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一类非线性时滞系统的自适应动态面控制

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摘要: 本文对于一类含不确定输入时滞和干扰的非线性系统的跟踪控制问题提出了一种自适应动态面控制方案. 利用动态面控制方法避免了传统的后推设计中存在的复杂度爆炸问题. 分别构造了一个滤波器和一个虚拟观测器来产生辅助信号. 采用神经网络来逼近未知的连续函数. 跟踪误差被证明最终收敛到一个足够小的紧集. 给出了一个数字仿真示例验证了理论结果.

关键词: 动态面控制; 输入时滞; 神经网络; 自适应控制
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Adaptive dynamic surface control for a class of nonlinear time-delay systems

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Abstract: Adaptive dynamic surface control is developed for the tracking control problem of a class of nonlinear systems with uncertain input delay and disturbances. The explosion of complexity in traditional backstepping design is avoided by utilizing dynamic surface control. A filter and a virtual observer are constructed respectively to produce the auxiliary signal. Neural networks are employed to approximate the unknown continuous functions. It is proved that the tracking error ultimately converges to an adequately small compact set. The theoretical result is illustrated through a simulation example.

Key words: dynamic surface control; input delay; neural network; adaptive control

1 Introduction

Time delays are frequently encountered in practical control systems, such as aircraft, chemical or process control systems, etc. The existence of the time delays, either in the input or in the state, may be the source of instability or serious deterioration in the performance of the closed-loop systems[1–3]. Due to the inherent controller delay and remote transfer delay, the input delay problem is particularly serious. Therefore, the stability issue and the performance of input delayed control systems are of both theoretical and practical importance.

In the past decade, tremendous strides have been made in the area of controller design for uncertain nonlinear systems[1–22]. Intelligent control techniques including neural network, fuzzy system, often combined with adaptive control, have been successfully applied in this area, and there are a lot of research results about them[1–11]. An adaptive neural controller is presented for a class of strict-feedback nonlinear systems with unknown time delays. The unknown time delays are compensated for using appropriate Lyapunov-Krasovskii functionals in the design. It is proved that the proposed design method is able to guarantee semi-globally uniformly ultimate boundedness (SGUUB) of all the signals in the closed-loop system and the tracking error is proven to converge to a small neighborhood of the origin[1–2]. Several neural control schemes are proposed for different nonlinear systems, such as multi-input-multi-output (MIMO) systems, strict-feedback systems, systems with input saturation, etc.[3–11].

On the other hand, backstepping is evolved as an effective methodology. It provides a systematic framework for the controller design of a large class of nonlinear systems[12–17]. The main advantages of backstepping methodology include: a) global stability can be achieved with ease; b) the transient performance can be guaranteed and explicitly analyzed. However, an obvious draw-

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back in the traditional backstepping is the problem of 'explosion of complexity', which is caused by the repeated differentiations of certain nonlinear functions. To overcome the 'explosion of complexity', dynamic surface control (DSC) is proposed by introducing first-order filtering of the synthetic virtual control input at each step of traditional backstepping approach[18–22]. A robust adaptive tracking control approach is presented for a class of nonlinear systems. By employing radial-basis-function (RBF) neural networks to account for system uncertainties, the proposed scheme is developed by combining DSC and minimal learning parameter (MLP) techniques[18]. DSC methodology is also utilized in systems with periodic disturbance, systems with unknown dead zone, magnetic levitation system and servo mechanisms[19–22].

In this note, we deal with the tracking control problem for a class of uncertain nonlinear systems with uncertain input delay and time-varying disturbances. The state feedback control scheme combined with DSC, neural networks and adaptive control is proposed. Compared with the previous works, the main contributions of the paper lie in: 1) The novel definition of the dynamic surface variable including the input integral term is presented. By means of the definition, the input delayed system is converted to a non-delayed system. 2) DSC technique is introduced to improve the traditional backstepping method in the control scheme of input delayed nonlinear systems. 3) A filter and a virtual observer are constructed to replace the system state. The theoretic result is illustrated through a simulation example.

2 Problem statement

Consider the nonlinear input delay system described by

$$\begin{cases} \dot{x}_i = f_i(\bar{x}_i) + x_{i+1} + h_i(x,t),\ 1 \leqslant i \leqslant n-1, \\ \dot{x}_n = f_n(x) + g(x)u(t-\tau(t)) + h_n(x,t), \\ y(t) = x_1(t), \end{cases} \tag{1}$$

where $x = [x_1\ \cdots\ x_n]^{\mathrm{T}} \in \mathbb{R}^n$ is the measurable system state and $\bar{x}_i = [x_1\ \cdots\ x_i]^{\mathrm{T}}$, $y(t)$ is the system output, $u(t-\tau(t)) \in \mathbb{R}$ is the control input, $u(t) = 0$, $t < 0$, $g(x)$ is the uncertain control gain function, $f_i(\bar{x}_i)$ are unknown continuous functions, $h_i(x,t)$ are time-varying disturbances. System state $x(t)$ is bounded when $t = 0$. The control objective is that the output $y(t)$ follows the reference signal $y_{\mathrm{r}}(t)$.

The following assumptions and lemma are made throughout the paper.

Assumption 1 The input delay meets the following restrictions:

$\tau(t) = \tau + \Delta_\tau(t)$, $|\Delta_\tau(t)| < \delta_\tau$, $\tau(t) \geqslant 0$, $\tau \geqslant 0$,

where τ is a known constant and δ_τ is an unknown small constant.

