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有界随机测量时滞的网络控制系统的最优估计

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摘要: 在假设测量没有丢包的情况下, 研究了带有随机测量时滞的网络控制系统的最优估计问题. 利用已知的时滞分布概率, 建立新的模型来描述随机时滞测量. 进一步将带有时滞的测量等价成每个通道是单时滞的多通道测量, 从而利用新息重组方法, 通过求解黎卡提方程求解最优估计器. 最后给出仿真实例验证了该算法的有效性.

关键词: 最优估计; 随机测量时滞; 网络控制系统; 新息分析法; Riccati方程
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Optimal estimation for networked control systems with bounded random measurement delays

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Abstract: The optimal estimation problem is studied for networked control systems subject to random measurement delays and without packet loss. With the given probability distribution of the delays, a new model is proposed to describe the delay measurements. In order to solve the estimation problem, the measurement model with delays is written as an equivalent multiple channel measurement with each channel has a single constant delay. The optimal estimator is derived by using a reorganized innovation analysis approach, and is given in terms of Riccati difference equations. A simulation example illustrates the efficiency of the proposed method.

Key words: optimal estimation; random measurement delays; networked control systems; innovation analysis; Riccati equation

1 Introduction

In recent years, the research on networked control systems has gained a lot of attention in communication networks, control and state estimation[1–4]. In a networked control system, the sensor measures outputs of the system at every sampling time and transmits the measurement to a data processing center. The random communication delays, out-of-sequence measurements, and packet losses are inevitable in networked systems by unreliable communication networks from sensors to a processing center and from the processing center to end users. The data available in control and estimation may not be up-to-date due to stochastic delays or packet dropouts. These problems should be properly handled in order to achieve satisfactory estimation and control performance[5–7].

The estimation problem for networked control systems with random delays or packet dropouts has received many results during the past few years[8–12]. Nahi[13] investigates the estimation problem for system with an uncertain measurement, where sensor data that are simply the measurement noises at some samples are used for updating the estimate. Yaz et al.[14] discussed the least mean square filtering problem with random sensor delays or packet dropouts. It is well known, the time stamp is necessary to reorder the packets when the measurements arrive out of order. Schenato[15] proposed the estimators subject to simultaneous random packet delay and packet dropped as measurements are time-stamped and can be re-ordered at the estimator site. Zhang et al.[16] studied the optimal estimation problem for discrete-time systems with time delay in the measurement channel, and the measurements are time-stamped which can only take one value at time instant t. Sun[17] investigated the estimation problem for the systems with bounded random measurement delays and packet dropouts, which are described by some binary distributed random variables with known probabilities. In [17], the measurement model is received only one or no measurement at time instant t, and one measurement can be received for multiple times. However, the network transmission has limited capabilities, one measurement should not be re-received. The measurement model does not represent practical communication systems, because it allows the same measurement to be received for multiple times and/or generates too much packet loss. In [18], the measurement model guaranteed

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the packet could be received at most one time with time stamp, but too much packet loss also possibly happens.

Motivated by the discussion above, with the assumption of no packet loss, we present a more precise measurement model, in which the measurement at the present time instant is correlated with the previous time instant. The new measurement model is consistent with practical communication protocols, which guarantees each measurement can be received only once and also allows to receive multi-measurements at a time. Then we model the delayed measurements as multiple measurement channels, and the optimal estimation problem is investigated by using the reorganized innovation[19]. The designed estimator only depends on the probabilities of the delay at each instant but do not need to know if a measurement is delayed or received at a particular instant.

This paper is organized as follows: Problem formulation is given, and the measurement model is proposed in Section 2. Section 3 presents the solution for the optimal estimation based on the reorganized innovation analysis approach. A numerical example is given in Section 4 to illustrate the main results. Finally, Section 5 draws some conclusions of this paper.

2 Problem formulation

Consider the discrete linear stochastic system

$$x(t+1) = \Phi x(t) + \Gamma\omega(t), \tag{1}$$

$$z(t) = Hx(t) + \upsilon(t), \tag{2}$$

where $x(t) \in \mathbb{R}^n$ is the state, $z(t) \in \mathbb{R}^m$ is the output that is the measured signal at time t, and the received signal after network transmission is noted by $y(t)$. $\omega(t) \in \mathbb{R}^n$ and $\upsilon(t) \in \mathbb{R}^m$ are white noises. Φ, Γ, H are matrices with suitable dimensions. The initial state $x(0)$ and $\omega(t), \upsilon(t)$ are Gaussian, uncorrelated, white, with mean $(\bar{x}_0, 0, 0)$ and covariance (P_0, Q, R), respectively.

