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多变量模糊系统控制设计及其在并行混合电动汽车中的应用

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摘要: 利用矩阵半张量积方法研究了多变量模糊系统模糊逻辑控制器的设计, 并得到了若干新的结果. 首先给出了模糊规则新的表示形式, 基于该表示形式, 构造了模糊逻辑控制器的结构矩阵, 将复杂的模糊推理转变成了简单的代数等式. 然后当模糊控制规则不完全时, 建立了最小入度控制算法; 当模糊控制规则不一致时, 给出了相应的处理方法. 最后将得到的结果应用到并行混合电动汽车(PHEV)能量管理和控制策略的模糊控制器设计.

关键词: 模糊逻辑控制; 最小入度模糊控制; 半张量积
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Control design for multi-variable fuzzy systems with application to parallel hybrid electric vehicles

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Abstract: This paper studies the fuzzy logic controller (FLC) design for multi-variable fuzzy systems based on the semi-tensor product of matrices, and presents several new results. A new expression of fuzzy rules for multi-variable FLC is introduced, which is very convenient to use in fuzzy logic inference. Based on the new expression of fuzzy rules, the complex fuzzy reasoning is converted into simple algebraic equations by constructing structural matrices of the FLC. A simulation example is given to demonstrate the effectiveness of the proposed approach. A set of least in-degree controls that remove possible fabricated variables are constructed, and an algorithm is given to design the least in-degree controls when the control rules are incomplete. Principles are proposed for dealing with the inconsistency of fuzzy control rules. Finally, the results obtained in this paper are applied to the design of fuzzy controller for energy management and control strategy of parallel hybrid electric vehicles (PHEV).

Key words: fuzzy logic control; least in-degree fuzzy control; semi-tensor product

1 Introduction

The fuzzy logic control techniques were originally introduced by Zadeh[1–2] as a means of both collecting human knowledge or experience and dealing with uncertainties in the control process. Since then, the fuzzy logic control has attracted a great deal of attention and a lot of works have been published in the field. The fuzzy logic control has proved to be a successful control approach to many complex nonlinear systems or even nonanalytic systems[3]. However, it is difficult to infer the proper control input for a multi-variable system since the dimension of its relation matrix is very large. The high dimensionality of the relation matrix might lead to not only computational difficulties but also memory overload when the physical control system is implemented on a computer. To solve this problem, Gupta[4] et al. proposed a fuzzy control algorithm by which the multi-variable fuzzy system is decomposed into a set of one-dimensional systems. The decomposition of control rules is preferable since it alleviates the complexity of the problem.

Recently, the semi-tensor product (STP) of matrices was proposed in [5], and up to now, it has been widely applied in many fields and lots of fundamental results have been presented[5–14]. By this method, it is very convenient to convert a logic expression into an algebraic form by constructing its structural matrix. It is noted that the fuzzy logic can be considered as an extended mix-valued logic in which the truth-values are the values of memberships of all the elements in a fuzzy set, and by the STP method, the complex reasoning process can be converted into a problem of solving a set of algebraic equations, which greatly simplifies the process of logical reasoning[6–8]. In [6], the authors considered the adequate sets and the normal forms of general logical mappings, and several interesting results

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were obtained for the fuzzy logic and the resolution of fuzzy relation equations by STP.

In this paper, we deal with the fuzzy logic controller (FLC) analysis and design for multi-input multi-output fuzzy systems based on the semi-tensor product. The main contributions of this paper are as follows: i) A new expression of fuzzy control rules for multi-variable FLC is obtained by STP. By the new expression, the complex fuzzy reasoning is converted into algebraic equations by constructing structural matrices of the FLC. Then a new framework is established to study multi-variable FLC. Compared with the existing fuzzy inference, the algebraic expression of fuzzy reasoning is easier. Simulation results demonstrate the effectiveness of the proposed method. ii) Through the analysis of the structural matrices of the FLC, a set of least in-degree fuzzy controls can be obtain. Moreover, a least in-degree algorithm is presented for the controller design when the control rules are incomplete. iii) When the fuzzy rules are not consistent, some principles are proposed to deal with fuzzy controls with inconsistency.

The rest of the paper is organized as follows. Section 2 presents some necessary preliminaries on the semi-tensor product of matrices and the expression of logical function and logical variables. We present the main results of this paper in Section 3. In Section 4, the results obtained in this paper are applied to the design of fuzzy controller for energy management and control strategy of parallel hybrid electric vehicles (PHEV), which is followed by the conclusion in Section 5.

2 Preliminaries

First, we introduce some notations, which will be used in this paper.

• δ_k^i: the i-th column of the identity matrix I_k.

• $\varDelta_k := \{\delta_k^i | i = 1, 2, \cdots, k\}$. Especially, $\varDelta := \varDelta_2$.

• $\mathcal{D} := \{1,\ 0\}$. To use matrix expression, '1' and '0' can be expressed with the following vectors, respectively:

$$1 \sim \delta_2^1,\ 0 \sim \delta_2^2.$$

• $\mathcal{D}_k := \{1, \frac{k-2}{k-1}, \frac{k-3}{k-1}, \cdots, 0\},\ k \geqslant 2.$

• A matrix $L \in \mathbb{R}^{m\times n}$ is called a logical matrix if the columns of L, denoted by $\mathrm{col}(L)$, are of the form of δ_n^k, that is, $\mathrm{col}(L) \subset \varDelta_n$.