Assumption 2 The control gain function meets the following restrictions:

$g(x) = g_1(\bar{x}_{n-1}) + \Delta_g(x)$, $|\Delta_g(x)| < \delta_g$,

$g(x) \geqslant g_0$, $g_1(\bar{x}_{n-1}) \geqslant g_0$,

where $g_1(\bar{x}_{n-1})$ is a known differentiable function and δ_g, g_0 are unknown small positive constants. Substitute $g_1(x)$ for $g_1(\bar{x}_{n-1})$ throughout the paper.

Assumption 3 The unknown disturbances $h_i(\cdot)$ $(1 \leqslant i \leqslant n)$ satisfy $|h_i(\cdot)| \leqslant \bar{h}_i$, where $\bar{h}_i$ are unknown constants, respectively.

Assumption 4 The reference signal $y_{\mathrm{r}}(t)$ is differentiable. $y_{\mathrm{r}}(t) \leqslant \bar{y}_{\mathrm{r}}$, $\dot{y}_{\mathrm{r}}(t) \leqslant \bar{y}_{\mathrm{r}}$, where $\bar{y}_{\mathrm{r}}$ is an unknown constant.

Lemma 1 For the continuous function $q_i(\lambda_i)$ and the bounded closed set C_{λ_i}, there is a neural network satisfying[25]

$$q_i(\lambda_i) = W_i^{\mathrm{T}} S_i(\lambda_i) + \varepsilon_{w_i}(\lambda_i),\ \forall \lambda_i \in C_{\lambda_i},$$

where $S_i(\lambda_i) = [S_{i,1}(\lambda_i)\ \cdots\ S_{i,m_i}(\lambda_i)]^{\mathrm{T}}$. The Gaussian basis function is selected as $S_{i,j}(\lambda_i) = \mathrm{e}^{(\frac{-\|\lambda_i-\mu_{i,j}\|^2}{2\sigma_{i,j}^2})}$. $W_i \in \mathbb{R}^{m_i}$ is the weight vector of the neural network. $|\varepsilon_{w_i}(\lambda_i)| \leqslant \bar{\varepsilon}_{w_i}$ is the estimation error. Denote the best weight vector as

$$W_i^* := \arg \min_{W_i \in \mathbb{R}^{m_i}} \{ \sup_{\lambda_i \in C_{\lambda_i}} |W_i^{\mathrm{T}} S_i(\lambda_i) - h_i(\lambda_i)| \}.$$

Define $\hat{W}_i$ as the estimation of W_i^*.

The conclusion can be easily promoted to the vector functions. For the continuous vector function $f(x)$ and the bounded closed set C_x, by the estimate ability of RBF neural networks, there is a perfect RBF neural network which satisfies

$$f(x) = \phi(x)\theta + \varepsilon_f(x),\ \forall x \in C_x,$$

where $\phi(x) = [\phi_1^{\mathrm{T}}(x)\ \cdots\ \phi_n^{\mathrm{T}}(x)]^{\mathrm{T}}$, $\phi_i(x) = [\phi_{i,1}(x)\ \cdots\ \phi_{i,q}(x)]$. The Gaussian basis function is selected as $\phi_{i,j}(x) = \mathrm{e}^{\frac{-\|x-\mu_{i,j}\|^2}{2\sigma_{i,j}^2}}$. $\theta \in \mathbb{R}^q$ is the weight vector of the neural network. $\|\varepsilon_f(x)\| \leqslant \bar{\varepsilon}_f$ is the estimation error. Denote the best weight vector as

$$\theta^* := \arg \min_{\theta \in \mathbb{R}^q} \{ \sup_{x \in C_x} \|\phi(x)\theta - f(x)\| \}.$$

For convenience, θ is used to denote θ^* in some equations below without confusion, and W is used to denote W^* in some equations below too.

Notation $\|\cdot\|$ denotes the 2-norm of a vector or the Frobenius norm of a matrix.

3 Main result

By Lemma 1, system (1) can be denoted as

$$\begin{cases} \dot{x} = Ax + Bg(x)u(t-\tau(t)) + \phi(x)\theta + \\ \qquad \varepsilon_f(x) + h(x,t), \\ y = Cx, \end{cases} \tag{2}$$

where

$$A = \begin{bmatrix} 0\,1 & & \\ \vdots & \ddots & \\ 0 & & 1 \\ 0\,0 \cdots 0 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix},\ C = [1\ 0\ \cdots\ 0],$$

$$\phi(x) = \begin{bmatrix} \phi_1(x_1) \\ \phi_2(\bar{x}_2) \\ \vdots \\ \phi_n(x) \end{bmatrix},\ \varepsilon_f(x) = \begin{bmatrix} \varepsilon_{f_1}(x_1) \\ \varepsilon_{f_2}(\bar{x}_2) \\ \vdots \\ \varepsilon_{f_n}(x) \end{bmatrix},\ \theta = \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_q \end{bmatrix},$$

$\phi_i(\bar{x}_i) \in \mathbb{R}^{1\times q}$, $h(x,t) = [h_1(x,t)\ \cdots\ h_n(x,t)]^{\mathrm{T}}$, $\|\varepsilon_f(x)\| \leqslant \bar{\varepsilon}_f$.

Substitute h_i, ε_f for $h_i(x,t), \varepsilon_f(x)$ respectively throughout the paper.