In the networked systems, the sensor measures the outputs of the system at every time instant and transmits the measurement to the estimator. Time-delay is unavoidable due to unreliable network communication. Since the packet delay is random, it is possible that between two consecutive sampling periods no packet or multiple packets are delivered. In this paper, we assume that there is no packet loss and the largest delay in data transmission are no more than N. On the other hand, the measured packets are not independent and they are correlated with the previous time. Therefore, let $\xi_{k,i}(k = t, t-1, \cdots, t-N;\ i = 0, 1, \cdots, N)$ be the indicator function for $z(k-i)$ with the sample delay i at time k, then the following model for the measurement received by the estimator is adopted:

$$y(t) = \begin{bmatrix} \xi_{t,0}z(t) \\ (1-\xi_{t-1,0})\xi_{t,1}z(t-1) \\ (1-\xi_{t-2,0})(1-\xi_{t-1,1})\xi_{t,2}z(t-2) \\ \vdots \\ (1-\xi_{t-N,0})\cdots(1-\xi_{t-1,N-1})z(t-N) \end{bmatrix}, \tag{3}$$

where $\xi_{k,i}$ are mutually independent scalar binary distributed random variables with the known distributions. We assume at any time $k = t, t-1, \cdots, t-N$, $\mathrm{P}\{\xi_{k,i} = 1\} = \alpha_i$ and $\mathrm{P}\{\xi_{k,i} = 0\} = 1 - \alpha_i (0 \leqslant i \leqslant N)$ with $\alpha_0 + \alpha_1 + \cdots + \alpha_N = 1$, i.e., the probability of received packet is α_i with the sample delay i.

For simplicity, we let

$$a_0(t) = \xi_{t,0},$$

$$a_i(t) = \prod_{k=0}^{i-1}(1-\xi_{t-(i-k),k})\xi_{t,i},\ 0 < i < N,$$

$$a_N(t) = \prod_{k=0}^{N-1}(1-\xi_{t-(N-k),k}),$$

then Eq.(3) is written as

$$y(t) = \begin{bmatrix} a_0(t)z(t) \\ a_1(t)z(t-1) \\ a_2(t)z(t-2) \\ \vdots \\ a_N(t)z(t-N) \end{bmatrix}. \tag{4}$$

Remark 1 In the model (4), the received measurement signal $y(t)$ is a multi-channel signal with number of variable ranging from 0 to N. For $i = 0, \cdots, N$, we have

$$a_i(t)z(t-i) = \begin{cases} z(t-i) \text{ if } a_i(t) = 1, \\ 0, \text{ i.e. } z(t-i) \text{ is not received if } a_i(t) = 0. \end{cases}$$

First, we explain the measurement model (4).

For $N = 1$, at time t, then when $i = 0$, $y(t) = \xi_{t,0}z(t)$; when $i = 1$, $y(t) = (1-\xi_{t-1,0})z(t-1)$. The measurements are not independent but are correlated with the one in previous time instance $t-1$. That is if $z(t-1)$ is received at time $t-1$, then it can not be received at time t; but if $z(t-1)$ is not received at time $t-1$, it must be received at time t. So, whether $y(t)$ contains $z(t-1)$ or not is conditioned on whether $z(t-1)$ was received previously. Then, if $y(t-1)$ contains $z(t-1)$, then $y(t) = z(t)$ with probability α_0 and $y(t) = 0$ with probability $1-\alpha_0$; otherwise, $y(t-1)$ does not contain $z(t-1)$, then $y(t) = [z(t)\ \ z(t-1)]$ is with probability α_0 and $y(t) = z(t-1)$ is with probability $1-\alpha_0$.

We have known that $\xi_{t,0}$ and $\xi_{t-1,0}$ are stochastic parameters, and at each time it equals 0 or 1 with the probability $1-\alpha_0$ or α_0, respectively. Thus at every time in the model (4), one possible received measurement $y(t)$ can be shown as in the following Table 1.

Table 1 Data transmission in network

t	1	2	3	4	5	6	7	8	9	10	$\cdots$
$\xi_{t,0}$	1	1	0	1	0	1	1	0	0	0	$\cdots$
$\xi_{t-1,0}$	0	1	1	0	1	0	1	1	0	0	$\cdots$
$y(t)$	$z(1)$	$z(2)$	0	$z(3), z(4)$	0	$z(5), z(6)$	$z(7)$	0	$z(8)$	$z(9)$	$\cdots$

From Table 1, we see that $z(1)$, $z(2)$, $z(4)$, $z(6)$, $z(7)$ are received on time; $z(3)$, $z(5)$, $z(8)$ and $z(9)$ are delayed. Furthermore, $z(3)$, $z(4)$ and $z(5)$, $z(6)$ are respectively received at the same time. Compared to the [17], it is worthy noting that the new model not only guarantees one packet to be received only once but also satisfies multiple packets to be received in the actual network transmission.

Substituting Eqs.(1)–(2) into Eq.(4), we find that the system is equivalent to

$$x(t+1) = \Phi x(t) + \Gamma\omega(t), \tag{5}$$

$$y(t) = \begin{bmatrix} z_0'(t) & z_1'(t) & \cdots & z_N'(t) \end{bmatrix}', \tag{6}$$

where

$$z_i'(t) = a_i(t)Hx(t-i) + a_i(t)\upsilon(t-i),\quad i = 0, \cdots, N. \tag{7}$$

We let $z_i'(t) \equiv 0$ for $0 < t < N$ in Eq.(7), that is for $t < N$, the measurement (6) is written as

$$y(t) = \begin{bmatrix} z_0'(t) & z_1'(t) & \cdots & z_t'(t) & 0 & \cdots & 0 \end{bmatrix}'. \tag{8}$$

Remark 2 In view of the distribution of $\xi_{t,i}$, we have these properties

$\mathrm{E}[\xi_{t,i}] = \alpha_i,\ \mathrm{E}[{\xi_{t,i}}^2] = \alpha_i,$

$\mathrm{Cov}[\xi_{t,i}] = \alpha_i(1-\alpha_i),\ \mathrm{E}[(1-\xi_{t,i})^2] = 1-\alpha_i,$

