁棒耗散控制 [J]. 控制理论与应用, 2007, 24(6): 1038 – 1042.)

[12] MAHMOUDA M S, SHI Y, AL-SUNNI F M . Dissipativity analysis and synthesis of a class of nonlinear systems with time-varying delays [J]. *Journal of the Franklin Institute*, 2009, 346(6): 570 – 592.

[13] MAHMOUDA M S, NOUNOUB H N, XIA Y Q. Robust dissipative control for internet-based switching systems [J]. *Journal of the Franklin Institute*, 2010, 347(1): 154 – 172.

[14] HILL D J, MOYLAN P J. Dissipative dynamical systems: basic input-output and state properties [J]. *Journal of the Franklin Institute*, 1980, 309(5): 327 – 357.

[15] XIE S L, XIE L H, DE SOUZA C E. Robust dissipative control for linear systems with dissipative uncertainty [J]. *International Journal of Control*, 1998, 70(2): 169 – 191.

[16] CHOI H H, CHUNG M J. An LMI approach to H_∞ controller design for linear time-delay systems [J]. *Automatica*, 1997, 33(4): 737 – 739.

[17] JIANG X F, HAN Q L. On H_∞ control for linear systems with interval time-varying delay [J]. *Automatica*, 2005, 41(12): 2099 – 2106.

[18] YU Li, CHEN Guoding. Passive control of linear time-delay systems [J]. *Control Theory & Applications*, 1999, 16(1): 130 – 133.
(俞立, 陈国定. 线性时滞系统的无源控制 [J]. 控制理论与应用, 1999, 16(1): 130 – 133.).

[19] CUI B T, HUA M G. Robust passive control for uncertain discrete-time systems with time-varying delays [J]. *Chaos, Solitons & Fractals*, 2006, 29(2): 331 – 341.

[20] JIANG X, HAN Q L, YU X. Stability criteria for linear discrete-time systems with interval-like time-varying delay [C] //*Proceedings of the American Control Conference*. Portland: IEEE, 2005: 2817 – 2822.

[21] WANG Y Y, XIE L H, DE SOUZA C E. Robust control of a class of uncertain nonlinear systems [J]. *Systems & Control Letters*, 1992, 19(2): 139 – 149.

[22] BOUKAS E K. Static output feedback control for stochastic hybrid systems: LMI approach [J]. *Automatica*, 2006, 42(1): 183 – 188.

作者简介:

李　媛　(1974–), 女, 副教授, 博士研究生, 目前研究方向为网络控制、鲁棒控制、随机系统等, E-mail: syliyy@163.com;

张庆灵　(1956–), 男, 教授, 博士生导师, 目前研究方向为广义大系统的鲁棒控制、网络控制等, E-mail: qlzhang@mail.neu.edu.cn;

邱占芝　(1960–), 女, 教授, 博士生导师, 目前研究方向为计算机远程监控、网络控制等, E-mail: zhanzhiqiuok@163.com;

蔡　敏　(1964–), 男, 教授, 目前研究方向为无线通信信号处理、功率控制、网络控制的等, E-mail: dlcaim@126.com.