Construct a filter as[23]

$$\begin{cases} \dot{\xi} = A_0\xi + Ky + Bg_1(x)u(t-\tau), \\ \dot{\Omega} = A_0\Omega + \phi(x), \end{cases} \tag{3}$$

and a virtual observer as[24]

$$\hat{x} = \xi + \Omega\theta, \tag{4}$$

where

$$\begin{cases} K = \begin{bmatrix} k_1 \\ \vdots \\ k_n \end{bmatrix}, \xi = \begin{bmatrix} \xi_1 \\ \vdots \\ \xi_n \end{bmatrix}, \hat{x} = \begin{bmatrix} \hat{x}_1 \\ \vdots \\ \hat{x}_n \end{bmatrix}, \Omega = \begin{bmatrix} \Omega_1 \\ \vdots \\ \Omega_n \end{bmatrix}, \\ \Omega_i(t) \in \mathbb{R}^{1\times q}, \end{cases} \tag{5}$$

and $k_i > 0 (i = 1, \cdots, n)$ is a design constant which satisfies that $A_0 = A - KC$ is Hurwitz. The observer $\hat{x}$ is called 'virtual', because it is unimplementable when the optimal value θ is unknown. The purpose of introducing a filter and an observer here is to be discussed later as a remark.

Define

$$e = x - \hat{x}, \tag{6}$$

where $e = [e_1 \ \cdots \ e_n]^{\mathrm{T}}$. Therefore, we get

$$\dot{e} = A_0 e + (B\Delta_u + \varepsilon_f(x) + h(x,t)), \tag{7}$$

where

$$\Delta_u = g(x)u(t-\tau(t)) - g_1(x)u(t-\tau). \tag{8}$$

In equation (7), if we take the whole item $B\Delta_u + \varepsilon_f(x) + h(x,t)$ as the system control input, and let an identity matrix to be the control gain matrix, the error system is a linear time invariant (LTI) system. Because A_0 is Hurwitz, according to the linear system theory, the system is input state stable (ISS). It is assumed that $\|e(t)\| \leqslant \bar{e}$. Similarly by equations (3), Ω is ISS with respect to $\phi(x)$. Therefore, $\|\Omega_i(t)\| \leqslant \bar{\Omega}_i$.

Define one-order filters as

$$\iota_i \dot{z}_i = -z_i + \alpha_{i-1}, \ 2 \leqslant i \leqslant n, \tag{9}$$

where ι_i are positive design parameters. The virtual controls α_i are defined in the following.

Define dynamic surfaces as

$$\begin{cases} s_1 = x_1 - y_{\mathrm{r}}, \\ s_i = x_i - z_i, \ 2 \leqslant i \leqslant n-1, \\ s_n = \xi_n - z_n + g_1(x)\int_{-\tau}^{0} u(t+v)\mathrm{d}v. \end{cases} \tag{10}$$

Define assistant functions

$$\begin{cases} q_1(\lambda_1) = \phi_1(x_1)\theta - \dot{y}_{\mathrm{r}} + (\eta_1^2+1)s_1, \\ q_i(\lambda_i) = \phi_i(\bar{x}_i)\theta - \dot{z}_i + (\eta_i^2+1.5)s_i, \\ \qquad 2 \leqslant i \leqslant n-1, \\ q_n(\lambda_n) = k_n(x_1 - \xi_1) - \dot{z}_n + \sum\limits_{j=1}^{n-1} \dfrac{\partial g_1}{\partial x_j} \cdot \\ \qquad \int_{-\tau}^{0} u(t+v)\mathrm{d}v[x_{j+1} + \phi_j(x)\theta] + \\ \qquad \gamma^2 \sum\limits_{j=1}^{n-1} (\dfrac{\partial g_1}{\partial x_j}\int_{-\tau}^{0} u(t+v)\mathrm{d}v)^2 s_n + \\ \qquad (\eta_n^2 + 0.5)s_n, \end{cases} \tag{11}$$

where $\gamma > 0$, $\eta_i > 0 (i = 1, \cdots, n)$ are design constants and k_n is a positive constant which defined by equations (5).

By Lemma 1, the RBF neural network is constructed to approximate the continuous function $q_i(\lambda_i)$ for each subsystem.

$$q_i(\lambda_i) = W_i^{\mathrm{T}} S_i(\lambda_i) + \varepsilon_{w_i}(\lambda_i), \ \forall \lambda_i \in C_{\lambda_i}. \tag{12}$$

Define the virtual controls

$$\alpha_i = -c_i^2 s_i - \hat{W}_i^{\mathrm{T}} S_i(\lambda_i), \ 1 \leqslant i \leqslant n, \tag{13}$$

where c_i are design parameters,

$$\begin{cases} \lambda_1 = [x_1 \ y_{\mathrm{r}} \ \dot{y}_{\mathrm{r}}]^{\mathrm{T}}, \\ \lambda_i = [\bar{x}_i^{\mathrm{T}} \ z_i \ \dot{z}_i]^{\mathrm{T}}, \ 2 \leqslant i \leqslant n-1, \\ \lambda_n = [x^{\mathrm{T}} \ z_n \ \dot{z}_n \ \xi_1 \ \xi_n \ \dfrac{\partial g_1}{\partial x_1} \ \cdots \ \dfrac{\partial g_1}{\partial x_{n-1}} \\ \qquad \int_{-\tau}^{0} u(t+v)\mathrm{d}v]^{\mathrm{T}}. \end{cases} \tag{14}$$

The following theorem is presented.