$\mathrm{E}[\xi_{t,i}(1-\xi_{t,i})] = 0,\ \mathrm{E}[\xi_{t,i}(1-\xi_{k,i})] = \alpha_i(1-\alpha_i),$

$\mathrm{E}[\xi_{t,i}\xi_{k,j}] = \alpha_i\alpha_j,\ k \neq t,\ i \neq j.$

Lemma 1 Random variable $a_i(t)$ has the following properties with the delayed probabilities α_i:

$$\bar{a}_0 = \mathrm{E}[a_0(t)] = \alpha_0, \tag{9}$$

$$\bar{a}_i = \mathrm{E}[a_i(t)] = \prod_{k=0}^{i-1}(1-\alpha_k)\alpha_i,\ 0 < i < N, \tag{10}$$

$$\bar{a}_N = \mathrm{E}[a_N(t)] = \prod_{k=0}^{N-1}(1-\alpha_k), \tag{11}$$

$$\mathrm{E}[a_i(t) - \bar{a}_i] = 0 \ \text{ for all } \ i, \tag{12}$$

$$\mathrm{E}[a_i(t)a_j(t)] = \begin{cases} \bar{a}_i, & i = j, \\ \bar{a}_i\bar{a}_j, & i \neq j, \end{cases} \tag{13}$$

$$A_{ij} = \mathrm{E}[(a_i(t)-\bar{a}_i)(a_j(t)-\bar{a}_j)] = \begin{cases} \bar{a}_i(1-\bar{a}_i), & i = j, \\ 0, & i \neq j. \end{cases} \tag{14}$$

Proof Eqs.(9)–(11) can be obtained directly from Remark 2. Then we have

$$\mathrm{E}[a_i(t) - \bar{a}_i] = \mathrm{E}[a_i(t)] - \bar{a}_i = 0,$$

and Eq.(12) is obtained. In Eq.(13),

$$\mathrm{E}[a_i(t)a_j(t)] = \mathrm{E}\{\prod_{k=0}^{i-1}(1-\xi_{t-(i-k),k})\xi_{t,i}\prod_{k=0}^{j-1}(1-\xi_{t-(j-k),k})\xi_{t,j}\},$$

from Remark 2 we know, when $i = j$,

$$\mathrm{E}[{\xi_{t,i}}^2] = \alpha_i,\ \mathrm{E}[(1-\xi_{t,i})^2] = 1-\alpha_i,$$

and when $i \neq j$,

$$\mathrm{E}[\xi_{t,i}\xi_{t,j}] = \alpha_i\alpha_j,$$
$$\mathrm{E}[(1-\xi_{t,i})(1-\xi_{t,j})] = (1-\alpha_i)(1-\alpha_j),$$

then we get Eq.(13).

For Eq.(14), we first prove the case $0 < i < N$: when $i = j$, we have

$$A_{ii} = \mathrm{E}[(a_i(t)-\bar{a}_i)^2] = \mathrm{E}[a_i^2(t)] - \bar{a}_i^2 = \mathrm{E}[\prod_{k=0}^{i-1}(1-\xi_{t-(i-k),k})^2\xi_{t,i}^2] - \bar{a}_i^2 = \bar{a}_i(1-\bar{a}_i),$$

when $i \neq j$,

$$A_{ij} = \mathrm{E}[(a_i(t)-\bar{a}_i)(a_j(t)-\bar{a}_j)] = \mathrm{E}[a_i(t)a_j(t)] - \bar{a}_i\bar{a}_j = 0.$$

For the cases $i = 0$ and $i = N$, the Eq.(14) also holds, and the proof is similar to the above. For simplicity, we will denote A_{ii} to be A_i.

Lemma 2 For the system (1), we have the state covariance matrix satisfies the following recursion:

$$S(t+1) = \Phi S(t)\Phi' + \Gamma Q\Gamma', \tag{15}$$

where $S(t+1) = \mathrm{E}[x(t+1)x'(t+1)]$, and $S(0) = \mathrm{E}[x(0)x'(0)] = P_0$.

Our purpose is to find the linear minimum variance optimal estimator $\hat{x}(t|t)$ of the state $x(t)$ based on the sensor measurements $(y(t), y(t-1), \cdots, y(0))$, i.e. to minimize the performance

$$J = \mathrm{E}_x\mathrm{E}_\xi[(x(t)-\hat{x}(t|t))(x(t)-\hat{x}(t|t))^{\mathrm{T}}|\tilde{\boldsymbol{y}}(t), \xi, \bar{x}_0, P_0], \tag{16}$$

where E_x is the expectation with respect to $x(t)$, $\omega(t)$, $\upsilon(t)$; and E_ξ is the expectation with respect to $\xi_{t,i}$ considering the random aspect of the observation. Let $\tilde{\boldsymbol{y}}(t) = \{y(t), y(t-1), \cdots, y(0)\}$ and note that the values of $\xi_{t,i}$ are unknown except their probabilities α_i are known.

3 Optimal estimator design

In this section, we shall present a solution to the optimal estimation by reorganizing the measurements and applying the well-known projection method in [19].