• Denote by $\mathcal{L}_{n\times r}$ the set of $n \times r$ logical matrices. If $L \in \mathcal{L}_{n\times r}$, by definition, it can be expressed as

$$L = [\delta_n^{i_1}\ \delta_n^{i_2}\ \cdots\ \delta_n^{i_r}] := \delta_n[i_1\ i_2\ \cdots\ i_r].$$

Similarly, we identify each k-valued logical value with a vector as follows:

$$\frac{k-i}{k-1} \sim \delta_k^i,\ i = 1, 2, \cdots, k.$$

Then, $\mathcal{D}_k \sim \varDelta_k$, where $p \sim q$ denotes the logic equivalence of p and q.

In the following, we recall some definitions and basic properties about the semi-tensor product[5–6].

Definition 1 The semi-tensor product of two matrices $A \in \mathbb{R}^{m\times n}$ and $B \in \mathbb{R}^{p\times q}$ is defined as

$$A \ltimes B = (A \otimes I_{\frac{\alpha}{n}})(B \otimes I_{\frac{\alpha}{p}}), \tag{1}$$

where $\alpha = lcm(n, p)$ is the least common multiple of n and p, and $\otimes$ is the Kronecker product.

Remark 1 It is noted that when $n = p$, the STP of A and B becomes the conventional matrix product. Hence, the STP is a generalization of the conventional matrix product and remains all the fundamental properties of the latter. Because of this, we can omit the sign '$\ltimes$' without confusion.

Definition 2 A swap matrix $W_{[m,n]}$ is an $mn\times mn$ matrix, defined as follows: Its rows and columns are labeled by double index (i, j), the columns are arranged by the ordered multi-index $Id(i, j; m, n)$, and the rows are arranged by the ordered multi-index $Id(j, i; n, m)$. Then the elements at position $[(I, J), (i, j)]$ is

$$w_{(I,J)(i,j)} = \delta_{i,j}^{I,J} = \begin{cases} 1,\ I = i \text{ and } J = j, \\ 0,\ \text{others.} \end{cases} \tag{2}$$

Lemma 1 Let $X \in \mathbb{R}^n$ and $Y \in \mathbb{R}^m$ be column vectors, then

$$W_{[n,m]}XY = YX.$$

Lemma 2 Let $X_i \in \mathbb{R}^{n_i}$, $i = 1, 2, \cdots, k$ be column vectors, then

$$[I_{n_1n_2\cdots n_{t-1}} \otimes W_{[n_t,n_{t+1}]}]X_1 \cdots X_{t-1}X_tX_{t+1} \cdots X_k = X_1 \cdots X_{t-1}X_{t+1}X_t \cdots X_k.$$

Let $x_i \in \mathcal{D}_{k_i}$, $i = 1, \cdots, n$ and $z_j \in \mathcal{D}_{s_j}$, $j = 1, \cdots, m$. Assume that a logic mapping

$$F : \mathcal{D}_{k_1} \times \cdots \times \mathcal{D}_{k_n} \to \mathcal{D}_{s_1} \times \cdots \times \mathcal{D}_{s_m} \tag{3}$$

can be expressed as

$$\begin{cases} z_1 = f_1(x_1, x_2, \cdots, x_n), \\ z_2 = f_2(x_1, x_2, \cdots, x_n), \\ \quad\vdots \\ z_m = f_m(x_1, x_2, \cdots, x_n), \end{cases} \tag{4}$$

where $f_j : \mathcal{D}_{k_1} \times \cdots \times \mathcal{D}_{k_n} \to \mathcal{D}_{s_j},\ j = 1, \cdots, m.$

Lemma 3 Any logical function $z = F(x_1, x_2, \cdots, x_n)$ can be uniquely expressed into the multi-linear form of

$$z = F(x_1, x_2, \cdots, x_n) = M_F \ltimes_{i=1}^{n} x_i, \tag{5}$$

where $M_F \in \mathcal{L}_{s\times k}$ is called the structural matrix of F, $z \in \varDelta_s$, $s = s_1s_2 \cdots s_m$, and $k = k_1k_2 \cdots k_n$.

Lemma 4 Consider Eq.(5). For any $1 \leqslant i \leqslant n$, we split $M_FW_{[k_i, \prod_{p=1}^{i-1} k_p]}$ into k_i equal-size blocks as

$$[Blk_1(M_FW_{[k_i, \prod_{p=1}^{i-1} k_p]}),\ \cdots,\ Blk_{k_i}(M_FW_{[k_i, \prod_{p=1}^{i-1} k_p]})].$$

If all the blocks are the same, then x_i is a fabricated variable. Moreover, z can be replaced by $z = M'_Fx_1 \cdots x_{i-1}x_{i+1} \cdots x_n$, where

$$M'_F = Blk_1(M_FW_{[k_i, \prod_{p=1}^{i-1} k_p]}) = M_FW_{[k_i, \prod_{p=1}^{i-1} k_p]}\delta_{k_i}^1,$$

and $Blk_q(M)$ denotes the q-th block of the matrix M.

3 Main results

This section studies the controller design of multi-variable FLC via STP, and presents the main results.