Theorem 1 Consider the system (1) under Assumptions 1–4. The virtual controls α_i are defined by equation (13). If the control law is selected as

$$u(t) = -g_1^{-1}(x)\alpha_n, \tag{15}$$

and the adaptive law is selected as

$$\dot{\hat{W}}_i = \Gamma_i(S_i(\lambda_i)s_i - \sigma_i^2 \hat{W}_i), \ 1 \leqslant i \leqslant n, \tag{16}$$

then the system tracking error is bounded and ultimately converges to an adequately small compact set:

$$|y - y_{\mathrm{r}}| \leqslant \sqrt{2\bar{V}}, \ \lim_{t\to\infty} |y - y_{\mathrm{r}}| \leqslant k^*\delta^*, \tag{17}$$

where k^* is the design parameter and $\bar{V}$, δ^* are constants. The closed-loop system is SGUUB.

Proof Define variables ϱ_i as

$$\varrho_i = z_i - \alpha_{i-1}, \ 2 \leqslant i \leqslant n. \tag{18}$$

Referring to equations (9) and (13), we have

$$\dot{\varrho}_i = \dot{z}_i - \dot{\alpha}_{i-1} = -\iota_i^{-1}\varrho_i - f_{\varrho_i}(\lambda_{i-1}), \tag{19}$$

where

$$f_{\varrho_i}(\lambda_{i-1}) = -c_{i-1}^2 \dot{s}_{i-1} - \dot{\hat{W}}_{i-1}^{\mathrm{T}} S_{i-1}(\lambda_{i-1}) - \hat{W}_{i-1}^{\mathrm{T}} \frac{\partial S_{i-1}}{\partial \lambda_{i-1}} \dot{\lambda}_{i-1}. \tag{20}$$

Each $f_{\varrho_i}(\lambda_{i-1})$ is a continuous function. Therefore, $|f_{\varrho_i}(\lambda_{i-1})|$ has a maximum $\bar{f}_{\varrho_i}$ on a compact set $C_{\lambda_{i-1}}$. The explicit definition of C_{λ_i} is to be illustrated in the following.

Define the Lyapunov function $V(t)$ as

$$V(t) = \sum_{i=1}^{n} V_i(t), \tag{21}$$

$$V_i(t) = \frac{1}{2}s_i^2 + \frac{1}{2}\tilde{W}_i^{\mathrm{T}}\Gamma_i^{-1}\tilde{W}_i + \frac{\gamma_{\varrho_{i+1}}^2}{2}\varrho_{i+1}^2, \quad 1 \leqslant i \leqslant n-1, \tag{22}$$

$$V_n(t) = \frac{1}{2}s_n^2 + \frac{1}{2}\tilde{W}_n^{\mathrm{T}}\Gamma_n^{-1}\tilde{W}_n, \tag{23}$$

where $\tilde{W}_i = W_i^* - \hat{W}_i$.

Step 1 For the first subsystem, by equations (2)–(3) and (9), it can be obtained that

$$\dot{s}_1=\dot{x}_1-\dot{y}_{\rm r}=x_2+\phi_1(x_1)\theta+\varepsilon_{f_1}+h_1-\dot{y}_{\rm r}= x_2+W_1^{\rm T}S_1(\lambda_1)+\varepsilon_{w_1}-(\eta_1^2+1)s_1+\varepsilon_{f_1}+h_1= x_2-c_1^2s_1-\hat{W}_1^{\rm T}S_1(\lambda_1)-\alpha_1+W_1^{\rm T}S_1(\lambda_1)+ \varepsilon_{w_1}(\lambda_1)-(\eta_1^2+1)s_1+\varepsilon_{f_1}+h_1= -c_1^2s_1+x_2-\alpha_1+\tilde{W}_1^{\rm T}S_1(\lambda_1)+\varepsilon_{w_1}(\lambda_1)+\varepsilon_{f_1}+ h_1-(\eta_1^2+1)s_1. \tag{24}$$

Using Young's Inequality, we get

$$s_i\varepsilon_i\leqslant\eta_i^2s_i^2+\frac{1}{4\eta_i^2}\varepsilon_i^2, \tag{25}$$

$$\tilde{W}_i\hat{W}_i\leqslant-\frac{1}{2}\|\tilde{W}_i\|^2+\frac{1}{2}\|W_i\|^2,\ 1\leqslant i\leqslant n, \tag{26}$$

$$s_1(s_2+\varrho_2)\leqslant s_1^2+\frac{1}{2}s_2^2+\frac{1}{2}\varrho_2^2, \tag{27}$$

and

$$-\gamma_{\varrho_2}^2\varrho_2f_{\varrho_2}(\lambda_1)\leqslant\zeta^2\gamma_{\varrho_2}^2\varrho_2^2+\frac{1}{4\zeta^2}\gamma_{\varrho_2}^2f_{\varrho_2}^2(\lambda_1). \tag{28}$$

Differentiating the Lyapunov function V_1 (22) along the track (24) and substituting virtual control (13), adaptive law (16) and Inequalities (25)–(28) into it, it is easy to have