3.1 Reorganized measurements

Because $y(t)$ has its entries associated with states at different time instants due to the delays, the standard Kalman filtering is not applicable to the estimation problem. In this section, we adopt a reorganized innovation approach to the above optimal estimation problem. We will define another new observation sequence which is delay-free and contains the same amount of information as $\tilde{\boldsymbol{y}}(t)$, so that the Kalman filtering formulation can be applied to solve the estimation problem.

Let

$$\tilde{y}_N(s) \triangleq \begin{bmatrix} z_0(s) \\ z_1(s+1) \\ \vdots \\ z_N(s+N) \end{bmatrix},\ 0 \leqslant s \leqslant t-N, \tag{17}$$

$$\tilde{y}_{t-s}(s) \triangleq \begin{bmatrix} z_0(s) \\ z_1(s+1) \\ \vdots \\ z_{t-s}(t) \end{bmatrix},\ t-N \leqslant s \leqslant t. \tag{18}$$

Now the measurements $\tilde{y}(\cdot)$ are delay-free and shown in the following:

$$\tilde{y}_N(s) = H_N(s)x(s) + \upsilon_N(s),\ 0 \leqslant s \leqslant t-N, \tag{19}$$

$$\tilde{y}_{t-s}(s) = H_{t-s}(s)x(s) + \upsilon_{t-s}(s),\ t-N \leqslant s \leqslant t, \tag{20}$$

where

$$H_N(s) = \begin{bmatrix} a_0(s)H \\ a_1(s+1)H \\ \vdots \\ a_N(s+N)H \end{bmatrix}, \tag{21}$$

$$\upsilon_N(s) = \begin{bmatrix} a_0(s)\upsilon(s) \\ a_1(s+1)\upsilon(s) \\ \vdots \\ a_N(s+N)\upsilon(s) \end{bmatrix}, \tag{22}$$

$$H_{t-s}(s) = \begin{bmatrix} a_0(s)H \\ a_1(s+1)H \\ \vdots \\ a_{t-s}(t)H \end{bmatrix}, \tag{23}$$

$$\upsilon_{t-s}(s) = \begin{bmatrix} a_0(s)\upsilon(s) \\ a_1(s+1)\upsilon(s) \\ \vdots \\ a_{t-s}(t)\upsilon(s) \end{bmatrix}, \tag{24}$$

with $\upsilon_N(s),\ \upsilon_{t-s}(s)$ are white noise of zero mean and by the Remark 2, the covariance matrix is

$$R_N(s) = \mathrm{E}[\upsilon_N(s)\upsilon'_N(s)] = E\begin{bmatrix} a_0(s)\upsilon(s) \\ a_1(s+1)\upsilon(s) \\ \vdots \\ a_N(s+N)\upsilon(s) \end{bmatrix} \cdot \begin{bmatrix} a_0(s)\upsilon'(s) \\ a_1(s+1)\upsilon'(s) \\ \vdots \\ a_N(s+N)\upsilon'(s) \end{bmatrix}^{\mathrm{T}} = \mathrm{diag}\{\bar{a}_0R, \bar{a}_1R, \cdots, \bar{a}_NR\}. \tag{25}$$

In the similar way, we get

$$R_{t-s}(s) = \mathrm{diag}\{\bar{a}_0R, \bar{a}_1R, \cdots, \bar{a}_{t-s}R\}. \tag{26}$$

It is clear that the original measurement set $\tilde{\boldsymbol{y}}(t)$ contains the same amount of information about the system as the set $\{\{\tilde{y}_N(s)\}_{s=0}^{t-N};\ \{\tilde{y}_{t-s}(s)\}_{s=t-N+1}^{t}\}$. Thus, the optimal linear estimation problem to be addressed in this paper can be restated as: given the measurement sequence $\{\{\tilde{y}_N(s)\}_{s=0}^{t-N};\ \{\tilde{y}_{t-s}(s)\}_{s=t-N+1}^{t}\}$, to minimize Eq.(16) in order to get the estimator $\hat{x}(t|t)$ of the state $x(t)$.

3.2 Reorganized innovation sequence

In this subsection we shall define the innovation associated with the reorganized measurements.

Definition 1 Consider the reorganized measurement $\{\{\tilde{y}_N(s)\}_{s=0}^{t-N};\ \{\tilde{y}_{t-s}(s)\}_{s=t-N+1}^{t}\}$, and define the following:

For $0 \leqslant s \leqslant t-N$,

$$\eta_N(s) \triangleq \tilde{y}_N(s) - \hat{\tilde{y}}_N(s),\ s = 0, \cdots, t-N, \tag{27}$$

where $\hat{\tilde{y}}_N(s)$ is the projection (optimal estimation) of $\tilde{y}_N(s)$ based on the measurements $\{\tilde{y}_N(0), \cdots, \tilde{y}_N(s-1)\}$, and

$$\hat{\tilde{y}}_N(s) = \mathrm{E}[\tilde{y}_N(s)] = \mathrm{E}[H_N(s)x(s) + \upsilon_N(s)] = \tilde{H}_N(s)\hat{x}(s,N), \tag{28}$$

with $\tilde{H}_N(s) = [\bar{a}_0H'\ \bar{a}_1H'\ \cdots\ \bar{a}_NH']'$.