Consider the linguistic control rules of the multi-variable fuzzy system

$$R^l: \text{IF } x_1 \text{ is } A_1^l, \cdots, \text{and } x_n \text{ is } A_n^l, \quad \text{THEN } y_1 \text{ is } B_1^l, \cdots, \text{and } y_m \text{ is } B_m^l, \tag{6}$$

where x_i and y_j are linguistic variables representing the process state and the control variable, respectively. R^l denotes the l-th fuzzy inference rule, where $l \in \{1, \cdots, L\}$, and L is the number of fuzzy rules. A_i, $i = 1, \cdots, n$ and B_j, $j = 1, \cdots, m$ are the normalized fuzzy set of linguistic values on universes of discourse X_i and Y_j, respectively.

3.1 Controller design of multi-variable fuzzy system

The fuzzy control rules considered in this subsection are both consistent and correct. Next, we study the fuzzy control rules and the fuzzy reasoning via STP.

For the n inputs and m outputs fuzzy controller (6), let the number of the linguistic values of x_i and y_j be, k_i and s_j respectively, that is

$$x_i \in \mathcal{D}_{k_i},\ A_i = \{A_i^1, \cdots, A_i^{k_i}\},\ i = 1, \cdots, n,$$
$$y_j \in \mathcal{D}_{s_j},\ B_j = \{B_j^1,\ \cdots,\ B_j^{s_j}\},\ j = 1, \cdots, m.$$

We identify

$$A_1^{i_1} \sim \delta_{k_1}^{i_1};\ \cdots;\ A_n^{i_n} \sim \delta_{k_n}^{i_n},$$
$$B_1^{j_1} \sim \delta_{s_1}^{j_1};\ \cdots;\ B_m^{j_m} \sim \delta_{s_m}^{j_m},$$

where, $i_1 = 1, \cdots, k_1;\ \cdots;\ i_n = 1, \cdots, k_n;\ j_1 = 1, \cdots, s_1;\ \cdots;\ j_m = 1, \cdots, s_m$. Then, rule (6) can be written as

$$R^l: \text{IF } x_1 = \delta_{k_1}^{i_1},\ \cdots, \text{ and } x_n = \delta_{k_n}^{i_n}, \quad \text{THEN } y_1 = \delta_{s_1}^{j_1},\ \cdots, \text{ and } y_m = \delta_{s_m}^{j_m}. \tag{7}$$

Using the vector form of logical variables, we express the fuzzy controller as

$$\begin{cases} y_1 = M_1 x, \\ y_2 = M_2 x, \\ \ \vdots \\ y_m = M_m x, \end{cases} \tag{8}$$

and

$$y = Mx, \tag{9}$$

where $y := \ltimes_{j=1}^{m} y_j$, $x := \ltimes_{i=1}^{n} x_i$, $M_j \in \mathcal{L}_{s_j \times k}$, $j = 1, \cdots, m$ and $\mathrm{col}_i(M) = \mathrm{col}_i(M_1) \ltimes \cdots \ltimes \mathrm{col}_i(M_n)$, where $\mathrm{col}_i(M)$ denotes the i-th column of matrix M. For rules l and $y_j = M_j x$, since $x = \ltimes_{i=1}^{n} x_i = \delta_{k_1}^{i_1} \ltimes \cdots \ltimes \delta_{k_n}^{i_n} = \delta_k^i$, $y_j = \delta_{s_j}^{j_j}$, we have $\mathrm{col}_i(M_j) = \delta_{s_j}^{j_j}$. If the fuzzy rules are complete, all the columns of M_j, $j = 1, \cdots, m$ can be obtained. Then, we have the following result.

Theorem 1 The structural matrices M_j, $j = 1, \cdots, m$ and M of the fuzzy controller can be uniquely determined, if and only if the fuzzy rules of the fuzzy controller are complete.

Proof Sufficiency. For the fuzzy rules (7), let $x = x_1 \ltimes \cdots \ltimes x_n$. Assume the fuzzy rules of the fuzzy controller are complete, that is, there are k fuzzy rules. For the l-th, $l = 1, \cdots, k$, fuzzy rule, we have $x = \delta_k^i$ and $y_1 = \delta_{s_1}^{j_1}$. Then the i-th column of M_1 can be obtained as

$$\mathrm{col}_i(M_1) = \delta_{s_1}^{j_1}. \tag{10}$$

Repeating this procedure, one can obtain all the columns of M_1 if the fuzzy rules are complete. Similarly, all $M_2, \cdots, M_m$ and M can be determined.

Necessity. If the structural matrices M_i and M of the fuzzy controller are uniquely determined, then all the columns of M_i and M are uniquely determined. Because one column of M can generate one fuzzy rule, one can obtain k fuzzy rules from k columns of M_i or M, that is, the fuzzy rules are complete.

Remark 2 If the rules are not complete, some columns of M_i and M can be determined. In this case, the model is not unique. In addition, uncertain column(s) of M_i and M can be chosen arbitrarily.

Now we give the following example of binary distillation column control in computer simulation to verify the effectiveness of this new method.