$$\begin{aligned}\dot{V}_1=\ & s_1[-c_1^2s_1+x_2-\alpha_1+\tilde{W}_1^{\rm T}S_1(\lambda_1)+\varepsilon_{w_1}(\lambda_1)+ h_1-(\eta_1^2+1)s_1]-\tilde{W}_1^{\rm T}(S_1(\lambda_1)s_1-\sigma_1^2\hat{W}_1)+ \gamma_{\varrho_2}^2\varrho_2\dot{\varrho}_2+\varepsilon_{f_1}=\\ & s_1[-c_1^2s_1+x_2-\alpha_1+\varepsilon_{w_1}(\lambda_1)+\varepsilon_{f_1}+h_1- (\eta_1^2+1)s_1]+\sigma_1^2\tilde{W}_1^{\rm T}\hat{W}_1+\gamma_{\varrho_2}^2\varrho_2\dot{\varrho}_2\leqslant\\ & -c_1^2s_1^2+s_1(x_2-\alpha_1)+|s_1|(\bar{\varepsilon}_{w_1}+\bar{\varepsilon}_{f_1}+\bar{h}_1)- (\eta_1^2+1)^2s_1^2+\sigma_1^2\tilde{W}_1^{\rm T}\hat{W}_1+\gamma_{\varrho_2}^2\varrho_2\dot{\varrho}_2\leqslant\\ & -c_1^2s_1^2+s_1(x_2-\alpha_1)+\frac{1}{4\eta_1^2}(\bar{\varepsilon}_{w_1}+\bar{\varepsilon}_{f_1}+\bar{h}_1)^2- s_1^2+\sigma_1^2(\frac{-1}{2}\|\tilde{W}_1\|^2+\frac{1}{2}\|W_1\|^2)+ \gamma_{\varrho_2}^2\varrho_2(-\iota_2^{-1}\varrho_2-f_{\varrho_2}(\lambda_1))\leqslant\\ & -c_1^2s_1^2-\frac{\sigma_1^2}{2}\|\tilde{W}_1\|^2-\frac{\gamma_{\varrho_2}^2}{\iota_2}\varrho_2^2-s_1^2+s_1(s_2+\varrho_2)+ \frac{1}{4\eta_1^2}(\bar{\varepsilon}_{w_1}+\bar{\varepsilon}_{f_1}+\bar{h}_1)^2+\frac{\sigma_1^2}{2}\|W_1\|^2-\gamma_{\varrho_2}^2\varrho_2f_{\varrho_2}\leqslant\\ & -c_1^2s_1^2-\frac{\sigma_1^2}{2}\|\tilde{W}_1\|^2-\frac{\gamma_{\varrho_2}^2}{\iota_2}\varrho_2^2-s_1^2+ \frac{1}{4\eta_1^2}(\bar{\varepsilon}_{w_1}+\bar{\varepsilon}_{f_1}+\bar{h}_1)^2+\frac{\sigma_1^2}{2}\|W_1\|^2+s_1^2+\frac{1}{2}s_2^2+ \frac{1}{2}\varrho_2^2+\zeta^2\gamma_{\varrho_2}^2\varrho_2^2+\frac{1}{4\zeta^2}\gamma_{\varrho_2}^2f_{\varrho_2}^2(\lambda_1)\leqslant\\ & -k_{\rm v1}V_1+b_{\rm v1}+\varDelta_{\rm v1},\end{aligned} \tag{29}$$

where

$$\begin{cases}k_{\rm v1}=\min\{2c_1^2,(\dfrac{2}{\iota_2}-\dfrac{1}{\gamma_{\varrho_2}^2}-2\zeta^2),\dfrac{\sigma_1^2}{\lambda_{\max}(\varGamma_1^{-1})}\},\\ b_{\rm v1}=\dfrac{1}{4\eta_1^2}(\bar{\varepsilon}_{w_1}+\bar{\varepsilon}_{f_1}+\bar{h}_1)^2+\dfrac{\sigma_1^2}{2}\|W_1\|^2+\dfrac{\gamma_{\varrho_2}^2}{4\zeta^2}\bar{f}_{\varrho_2}^2,\\ \varDelta_{\rm v1}=\dfrac{1}{2}s_2^2.\end{cases} \tag{30}$$

Step i For the ith ($2\leqslant i\leqslant n-1$) subsystem, by equations (2)(3) and (9), it can be obtained that

$$\dot{s}_i=\dot{x}_i-\dot{z}_i=x_{i+1}+\phi_i(\bar{x}_i)\theta+\varepsilon_{f_i}+h_i-\dot{z}_i= x_{i+1}+W_i^{\rm T}S_i(\lambda_i)+\varepsilon_{w_i}-(\eta_i^2+1.5)s_i+\varepsilon_{f_i}+h_i= x_{i+1}-c_i^2s_i-\hat{W}_i^{\rm T}S_i(\lambda_i)-\alpha_i+W_i^{\rm T}S_i(\lambda_i)+\varepsilon_{w_i}= -(\eta_i^2+1.5)s_i+\varepsilon_{f_i}+h_i-c_i^2s_i+x_{i+1}-\alpha_i+ \tilde{W}_i^{\rm T}S_i(\lambda_i)+\varepsilon_{w_i}+\varepsilon_{f_i}+h_i-(\eta_i^2+1.5)s_i. \tag{31}$$

Differentiating the Lyapunov function V_i (22) along the track (31) and substituting (13) into it, it is easy to have

$$\begin{aligned}\dot{V}_i=\ & s_i[-c_i^2s_i+x_{i+1}-\alpha_i+\tilde{W}_i^{\rm T}S_i(\lambda_i)+\varepsilon_{w_i}+ \varepsilon_{f_i}+h_i-(\eta_i^2+1.5)s_i]+\gamma_{\varrho_{i+1}}^2\varrho_{i+1}\dot{\varrho}_{i+1}- \tilde{W}_i^{\rm T}(S_i(\lambda_i)s_i-\sigma_i^2\hat{W}_i)\leqslant\\ & -k_{{\rm v}i}V_i+b_{{\rm v}i}+\varDelta_{{\rm v}i},\end{aligned} \tag{32}$$

where

$$\begin{cases}k_{{\rm v}i}=\min\{2c_i^2,(\dfrac{2}{\iota_i}-\dfrac{1}{\gamma_{\varrho_{i+1}}^2}-2\zeta^2),\dfrac{\sigma_i^2}{\lambda_{\max}(\varGamma_i^{-1})}\},\\ b_{{\rm v}i}=\dfrac{1}{4\eta_i^2}(\bar{\varepsilon}_{w_i}+\bar{\varepsilon}_{f_i}+\bar{h}_i)^2+\dfrac{\sigma_i^2}{2}\|W_i\|^2+ \dfrac{1}{4\zeta^2}\gamma_{\varrho_{i+1}}^2\bar{f}_{\varrho_{i+1}}^2,\\ \varDelta_{{\rm v}i}=-\dfrac{1}{2}s_i^2+\dfrac{1}{2}s_{i+1}^2.\end{cases} \tag{33}$$