For $s > t-N$,

$$\eta_{t-s}(s) \triangleq \tilde{y}_{t-s}(s) - \hat{\tilde{y}}_{t-s}(s), \tag{29}$$

where $\hat{\tilde{y}}_{t-s}(s)$ is the projection (optimal estimation) of $\tilde{y}_{t-s}(s)$ based on the measurements $\{\tilde{y}_N(0), \cdots, \tilde{y}_N(t-N); \tilde{y}_{N-1}(t-(N-1)), \cdots, \tilde{y}_{t-(s-1)}(s-1)\}$.

$$\hat{\tilde{y}}_{t-s}(s) = \mathrm{E}[\tilde{y}_{t-s}(s)] = \mathrm{E}[H_{t-s}(s)x(s) + \upsilon_{t-s}(s)] = \tilde{H}_{t-s}(s)\hat{x}(s,t-s), \tag{30}$$

with $\tilde{H}_{t-s}(s) = [\bar{a}_0H'\ \bar{a}_1H'\ \cdots\ \bar{a}_{t-s}H']'$.

The sequences $\{\eta_i(s)\}$ defined in above are white noise with zero mean and covariance $Q_{\eta i}(s)$ and span the same linear space as $\{\{\tilde{y}_N(s)\}_{s=0}^{t-N};\ \{\tilde{y}_{t-s}(s)\}_{s=t-N+1}^{t}\}$. As usual, the sequences $\eta_i(s)$ are termed as the innovation associated with the reorganized measurement $\{\{\tilde{y}_N(s)\}_{s=0}^{t-N};\ \{\tilde{y}_{t-s}(s)\}_{s=t-N+1}^{t}\}$. Thus the optimal estimator $\hat{x}(t|t)$ is the projection of the state $x(t)$ onto the linear space spanned by $\{\eta_N(0), \cdots, \eta_N(t-N); \eta_{N-1}(t-(N-1)), \cdots, \eta_0(t)\}$.

3.3 Optimal estimator $\hat{x}(t|t)$

In order to calculate the optimal estimator with the innovation sequence as in the above subsection, we firstly give the following definition.

Definition 2 Given time instant t,

For $0 \leqslant s \leqslant t-N$, let

$$P_N(s) \triangleq \mathrm{E}[\tilde{x}(s,N)\tilde{x}'(s,N)] \tag{31}$$

be the covariance matrix of the state estimation error, where

$$\tilde{x}(s,N) = x(s) - \hat{x}(s,N), \tag{32}$$

and $\hat{x}(s,N)$ is the projection of state $x(s)$ onto the linear space generated by

$$\mathcal{L}\{\tilde{y}_N(0), \cdots, \tilde{y}_N(s-1)\} = \mathcal{L}\{\eta_N(0), \cdots, \eta_N(s-1)\}. \tag{33}$$

For $s > t-N$, denote the covariance matrix of the state estimation error

$$P_{t-s}(s) \triangleq \mathrm{E}[\tilde{x}(s,t-s)\tilde{x}'(s,t-s)], \tag{34}$$

where

$$\tilde{x}(s,t-s) = x(s) - \hat{x}(s,t-s), \tag{35}$$

and $\hat{x}(s,t-s)$ is the projection of state $x(s)$ onto the linear space generated by

$$\mathcal{L}\{\tilde{y}_N(0), \cdots, \tilde{y}_N(t-N); \tilde{y}_{N-1}(t-(N-1)), \cdots, \tilde{y}_{t-(s-1)}(s-1)\} = \mathcal{L}\{\eta_N(0), \cdots, \eta_N(t-N); \eta_{N-1}(t-(N-1)), \cdots, \eta_{t-(s-1)}(s-1)\}. \tag{36}$$

With the definition, we get the solution to the optimal filtering problem by applying the reorganized innovation sequence and the Riccati equations.

Theorem 1 Consider the system (1)–(2) with the bounded random measurement delays, and the time-delay probability $\alpha_i (0 \leqslant i \leqslant N)$, the optimal estimator $\hat{x}(t|t)$ is given by

$$\hat{x}(t|t) = [I_n - P_1(t)\tilde{H}'_0(t)Q_{\eta_0}^{-1}(t)\tilde{H}_0(t)]\hat{x}(t,1) + P_1(t)\tilde{H}'_0(t)Q_{\eta_0}^{-1}(t)\tilde{y}_0(t), \tag{37}$$

where $\hat{x}(t,1)$ is as in Definition 2, which is computed by the following iteration:

$$\hat{x}(s+1,t-s) =$$

$$\Phi_{t-s}(s)\hat{x}(s,t-(s-1))+K_{t-s}(s)\tilde{y}_{t-s}(s), \quad (38)$$

where

$$\Phi_{t-s}(s)=\Phi-K_{t-s}(s)\tilde{H}_{t-s}(s), \quad (39)$$

$$K_{t-s}(s)=\Phi P_{t-s+1}(s)\tilde{H}'_{t-s}(s)Q^{-1}_{\eta_{t-s}}(s), \quad (40)$$

$$Q_{\eta_{t-s}}(s)=\mathrm{diag}\{A_0HS(s)H',A_1HS(s)H',\cdots,A_{t-s}HS(s)H'\}+\mathrm{diag}\{\bar{a}_0^2HP_{t-s+1}(s)H',\bar{a}_1^2HP_{t-s}(s+1)H',\cdots,\bar{a}_{t-s}^2HP_1(t)H'\}+R_{t-s}(s), \quad (41)$$

with the error covariance equation is

$$P_{t-s}(s+1)=\Phi P_{t-(s-1)}(s)\Phi'-K_{t-s}(s)Q_{\eta_{t-s}}(s)K'_{t-s}(s)+\Gamma Q\Gamma', \quad (42)$$

with the initial values $P_N(0)=P_0$. When $s\leqslant t-N$, we let $t-s=N$ in the above, then the estimator is obtained corresponding to the above.