Example 1 Consider the two-input-two-output (TITO) Takagi-Sugeno fuzzy controller[15]. According to the shapes of membership function and the rules table[15], we identify

$$e_1, r_1, e_2, r_2: P \sim \delta_2^1,\ P \sim \delta_2^2,$$
$$\begin{aligned} \Delta u_1: & ae_1 + br_1 + m_1(ce_2 + dr_2) \sim \delta_4^1, \\ & k_\alpha(ae_1 + br_1 + m_1(ce_2 + dr_2)) \sim \delta_4^2, \\ & k_\beta(ae_1 + br_1 + m_1(ce_2 + dr_2)) \sim \delta_4^3,\ 0 \sim \delta_4^4, \end{aligned}$$
$$\begin{aligned} \Delta u_2: & m_2(ae_1 + br_1) + ce_2 + dr_2 \sim \delta_4^1, \\ & k_\alpha[m_2(ae_1 + br_1) + ce_2 + dr_2] \sim \delta_4^2, \\ & k_\beta[(m_2(ae_1 + br_1) + ce_2 + dr_2] \sim \delta_4^3,\ 0 \sim \delta_4^4. \end{aligned}$$

By the STP, we obtain

$$\Delta u_j = M_j e_1 r_1 e_2 r_2,\ j = 1, 2, \tag{11}$$

where $M_1 = \delta_4 = [1\ 2\ 2\ 1\ 4\ 4\ 4\ 4\ 4\ 4\ 4\ 4\ 1\ 3\ 3\ 1]$ and $M_2 = \delta_4[1\ 4\ 4\ 1\ 2\ 4\ 4\ 3\ 2\ 4\ 4\ 3\ 1\ 4\ 4\ 1]$. If the values of the inputs of the fuzzy controller are known, the output changes of the fuzzy controller can be obtained through Eq.(11). Here the widely used centroid defuzzifier is employed to calculate the output changes of the fuzzy controller. It is obviously that the proposed method is easer to handle than the conventional method used in [15].

Here, we use the Wood-Berry model as the controlled object to simulate under the proposed fuzzy control method and the existing fuzzy control method. The model is Wood and Berry empirical model of a pilot-scale distillation column that is used to separate a methanol-water mixture[16]:

$$\begin{bmatrix} X_{\mathrm{D}}(s) \\ X_{\mathrm{B}}(s) \end{bmatrix} = \begin{bmatrix} \dfrac{12.8\mathrm{e}^{-s}}{16.7s+1} & \dfrac{-18.9\mathrm{e}^{-3s}}{21s+1} \\ \dfrac{6.6\mathrm{e}^{-7s}}{10.9s+1} & \dfrac{-19.4\mathrm{e}^{-3s}}{14.4s+1} \end{bmatrix} \begin{bmatrix} R(s) \\ S(s) \end{bmatrix}, \tag{12}$$

where $X_{\mathrm{D}}(s)$ and $X_{\mathrm{B}}(s)$ are the overhead and bottom compositions of methanol, respectively; $R(s)$ is the reflux flow rate and S is the steam flow rate. Fig.1 shows the responses of the controllers system with the fuzzy controller under the proposed method and the conventional method. The simulation results illustrate the effectiveness of the proposed method of fuzzy controller design. The parameter values for the fuzzy controller are $L = 0.5$, $k_\alpha = 1.5200$, $k_\beta = 9.5$, $m_1 = 1.476$, $m_2 = 0.34$, $a = 0.0104$, $b = 0.45$, $c = -0.0065$, $d = -0.0875$.

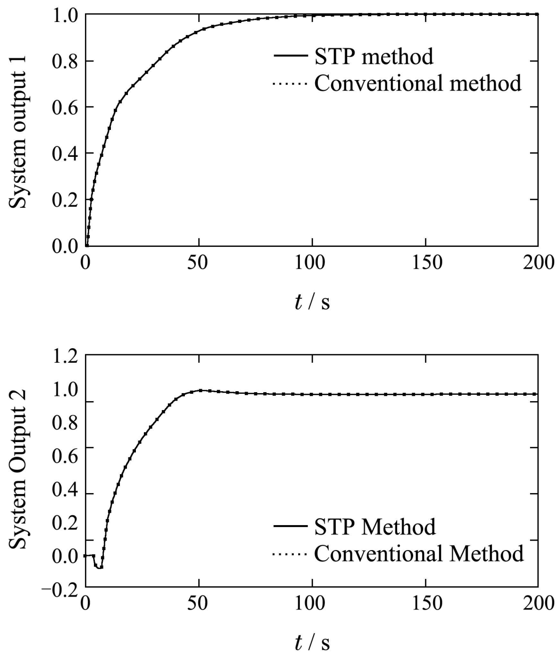

Fig. 1 Simulated performance comparisons of Wood-Berry model under TITO fuzzy control with the STP method and the convention method

3.2 Least in-degree control design

In this section, we consider the least in-degree control, and present a least in-degree algorithm for the fuzzy controller design when the fuzzy control rules are incomplete. The fuzzy control rules considered are also in accordance with consistency and correctness. We first define an incidence matrix to express the dynamic connection between the inputs and the outputs for a fuzzy controller.

Definition 3 Consider a fuzzy controller with m controls and n input variables. An $m \times n$ matrix, $\mathcal{J} = (r_{j,\,i}) \in \mathbb{R}^{m\times n}$, is called its incidence matrix, if

$$r_{j,i} = \begin{cases} 1, & y_j \text{ depends on } x_i, \\ 0, & \text{otherwise.} \end{cases}$$

Considerfuzzy controllers (7) and (8), the in-degree of the fuzzy controller output y_i, denoted by $d(y_j)$, is the number of fuzzy controller inputs which can influence y_j directly. From the incidence matrix of the fuzzy controller, we have

$$d(y_j) = \sum_{k=1}^{n} r_{jk},\ j = 1, \cdots, m. \tag{13}$$

Definition 4 A set of controllers (8) is said to be a feasible set of controllers with respect to rule (7), if rule (7) satisfies Eq.(8).