Step n For the nth subsystem, by Assumption 2,

$$\dot{s}_n=\dot{\xi}_n-\dot{z}_n+\frac{\rm d}{{\rm d}t}(g_1(x)\int_{-\tau}^0u(t+v){\rm d}v)= k_n(x_1-\xi_1)+g_1(x)u(t-\tau)-g_1(x)u(t-\tau)+ \sum_{j=1}^{n-1}\frac{\partial g_1}{\partial x_j}\dot{x}_j\int_{-\tau}^0u(t+v){\rm d}v+g_1(x)u(t)-\dot{z}_n. \tag{34}$$

By equation (2), we have

$$\begin{aligned}& s_n\sum_{j=1}^{n-1}\frac{\partial g_1}{\partial x_j}\dot{x}_j\int_{-\tau}^0u(t+v){\rm d}v=\\ & s_n\sum_{j=1}^{n-1}\frac{\partial g_1}{\partial x_j}\int_{-\tau}^0u(t+v){\rm d}v\cdot[x_{j+1}+\phi_j(x)\theta+ \varepsilon_{f_j}(x)+h_j(x,t)]\leqslant\\ & s_n\sum_{j=1}^{n-1}\frac{\partial g_1}{\partial x_j}\int_{-\tau}^0u(t+v){\rm d}v\cdot[x_{j+1}+\phi_j(x)\theta]+ \gamma^2s_n^2\sum_{j=1}^{n-1}(\frac{\partial g_1}{\partial x_j}\int_{-\tau}^0u(t+v){\rm d}v)^2+ \frac{1}{4\gamma^2}\sum_{j=1}^{n-1}(\bar{\varepsilon}_{f_j}+\bar{h}_j)^2,\end{aligned} \tag{35}$$

where γ is the design parameter in equation (11). Differentiating V_n (22) along the track (34) and substituting equations (15)–(16), and (35) into it, we can have

$$\begin{aligned}\dot{V}_n=\ & s_n[k_n(x_1-\xi_1)+\sum_{j=1}^{n-1}\frac{\partial g_1}{\partial x_j}\dot{x}_j\int_{-\tau}^0u(t+v){\rm d}v- \dot{z}_n+g_1(x)u(t)]-\tilde{W}_n^{\rm T}(S_n(\lambda_n)s_n-\sigma_n^2\hat{W}_n)\leqslant\\ & -c_n^2s_n^2-0.5s_n^2-\frac{\sigma_n^2}{2}\|\tilde{W}_n\|^2+s_n(-\alpha_n+g_1u)+ [\frac{\bar{\varepsilon}_{w_n}^2}{4\eta_n^2}+\frac{1}{4\gamma^2}\sum_{j=1}^{n-1}(\bar{\varepsilon}_{f_j}+\bar{h}_j)^2+\frac{\sigma_n^2}{2}\|W_n\|^2]\leqslant\end{aligned}$$

$$-k_{\mathrm{v}n}V_n + b_{\mathrm{v}n} + \varDelta_{\mathrm{v}n}, \tag{36}$$

where

$$\begin{cases} k_{\mathrm{v}n} = \min\{2c_n^2, \dfrac{\sigma_n^2}{\lambda_{\max}(\varGamma_n^{-1})}\}, \\ b_{\mathrm{v}n} = \dfrac{\bar{\varepsilon}_{w_n}^2}{4\eta_n^2} + \dfrac{1}{4\gamma^2}\sum\limits_{j=1}^{n-1}(\bar{\varepsilon}_{f_j} + \bar{h}_j)^2 + \dfrac{\sigma_n^2}{2}\|W_n\|^2, \\ \varDelta_{\mathrm{v}n} = -\dfrac{1}{2}s_n^2. \end{cases} \tag{37}$$

Therefore, $0 \leqslant V_n(t) \leqslant (V_{n0} - \delta_{\mathrm{v}n})\mathrm{e}^{-k_{\mathrm{v}n}t} + \delta_{\mathrm{v}n}$, $\delta_{\mathrm{v}n} = \dfrac{b_{\mathrm{v}n}}{k_{\mathrm{v}n}}$, $V_{n0} = V_n(0)$. Define $\bar{V}_n = \max\{V_{n0}, \delta_{\mathrm{v}n}\}$.

$$V_n \leqslant \bar{V}_n, \ \lim_{t\to\infty} V_n = \delta_{\mathrm{v}n}. \tag{38}$$

Therefore,

$$\begin{cases} |s_n| \leqslant \sqrt{2\bar{V}_n}, \ \|\tilde{W}_n\| \leqslant \dfrac{\sqrt{2\bar{V}_n}}{\sqrt{\lambda_{\min}(\varGamma_n^{-1})}}, \\ \|\hat{W}_n\| \leqslant \dfrac{\sqrt{2\bar{V}_n}}{\sqrt{\lambda_{\min}(\varGamma_n^{-1})}} + \|W_n\|. \end{cases} \tag{39}$$