Proof $\hat{x}(t|t)$ is the projection of the state $x(t)$ onto the linear space spanned by $\{\eta_N(0),\cdots,\eta_N(t-N);\ \eta_{N-1}(t-(N-1)),\cdots,\eta_0(t)\}$. Since this sequence is orthogonal, the estimator is calculated by using the projection formula as:

$$\hat{x}(t|t)=\mathrm{P}\{x(t)|\eta_N(0),\cdots,\eta_N(t-N);\ \eta_{N-1}(t-(N-1)),\cdots,\eta_1(t-1)\}+\mathrm{P}\{x(t)|\eta_0(t)\}=\hat{x}(t,1)+\mathrm{E}[x(t)\eta'_0(t)]\mathrm{E}[\eta_0(t)\eta'_0(t)]^{-1}\eta_0(t). \quad (43)$$

From Eqs.(30) and (20), we have

$$\eta_0(t)=\tilde{y}_0(t)-\hat{\tilde{y}}_0(t)=H_0(t)x(t)-\tilde{H}_0(t)\hat{x}(t,1)+\upsilon_0(t)=a_0(t)Hx(t)-\bar{a}_0H\hat{x}(t,1)+a_0(t)\upsilon(t). \quad (44)$$

According to Eq.(35) and the orthogonality, we have $\hat{x}(t,1)\perp\tilde{x}(t,1)$ and $x(t)\perp\upsilon(t)$, then substituting Eq.(44) into Eq.(43), we obtain

$$\mathrm{E}[x(t)\eta'_0(t)]=\mathrm{E}[x(t)(a_0(t)Hx(t)-\bar{a}_0H\hat{x}(t,1)+a_0(t)\upsilon(t))']=\mathrm{E}[x(t)(a_0(t)Hx(t)-\bar{a}_0H(x(t)-\tilde{x}(t,1))+a_0(t)\upsilon(t))']=\mathrm{E}[a_0(t)x(t)x'(t)H']-\bar{a}_0\mathrm{E}[x(t)x'(t)H']+\bar{a}_0\mathrm{E}[x(t)\tilde{x}'(t,1)H']=\bar{a}_0\mathrm{E}[\tilde{x}(t,1)\tilde{x}'(t,1)H']=\bar{a}_0P_1(t)H'=P_1(t)\tilde{H}'_0(t). \quad (45)$$

By the Lemma 1 and 2, we obtain the covariance of innovation $\eta_0(t)$ as

$$Q_{\eta_0}(t)=\mathrm{E}[\eta_0(t)\eta'_0(t)]=\mathrm{E}[(a_0(t)Hx(t)-\bar{a}_0H\hat{x}(t,1)+a_0(t)\upsilon(t))\cdot(a_0(t)Hx(t)-\bar{a}_0H\hat{x}(t,1)+a_0(t)\upsilon(t))']=\mathrm{E}[((a_0(t)H-\bar{a}_0H)x(t)+\bar{a}_0H\tilde{x}(t,1)+a_0(t)\upsilon(t))((a_0(t)H-\bar{a}_0H)x(t)+\bar{a}_0H\tilde{x}(t,1)+a_0(t)\upsilon(t))']=\mathrm{E}[(a_0(t)H-\bar{a}_0H)x(t)x'(t)(a_0(t)H-\bar{a}_0H)']+\mathrm{E}[(a_0(t)H-\bar{a}_0H)x(t)\tilde{x}'(t,1)\bar{a}_0H']+\mathrm{E}[\bar{a}_0H\tilde{x}(t,1)x'(t)(a_0(t)H-\bar{a}_0H)']+\bar{a}_0^2\mathrm{E}[H\tilde{x}(t,1)\tilde{x}'(t,1)H']+\mathrm{E}[a_0(t)\upsilon(t)\upsilon'(t)a_0(t)]=A_0H\mathrm{E}[x(t)x'(t)]H'+\bar{a}_0^2HP_1(t)H'+\bar{a}_0R=A_0HS(t)H'+\bar{a}_0^2HP_1(t)H'+\bar{a}_0R. \quad (46)$$

Substituting the Eqs.(45) and (46) into Eq.(43), we obtain Eq.(37).