Definition 5 A feasible set of controllers (8) with the in-degree $d^*(y_j),\ j = 1, \cdots, m$, is called a least in-degree feasible set, if for any other realization with in-degree $d(y_j),\ j = 1, \cdots, m$, we have

$$d^*(y_j) \leqslant d(y_j),\ j = 1, \cdots, m.$$

Remark 3 From the above definition, we know that all the equations of a least in-degree feasible set of controls contain no fabricated variables.

We can apply Lemma 4 to remove fabricated variables and obtain a least in-degree feasible set of controls when the fuzzy rules are complete. Since the structural matrix contains uncertain columns when the fuzzy rules are incomplete, the feasible set of controls may not be unique. Next, we will search for a particular feasible set, that is a least in-degree feasible set of controls. Assume a set of incomplete rules as

$$\begin{aligned} R^l:\ & \text{IF } x_1 = \delta_{k_1}^{i_1}, \cdots, \text{and } x_n = \delta_{k_n}^{i_n}, \\ & \text{THEN } y_1 = \delta_{s_1}^{j_1}, \cdots, \text{and } y_m = \delta_{s_m}^{j_m}, \\ & l \in \{1, \cdots, t\}, \end{aligned} \tag{14}$$

where R^l denotes the l-th fuzzy control rule, t is the number of control rules, and $t < k$. It is obvious that one needs much less data to obtain a set of least in-degree controls. Moreover, it is reasonable to assume a real practical fuzzy controller to be of least in-degree. In the following, we consider how to get a set of least in-degree controls.

Consider the controls $y_j = M_j x$. Using this set of fuzzy rules, some columns of the structural matrix M_j can be determined. For instance,

$$M_j = \delta_{s_j}[\star \cdots \star\ c_{j_1}\ \star \cdots \star \cdots \star\ c_{j_k}\ \star \cdots \star], \tag{15}$$

where $\star$ stands for the uncertain columns. Eq.(15) is called the uncertain structural matrix. Let

$$M_{j,\,i} := M_j W_{[k_i,\, \prod_{p=1}^{i-1} k_p]},\ i = 1, \cdots, n.$$

Then split it into k_i equal blocks as

$$M_{j,i} := [M_{j,i}^1\ \ M_{j,i}^2\ \ \cdots\ \ M_{j,i}^{k_i}].$$

According to Lemma 4, we have the following result.

Proposition 1 The fuzzy control y_j has an algebraic form which is independent of x_i, if and only if

$$M_{j,i}^1 = M_{j,i}^2 = \cdots = M_{j,i}^{k_i} \tag{16}$$

has a solution for uncertain elements.

Proof Sufficiency. Assume Eq.(16) holds. By Lemma 4, the fuzzy control y_j has an algebraic form which is independent of x_i.

Necessity. Assume the fuzzy control y_j is independent of x_i, then y_j remains invariant whenever $x_i = \delta_{k_i}^q$, $q = 1, \cdots, k_i$. Thus

$$M_j W_{[k_i,\, \prod_{p=1}^{i-1} k_p]} \delta_{k_i}^1 = \cdots = M_j W_{[k_i,\, \prod_{p=1}^{i-1} k_p]} \delta_{k_i}^{k_i},$$

which implies that Eq.(16) holds. Thus, the proof is completed.

Now, we give an algorithm to produce a least in-degree realization.

Algorithm 1 The process to produce a least in-degree realization is carried out based on the following steps.

Step 1 For each component-wise algebraic equation of fuzzy controls, using the incomplete rules to identify some columns of the structural matrix M_j as Eq.(15). Define the set as $P_i = \{1, 2, \cdots, n\}$, $i = 1, 2, \cdots, n$.

Step 2 When $i = 1$, consider x_1, we construct

$$M_j = [M_j^1 \ \ M_j^2 \ \ \cdots \ \ M_j^{k_1}] \tag{17}$$

to check whether

$$M_j^1 = M_j^2 = \cdots = M_j^{k_1} \tag{18}$$

has a solution or not. If it has a solution, we can ascertain some uncertain columns and update the fuzzy control as

$$y_j = M_j^1 \mathop{\ltimes}_{i=2}^{n} x_i.$$

Then go to the next step.

Step 3 ($i \leqslant n$) Check whether Eq.(16) has a solution. If it does, ascertain some uncertain columns and update the fuzzy control to

$$y_j = M_{j,i}^1 \mathop{\ltimes}_{1 \leqslant k \leqslant i-1,\ k \in P_i \backslash \{i\}} x_k \mathop{\ltimes}_{k=i+1}^{n} x_k. \tag{19}$$

In the following, we give an example to illustrate this algorithm.

Example 2 Consider a fuzzy controller, which has 4 inputs: $x_1, x_3 \in \mathcal{D}_2$, $x_2, x_4 \in \mathcal{D}_3$ and 2 outputs (controls): $y_1 \in \mathcal{D}_3$, $y_2 \in \mathcal{D}_4$.