By the property of Gaussian basis function, we have that

$$S_{ij}(\lambda_i) \leqslant 1, \ \|S_i(\lambda_i)\| \leqslant \sqrt{m_i}, \ 1 \leqslant i \leqslant n. \tag{40}$$

By Assumption 4, virtual control definition (13) and controller definition (15), we get

$$|u(t)| = |\frac{1}{g_1(x)}(-c_n^2 s_n - \hat{W}_n^{\mathrm{T}} S_n(\lambda_n))| \leqslant$$
$$\frac{1}{g_0}[c_n^2\sqrt{2\bar{V}_n} + (\frac{\sqrt{2\bar{V}_n}}{\sqrt{\lambda_{\min}(\varGamma_n^{-1})}} + \|W_n\|)\sqrt{m_n}] \leqslant \bar{u}, \tag{41}$$

where

$$\bar{u} = \frac{1}{g_0}[c_n^2\sqrt{2\bar{V}_n} + (\frac{\sqrt{2\bar{V}_n}}{\sqrt{\lambda_{\min}(\varGamma_n^{-1})}} + \|W_n\|)\sqrt{m_n}]. \tag{42}$$

Repeating the backstepping procedure, we can prove that for ith subsystem $(1 \leqslant i \leqslant n-1)$ the following result holds

$$V_i \leqslant \bar{V}_i, \ \lim_{t\to\infty} V_i = \delta_{\mathrm{v}i}, \tag{43}$$

$$|s_i| \leqslant \sqrt{2\bar{V}_i}, \ |\varrho_i| \leqslant \frac{\sqrt{2\bar{V}_i}}{\gamma_{\varrho_i}},$$

$$\|\tilde{W}_i\| \leqslant \frac{\sqrt{2\bar{V}_i}}{\sqrt{\lambda_{\min}(\varGamma_i^{-1})}}, \tag{44}$$

$$\|\hat{W}_i\| \leqslant \frac{\sqrt{2\bar{V}_i}}{\sqrt{\lambda_{\min}(\varGamma_i^{-1})}} + \|W_i\|, \tag{45}$$

$$|\alpha_i| \leqslant c_i^2\sqrt{2\bar{V}_i} + (\frac{\sqrt{2\bar{V}_i}}{\sqrt{\lambda_{\min}(\varGamma_i^{-1})}} + \|W_i\|)m_i^{\frac{1}{2}}, \tag{46}$$

$$|z_i| \leqslant |\varrho_i| + |\alpha_{i-1}| \leqslant \frac{\sqrt{2\bar{V}_i}}{\gamma_{\varrho_i}} + \bar{\alpha}_{i-1}, \tag{47}$$

$$|\dot{z}_i| = |-\iota_i^{-1}\varrho_i| \leqslant \frac{\sqrt{2\bar{V}_i}}{\gamma_{\varrho_i}\iota_i}, \tag{48}$$

$$|x_i| \leqslant |s_i| + |z_i| \leqslant$$
$$\sqrt{2\bar{V}_i} + \frac{\sqrt{2\bar{V}_i}}{\gamma_{\varrho_i}} + \bar{\alpha}_{i-1}, \ 1 \leqslant i \leqslant n, \tag{49}$$

where

$$\bar{\alpha}_i = c_i^2\sqrt{2\bar{V}_i} + (\frac{\sqrt{2\bar{V}_i}}{\sqrt{\lambda_{\min}(\varGamma_i^{-1})}} + \|W_i\|)m_i^{\frac{1}{2}}. \tag{50}$$

Step $n+1$ For the overall system, differentiating the Lyapunov function V, we have

$$\dot{V} = \sum_{i=1}^{n}\dot{V}_i(t) \leqslant -k_{\mathrm{v}}V(t) + b_{\mathrm{v}}, \tag{51}$$

where

$$k_{\mathrm{v}} = \min\{k_{\mathrm{v}1}, \cdots, k_{\mathrm{v}n}\}, \ b_{\mathrm{v}} = \sum_{i=1}^{n} b_{\mathrm{v}i}. \tag{52}$$

Therefore, $0 \leqslant V(t) \leqslant (V_0 - \delta_{\mathrm{v}})\mathrm{e}^{-k_{\mathrm{v}}t} + \delta_{\mathrm{v}}$, $\delta_{\mathrm{v}} = \dfrac{b_{\mathrm{v}}}{k_{\mathrm{v}}}$, $V_0 = V(0)$. Define $\bar{V} = \max\{V_0, \delta_{\mathrm{v}}\}$. Therefore, $\|s\| \leqslant \sqrt{2\bar{V}}$, $\|\hat{W}_i\| \leqslant \dfrac{\sqrt{2\bar{V}_i}}{\sqrt{\lambda_{\min}(\varGamma_i^{-1})}} + \|W_i\|$. Thus s, $\hat{W}_i$, α_i are bounded, respectively. It can be proved that x, $\varOmega$, ξ, e are all bounded. The closed-loop system is SGUUB. Because

$$V(t) \leqslant \bar{V}, \ \lim_{t\to\infty} V(t) = \delta_{\mathrm{v}}, \tag{53}$$

we get

$$|y - y_{\mathrm{r}}| \leqslant \sqrt{2\bar{V}}, \tag{54}$$

$$\lim_{t\to\infty}|y - y_{\mathrm{r}}| \leqslant \sqrt{2\delta_{\mathrm{v}}} = \sqrt{\frac{2}{k_{\mathrm{v}}}}\sqrt{b_{\mathrm{v}}} = k^*\delta^*, \tag{55}$$

where

$$k^* = \sqrt{\frac{2}{k_{\mathrm{v}}}}, \ \delta^* = \sqrt{b_{\mathrm{v}}}. \tag{56}$$

It means that the tracking error is bounded and ultimately converges to an adequately small compact set which can be adjusted by the design parameter k^*. The proof of Theorem 1 is completed.