For the case of $s>t-N$,

$$\hat{x}(s+1,t-s)=\mathrm{P}\{x(s+1)|\eta_N(0),\cdots,\eta_N(t-N);\ \eta_{N-1}(t-(N-1)),\cdots,\eta_{t-s}(s)\}=\Phi\hat{x}(s,t-s+1)+\mathrm{Proj}\{x(s+1)|\eta_{t-s}(s)\}=\Phi\hat{x}(s,t-s+1)+\Phi\mathrm{Proj}\{x(s)|\eta_{t-s}(s)\}. \quad (47)$$

With a similar discussion as above,

$$\mathrm{P}\{x(s)|\eta_{t-s}(s)\}=P_{t-s+1}(s)\tilde{H}'_{t-s}(s)Q^{-1}_{\eta_{t-s}}(s)\eta_{t-s}(s). \quad (48)$$

Substituting Eqs.(48)(29)–(30) into Eq.(47), we obtain

$$\hat{x}(s+1,t-s)=\Phi\hat{x}(s,t-s+1)+\Phi P_{t-s+1}(s)\tilde{H}'_{t-s}(s)Q^{-1}_{\eta_{t-s}}(s)\eta_{t-s}(s)=\Phi\hat{x}(s,t-s+1)+K_{t-s}(s)(\tilde{y}_{t-s}(s)-\tilde{H}_{t-s}(s)\hat{x}(s,t-s+1)=\Phi_{t-s}(s)\hat{x}(s,t-s+1)+K_{t-s}(s)\tilde{y}_{t-s}(s), \quad (49)$$

where $K_{t-s}(s)=\Phi P_{t-s+1}(s)\tilde{H}'_{t-s}(s)Q^{-1}_{\eta_{t-s}}(s)$ with $\Phi_{t-s}(s)=\Phi-K_{t-s}(s)\tilde{H}_{t-s}(s)$. $Q_{\eta_{t-s}}(s)$ is the innovation covariance as shown in the following:

$$Q_{\eta_{t-s}}(s)=\mathrm{E}[\eta_{t-s}(s)\eta'_{t-s}(s)]=\mathrm{E}[(\tilde{y}_{t-s}(s)-\hat{\tilde{y}}_{t-s}(s))(\tilde{y}_{t-s}(s)-\hat{\tilde{y}}_{t-s}(s))']=\mathrm{E}[(H_{t-s}(s)x(s)-\tilde{H}_{t-s}(s)\hat{x}(s,t-s+1)+\upsilon_{t-s}(s))(H_{t-s}(s)x(s)-\tilde{H}_{t-s}(s)\hat{x}(s,t-s+1)+\upsilon_{t-s}(s))']=\mathrm{E}[(H_{t-s}(s)-\tilde{H}_{t-s}(s))x(s)x'(s)(H_{t-s}(s)-\tilde{H}_{t-s}(s))']+\mathrm{E}[(H_{t-s}(s)-\tilde{H}_{t-s}(s))x(s)\tilde{x}'(s,t-s+1)\tilde{H}'_{t-s}(s)]+\mathrm{E}[\tilde{H}_{t-s}(s)\tilde{x}(s,t-s+1)x'(s)(H_{t-s}(s)-\tilde{H}_{t-s}(s))']+\mathrm{E}[\tilde{H}_{t-s}(s)\tilde{x}(s,t-s+1)\cdot\tilde{x}'(s,t-s+1)\tilde{H}'_{t-s}(s)]+R_{t-s}(s)=\mathrm{diag}\{A_0H\mathrm{E}[x(s)x'(s)]H',A_1H\mathrm{E}[x(s)x'(s)]H',\cdots,A_{t-s}H\mathrm{E}[x(s)x'(s)]H'\}+\mathrm{diag}\{\bar{a}_0^2HP_{t-s+1}(s)H',\bar{a}_1^2HP_{t-s}(s+1)H',\cdots,\bar{a}_{t-s}^2HP_1(t)H'\}+R_{t-s}(s)=\mathrm{diag}\{A_0HS(s)H',A_1HS(s)H',\cdots,A_{t-s}HS(s)H'\}+\mathrm{diag}\{\bar{a}_0^2HP_{t-s+1}(s)H',\bar{a}_1^2HP_{t-s}(s+1)H',$$

$$\cdots, \bar{a}_{t-s}^2 HP_1(t)H'\} + R_{t-s}(s). \tag{50}$$

From Eq.(35), we have

$$\tilde{x}(s+1,t-s) = x(s+1) - \hat{x}(s+1,t-s) =$$
$$\Phi\tilde{x}(s,t-s) + \Gamma\omega(s) - K_{t-s}(s)\eta_{t-s}(s), \tag{51}$$

since $\tilde{x}(s,t-s)$ is uncorrelated with $\omega(s)$, by Eqs.(51) and (50), we obtain the error covariance

$$P_{t-s}(s+1) = \mathrm{E}[\tilde{x}(s+1,t-s)\tilde{x}'(s+1,t-s)] =$$
$$\Phi P_{t-s+1}(s)\Phi' - K_{t-s}(s)Q_{\eta_{t-s}}(s)K'_{t-s}(s) + \Gamma Q\Gamma', \tag{52}$$

thus the recursive equation (42) is obtained.

In the case of $0 \leqslant s \leqslant t-N$, we just let $t-s=N$ in the above, then following the similar way, we obtain

$$\hat{x}(s+1,N) = \mathrm{P}\{x(s+1)|\eta_N(0),\cdots,\eta_N(s)\} =$$
$$\Phi\hat{x}(s,N) + \mathrm{P}\{x(s+1)|\eta_N(s)\} =$$
$$\Phi\hat{x}(s,N) + K_N(s)\eta_N(s), \tag{53}$$

then Eqs.(39)–(42) can be easily derived for $t-s=N$.

Remark 3 With the assumption of no packet loss, we propose the measurement model in which the received measurement is correlated with previous measurement. Thus our estimator just uses the probabilities of the delay at each time, and the obtained estimator is concerned with the state covariance matrix (15), this is the mainly different from [18].