In the vector form, we assume that there are a set of control rules as:

IF $x_1 = \delta_2^1,\ x_2 = \delta_3^1,\ x_3 = \delta_3^1$, and $x_4 = \delta_2^1$, THEN $y_1 = \delta_3^2$, and $y_2 = \delta_4^2$;

IF $x_1 = \delta_2^1,\ x_2 = \delta_3^2,\ x_3 = \delta_2^2$, and $x_4 = \delta_3^1$, THEN $y_1 = \delta_3^2$, and $y_2 = \delta_4^4$;

IF $x_1 = \delta_2^1,\ x_2 = \delta_3^3,\ x_3 = \delta_2^2$, and $x_4 = \delta_3^3$, THEN $y_1 = \delta_3^1$, and $y_2 = \delta_4^1$;

IF $x_1 = \delta_2^2,\ x_2 = \delta_3^1,\ x_3 = \delta_2^1$, and $x_4 = \delta_3^1$, THEN $y_1 = \delta_3^1$, and $y_2 = \delta_4^3$;

IF $x_1 = \delta_2^2,\ x_2 = \delta_3^2,\ x_3 = \delta_2^1$, and $x_4 = \delta_3^3$, THEN $y_1 = \delta_3^3$, and $y_2 = \delta_4^2$;

IF $x_1 = \delta_3^2,\ x_2 = \delta_3^3,\ x_3 = \delta_2^2$, and $x_4 = \delta_3^2$, THEN $y_1 = \delta_3^3$, and $y_2 = \delta_4^4$.

Now, we would like to get a least in-degree feasible set of controls. Some columns of M_1 and M_2 can be identified as

$$\begin{aligned} M_1 = \delta_3[\, & 2 \star \star \star \star \star \star \star \star 2 \star \star \star \star \star \star \star \ 1 \\ & \star \ 1 \star \star \star \star \star \star 3 \star \star \star \star \star \star \star \ 3 \ \star], \\ M_2 = \delta_4[\, & 2 \star \star \star \star \star \star \star \star 4 \star \star \star \star \star \star \star \ 1 \\ & \star \ 3 \star \star \star \star \star \star 2 \star \star \star \star \star \star \star \ 4 \ \star], \end{aligned}$$

where '$\star$' denotes the unknown element. Now, we check whether x_1 can be a fabricated variable of y_1. Split M_1 into two equal blocks as $M_1 = [M_1^1 \ \ M_1^2]$, and let $M_1^1 = M_1^2$ which yields the solution as

$$M_1^1 = M_1^2 = \delta_3[2\ 1\ \star\star\star\star \vdots \star\star\ 3\ 2 \star\star \vdots \star\star\star\star\ 3\ 1].$$

Hence, the control can be simplified as $y_1 = M_1^1 x_2 x_3 x_4$. Now, we check x_2. Splitting M_1^1 into three equal blocks as $[M_1^{11} \ M_1^{12} \ M_1^{13}]$ and letting $M_1^{11} = M_1^{12} = M_1^{13}$, it can be updated as

$$M_1^{11} = M_1^{12} = M_1^{13} = \delta_3[2 \ \ 1 \ \ 3 \ \ 2 \ \ 3 \ \ 1],$$

which satisfies $y_1 = M_1^{11} x_3 x_4$. Next, check x_3 and x_4. Since $M_1^{11} W_{[2,3]} = \delta_3[2\ 2\ 1\ 3\ 3\ 1]$, x_3 and x_4 are not fabricated variables. Finally, we obtain

$$y_1 = \delta_3[\ 2 \ \ 2 \ \ 1 \ \ 3 \ \ 3 \ \ 1] x_3 x_4.$$

Similarly, we have $y_2 = \delta_4[\ 2 \ \ 4 \ \ 3 \ \ 4 \ \ 2 \ \ 1] x_3 x_4$.

Hence, the least in-degree realization can finally be obtained as

$$\begin{cases} y_1 = \delta_3[\ 2 \ \ 2 \ \ 1 \ \ 3 \ \ 3 \ \ 1] x_3 x_4, \\ y_2 = \delta_4[\ 2 \ \ 4 \ \ 3 \ \ 4 \ \ 2 \ \ 1] x_3 x_4. \end{cases} \tag{20}$$

3.3 Inconsistency of fuzzy rules

In the previous parts, the fuzzy control rules are in accordance with consistency and correctness. In this subsection, some basic ideas are proposed to deal with inconsistency of rules.

If there are conflicting data caused by measurement or others, in extracting fuzzy rules there may be the fuzzy rules which are inconsistent. For example, it is obtained from the data that

$$\mathrm{col}_i(M_1) = \delta_{s_1}^{p_1}, \cdots, \mathrm{col}_i(M_m) = \delta_{s_m}^{p_m},\ a \text{ times},$$
$$\mathrm{col}_i(M_1) = \delta_{s_1}^{q_1}, \cdots, \mathrm{col}_i(M_m) = \delta_{s_m}^{q_m},\ b \text{ times},$$

where the number of $p_k = q_k,\ k = 1, \cdots, m$ are less than m.

1) If $a \gg b$, then ignore $\delta_{s_j}^{q_i}$ and let $\mathrm{col}_i(M_j) = \delta_{s_j}^{p_i},\ i = 1, \cdots, s_j,\ j = 1, \cdots, m$.

2) If $a \ll b$, then ignore $\delta_{s_j}^{p_i}$ and let $\mathrm{col}_i(M_j) = \delta_{s_j}^{q_i},\ i = 1, \cdots, s_j,\ j = 1, \cdots, m$.