4 Simulation example

In this section, we present a simple example to illustrate the previous theoretical results. Consider the system (1)–(2) with the following specifications:

$$\Phi = \begin{bmatrix} 0.5 & 0.05 \\ 0 & 0.86 \end{bmatrix},\ \Gamma = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix},\ H = [2\ \ 1],$$

and the white noises $\omega(t)$ and $v(t)$ with mean zero and their covariances are $Q=1$ and $R=1$, respectively. Since $\xi_{k,i}$ are mutually independent scalar binary distributed random variables with the known distributions, we assume that the maximal random time delay $N=2$, thus the probability of received packet is $\alpha_0+\alpha_1+\alpha_2=1$. Setting $\alpha_0=0.8$, $\alpha_1=0.1$, $\alpha_2=0.1$ and the initial state value $x(0)=[0\ \ 0]'$, as the initial conditions, we calculate the estimator by using Theorem 1 proposed in Section 3, Fig.1 gives the trace of the estimation error covariance by the proposed algorithm, and shows the comparison with the trace of estimation error covariance for the standard Kalman filtering assuming that there is no time delay. It can be seen that the proposed estimator in the paper has a better performance, and also the steady-state estimation error variance matrices is obtained as $P = \begin{bmatrix} 1.1034 & 0.58666 \\ 0.58666 & 0.3363 \end{bmatrix}$. The optimal estimator $\hat{x}(t|t)$ is shown in Fig.2 and Fig.3. It is also shown that the proposed estimation algorithm effectively tracks the states of the system.

Fig.4 shows the performance comparison of the sum of estimation error covariance in this paper and [18]. We see that the method in our paper is not as good as the optimal method in [18]. This is understandable because in the optimal method of [18] the time stamps are exactly known at each time, and it uses much more information in estimation than the probabilities of the delay we use in our paper.

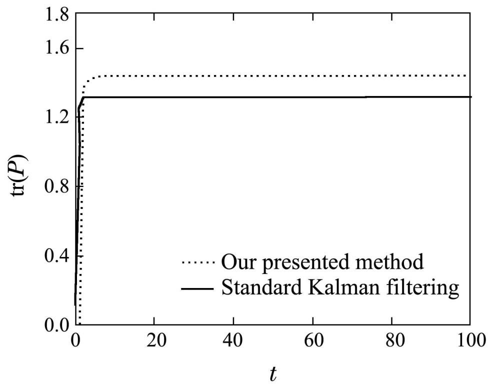

Fig. 1 The comparison of the trace of estimation error covariance

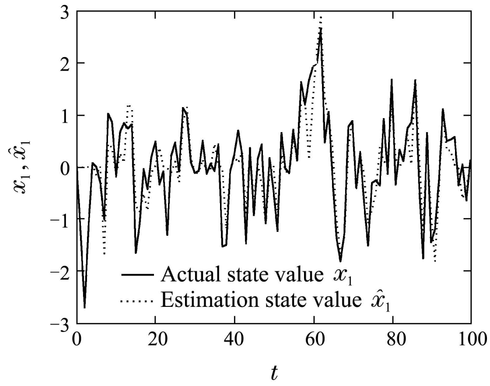

Fig. 2 The first state component $x_1(t)$ and the estimator $\hat{x}_1(t|t)$

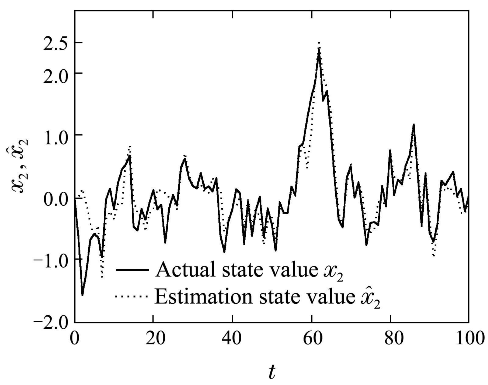

Fig. 3 The second state component $x_2(t)$ and the estimator $\hat{x}_2(t|t)$

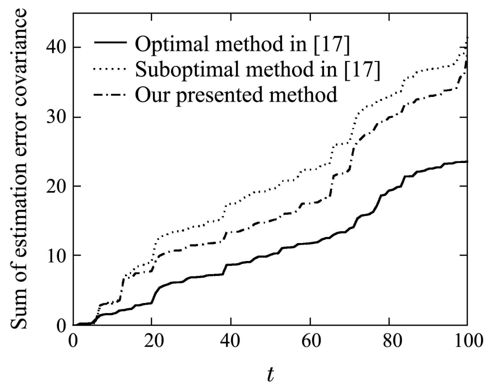

Fig. 4 The comparison of the sum of estimation error covariance with [18]

5 Conclusions

In this paper, for discrete-time stochastic linear systems with bounded random measurement delays, the optimal estimator is proposed. Firstly, we present a novel measurement model by using time stamps which are correlated with the previous time. The model ensures that each measurement is received and received only once, which is more precisely to describe the actual communication protocols. Then by applying the reorganized innovation analysis approach, the estimator is derived by solving a set of recursive discrete-time Riccati equations with the known probabilities of the delay. Our estimator can be computed off-line as it only depends on the data arrival probability at each time instant.

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