3) If $a \approx b$, then more data may be need or (when data are already enough) it is concluded the fuzzy rules are not acceptable so that $\mathrm{col}_i(M_i)$ can be considered as error columns and set $\mathrm{col}_i(M) = \star$, i.e., thinking of them as uncertain columns.

We would like to mention that, if a fuzzy controller is constructed and later on there are new rules via additional data, the controller could be updated in the following way: If the k-th control verifies new rules, it remains available. Otherwise, new identified columns could be added to the existing set and used to construct new structural matrix M_k. Then the new k-th control can be updated.

4 Application to PHEV

In the following, we use the proposed approach in the design of fuzzy controller based on energy management and control strategy of PHEV.

From the membership functions for input variables[17], we can get that the variable P_{driver} is represented by the linguistic terms ' normal' and 'high', SOC is represented by the linguistic terms 'too low', 'low', 'normal' and 'too high', ω_{EM} is represented by the linguistic terms ' low', 'optimal' and 'high'.

We identify

$$\begin{cases} \text{SOC} \sim x_1,\ P_{\text{driver}} \sim x_2,\ \omega_{\text{EM}} \sim x_3; \\ P_{\text{gen}} \sim y_1,\ \text{scale factor} \sim y_2; \\ \text{too low} \sim \delta_4^1,\ \text{low} \sim \delta_4^2, \\ \text{normal} \sim \delta_4^3,\ \text{too high} \sim \delta_4^4; \\ \text{normal} \sim \delta_2^1,\ \text{high} \sim \delta_2^2; \\ \text{low} \sim \delta_3^1,\ \text{optimal} \sim \delta_3^2,\ \text{high} \sim \delta_3^3; \\ 0\,\text{kW} \sim \delta_5^1,\ 5\,\text{kW} \sim \delta_5^2,\ 10\,\text{kW} \sim \delta_5^3, \\ 15\,\text{kW} \sim \delta_5^4,\ P_{\text{gen}_{\max}} \sim \delta_5^5; \\ 1 \sim \delta_2^1,\ 0 \sim \delta_2^2. \end{cases} \tag{21}$$

Then, the fuzzy rules[17] can be expressed into the following form:

IF $x_1 = \delta_4^4,\ x_2 = \delta_2^1,\ x_3 = \delta_3^1$, THEN $y_1 = \delta_5^1,\ y_2 = \delta_2^1$;
IF $x_1 = \delta_4^4,\ x_2 = \delta_2^1,\ x_3 = \delta_3^2$, THEN $y_1 = \delta_5^1,\ y_2 = \delta_2^1$;
IF $x_1 = \delta_4^4,\ x_2 = \delta_2^1,\ x_3 = \delta_3^3$, THEN $y_1 = \delta_5^1,\ y_2 = \delta_2^1$;
IF $x_1 = \delta_4^4,\ x_2 = \delta_2^2,\ x_3 = \delta_3^1$, THEN $y_1 = \delta_5^1,\ y_2 = \delta_2^1$;
IF $x_1 = \delta_4^4,\ x_2 = \delta_2^2,\ x_3 = \delta_3^2$, THEN $y_1 = \delta_5^1,\ y_2 = \delta_2^1$;
IF $x_1 = \delta_4^4,\ x_2 = \delta_2^2,\ x_3 = \delta_3^3$, THEN $y_1 = \delta_5^1,\ y_2 = \delta_2^1$;
IF $x_1 = \delta_4^3,\ x_2 = \delta_2^1,\ x_3 = \delta_3^2$, THEN $y_1 = \delta_5^3,\ y_2 = \delta_2^1$;
IF $x_1 = \delta_4^3,\ x_2 = \delta_2^1,\ x_3 = \delta_3^1$, THEN $y_1 = \delta_5^1,\ y_2 = \delta_2^1$;
IF $x_1 = \delta_4^3,\ x_2 = \delta_2^2,\ x_3 = \delta_3^1$, THEN $y_1 = \delta_5^1,\ y_2 = \delta_2^1$;
IF $x_1 = \delta_4^3,\ x_2 = \delta_2^1,\ x_3 = \delta_3^3$, THEN $y_1 = \delta_5^1,\ y_2 = \delta_2^1$;
IF $x_1 = \delta_4^3,\ x_2 = \delta_2^2,\ x_3 = \delta_3^3$, THEN $y_1 = \delta_5^1,\ y_2 = \delta_2^1$;
IF $x_1 = \delta_4^2,\ x_2 = \delta_2^1,\ x_3 = \delta_3^1$, THEN $y_1 = \delta_5^2,\ y_2 = \delta_2^1$;
IF $x_1 = \delta_4^2,\ x_2 = \delta_2^1,\ x_3 = \delta_3^2$, THEN $y_1 = \delta_5^4,\ y_2 = \delta_2^1$;
IF $x_1 = \delta_4^2,\ x_2 = \delta_2^1,\ x_3 = \delta_3^3$, THEN $y_1 = \delta_5^4,\ y_2 = \delta_2^1$;
IF $x_1 = \delta_4^1,\ x_2 = \delta_2^1,\ x_3 = \delta_3^1$, THEN $y_1 = \delta_5^5,\ y_2 = \delta_2^2$;
IF $x_1 = \delta_4^1,\ x_2 = \delta_2^1,\ x_3 = \delta_3^2$, THEN $y_1 = \delta_5^5,\ y_2 = \delta_2^2$;
IF $x_1 = \delta_4^1,\ x_2 = \delta_2^1,\ x_3 = \delta_3^3$, THEN $y_1 = \delta_5^5,\ y_2 = \delta_2^2$;
IF $x_1 = \delta_4^1,\ x_2 = \delta_2^2,\ x_3 = \delta_3^1$, THEN $y_1 = \delta_5^5,\ y_2 = \delta_2^2$;
IF $x_1 = \delta_4^1,\ x_2 = \delta_2^1,\ x_3 = \delta_3^2$, THEN $y_1 = \delta_5^5,\ y_2 = \delta_2^2$;
IF $x_1 = \delta_4^1,\ x_2 = \delta_2^1,\ x_3 = \delta_3^3$, THEN $y_1 = \delta_5^5,\ y_2 = \delta_2^2$;
IF $x_1 = \delta_4^2,\ x_2 = \delta_2^2,\ x_3 = \delta_3^1$, THEN $y_1 = \delta_5^1,\ y_2 = \delta_2^1$;
IF $x_1 = \delta_4^2,\ x_2 = \delta_2^2,\ x_3 = \delta_3^2$, THEN $y_1 = \delta_5^1,\ y_2 = \delta_2^1$;
IF $x_1 = \delta_4^2,\ x_2 = \delta_2^2,\ x_3 = \delta_3^3$, THEN $y_1 = \delta_5^1,\ y_2 = \delta_2^1$;
IF $x_1 = \delta_4^3,\ x_2 = \delta_2^2,\ x_3 = \delta_3^2$, THEN $y_1 = \delta_5^1,\ y_2 = \delta_2^1$.

From the above form of the fuzzy rules, we obtain

$$M_1 = \delta_5[\,5\,5\,5\,5\,5\,5\,2\,4\,4\,1\,1\,1\,1\,3\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1],$$
$$M_2 = \delta_2[\,2\,2\,2\,2\,2\,2\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1],$$

then $y_1 = M_1 x_1 x_2 x_3$, $y_2 = M_2 x_1 x_2 x_3$.

Now using Lemma 4, we check whether x_1, x_2 or x_3 is a fabricated variable of y_1. It is easy to verify that

$$M_1 = \\ \delta_5[5\,5\,5\,5\,5\,5 \vdots 2\,4\,4\,1\,1\,1 \vdots 1\,3\,1\,1\,1\,1 \vdots 1\,1\,1\,1\,1\,1],$$
$$M_1 W_{[2,4]} = \delta_5[\,5\ 5\ 5\ 2\ 4\ 4\ 1\ 3\ 1\ 1\ 1\ 1 \\ 5\ 5\ 5\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1],$$

and

$$M_1 W_{[3,8]} = \delta_5[\,5\ 5\ 2\ 1\ 1\ 1\ 1\ 1\ 5\ 5\ 4\ 1 \\ 3\ 1\ 1\ 1\ 5\ 5\ 4\ 1\ 1\ 1\ 1\ 1].$$

Obviously, we know x_1, x_2, x_3 are not fabricated variables of y_1. Similarly, we check whether x_1, x_2 or x_3 is a fabricated variable of y_2. Since

$$M_2 = \\ \delta_2[\,2\,2\,2\,2\,2\,2 \vdots 1\,1\,1\,1\,1\,1 \vdots 1\,1\,1\,1\,1\,1 \vdots 1\,1\,1\,1\,1\,1],$$
$$M_2 W_{[2,4]} = \delta_2[\,2\ 2\ 2\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1 \\ 2\ 2\ 2\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1],$$

and

$$M_2 W_{[3,8]} = \delta_2[\,2\ 2\ 1\ 1\ 1\ 1\ 1\ 1\ 2\ 2\ 1\ 1 \\ 1\ 1\ 1\ 1\ 2\ 2\ 1\ 1\ 1\ 1\ 1\ 1],$$

x_2 and x_3 are fabricated variables in the dynamical equation of y_2. Set $x_2 = \delta_2^1$ and $x_3 = \delta_3^1$, then

$$y_2 = M_2 W_{[3,8]} x_3 W_{[2,4]} x_2 x_1 = \delta_2[2\ 1\ 1\ 1]x_1.$$

Thus, the control can be simplified as $y_2 = M_2' x_1$, where $M_2' = \delta_3[2\ 1\ 1\ 1]$.

Assume that SOC is low, P_{driver} is normal and ω_{EM} is optimal, we have $y_1 = M_1 x_1 x_2 x_3 = M_1 \delta_4^2 \delta_2^1 \delta_3^2 = \delta_5^4$ and $y_2 = M_2' x_1 = \delta_2[2\ 1\ 1\ 1]\delta_4^2 = \delta_2^1$, which means $P_{\text{gen}} = 15\,\text{kW}$, and scale factor is 1.

5 Conclusion

In this paper, we have studied the analysis and design of multi-variable fuzzy logic controller based on the semi-tensor product of matrices. Using the basic properties of STP, we convert the fuzzy control rules and fuzzy logic inference into an algebraic form. When the control rules are not enough, we obtain a reasonable set of control rules and give a set of least in-degree rules by the structural matrix of the fuzzy controller. we introduce the consistency of fuzzy control rules, and propose some principles for dealing with fuzzy controls with inconsistency. Examples are proved to illustrate the effectiveness of the proposed method. Moreover, we have applied the approach in this paper to design of the fuzzy controller for energy management and control strategy of parallel hybrid vehicles.

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