Remark 1 In this remark, we demonstrate that the system state is in a compact set in detail. By Inequality (54), we have

$$|y| \leqslant \sqrt{2\bar{V}} + \bar{y}_{\mathrm{r}} \triangleq \bar{y}. \tag{57}$$

Because $\bar{x}_{n-1}$ is proved to be bounded, according to the property of continuous functions, it is held that

$$|g_1(\bar{x}_{n-1})| \leqslant \bar{g}_1, \tag{58}$$

where $\bar{g}_1$ is an unknown constant. By Assumption 2 and expression (8), it gives

$$g(x) \leqslant \bar{g}_1 + \delta_g, \tag{59}$$

$$\|\varDelta_u\| \leqslant (2\bar{g}_1 + \delta_g)\bar{u} \triangleq \bar{\varDelta}_u. \tag{60}$$

By equations (4)(7), we have

$$\xi(t) = \mathrm{e}^{A_0 t}\xi(0) + \int_0^t \mathrm{e}^{A_0(t-s)}(Ky(s) + Bg_1(x(s))u(s-\tau))\mathrm{d}s, \tag{61}$$

$$\varOmega(t) = \mathrm{e}^{A_0 t}\varOmega(0) + \int_0^t \mathrm{e}^{A_0(t-s)}\phi(y(s))\mathrm{d}s, \tag{62}$$

$$e(t) = \mathrm{e}^{A_0 t}e(0) + \int_0^t \mathrm{e}^{A_0(t-s)}[B\varDelta_u(s) + \varepsilon_f(x(s)) + h(x(s), s)]\mathrm{d}s. \tag{63}$$

Therefore,

$$\|\xi(t)\| \leqslant$$
$$\|\mathrm{e}^{A_0 t}\|\|\xi(0)\| + (\bar{y}\sqrt{\sum_{j=1}^{n} k_j^2} + \bar{g}_1\bar{u})\|\int_0^t \mathrm{e}^{A_0(t-s)}\mathrm{d}s\| \leqslant$$

$$\sqrt{n}\|P\|\|P^{-1}\|\|\xi(0)\| + (\bar{y}\sqrt{\sum_{j=1}^{n} k_j^2} + \bar{g}_1\bar{u})\sqrt{n}\|A_0^{-1}\|(\|P\|\|P^{-1}\|+1) \triangleq \bar{\xi}, \tag{64}$$

$$\|\Omega(t)\| \leqslant \|\mathrm{e}^{A_0 t}\|\|\Omega(0)\| + \sqrt{nq}\|\int_0^t \mathrm{e}^{A_0(t-s)}\mathrm{d}s\| \leqslant \sqrt{n}\|P\|\|P^{-1}\|\|\Omega(0)\| + n\sqrt{q}\|A_0^{-1}\|(\|P\|\|P^{-1}\|+1) \triangleq \bar{\Omega}, \tag{65}$$

$$\|e(t)\| \leqslant \|\mathrm{e}^{A_0 t}\|\|e(0)\| + (\bar{\Delta}_u + \sqrt{n}\bar{\varepsilon}_f + \sqrt{\sum_{j=1}^{n}\bar{h}_j^2})\|\int_0^t \mathrm{e}^{A_0(t-s)}\mathrm{d}s\| \leqslant \sqrt{n}\|P\|\|P^{-1}\|\|e(0)\| + (\bar{\Delta}_u + \sqrt{n}\bar{\varepsilon}_f + \sqrt{\sum_{j=1}^{n}\bar{h}_j^2})\sqrt{n}\|A_0^{-1}\| \times (\|P\|\|P^{-1}\|+1) \triangleq \bar{e}, \tag{66}$$

where the following results are used:

$$P^{-1}A_0P = \Lambda,\ \Lambda = \mathrm{diag}\{-\Lambda_1, \cdots, -\Lambda_n\},\ \Lambda_i > 0,$$
$$\|\mathrm{e}^{A_0 t}\| \leqslant \sqrt{n}\|P\|\|P^{-1}\|,$$
$$\int_0^t \mathrm{e}^{A_0(t-s)}\mathrm{d}s = -A_0^{-1}\mathrm{e}^{A_0(t-s)}\ |_0^t = A_0^{-1}(\mathrm{e}^{A_0 t} - I),$$
$$\|\int_0^t \mathrm{e}^{A_0(t-s)}\mathrm{d}s\| \leqslant \|A_0^{-1}(\mathrm{e}^{A_0 t} - I)\| \leqslant \|A_0^{-1}\|\sqrt{n}(\|P\|\|P^{-1}\|+1). \tag{67}$$

Finally, by equations (4)(6), we get

$$x = \xi + \Omega\theta + e. \tag{68}$$

By the above results, it shows that the system state $x(t)(t > 0)$ stays in a compact set C_x as

$$C_x = \{x \mid \|x\| \leqslant \bar{\xi} + \bar{\Omega}\|\theta\| + \bar{e}\}. \tag{69}$$

According to the property of continuous functions, we have

$$\exists \bar{p}_g,\ \text{s.t.}\ |\frac{\partial g_1}{\partial x_i}| \leqslant \bar{p}_g,\ 1 \leqslant i \leqslant n,\ 1 \leqslant j \leqslant n. \tag{70}$$

By equation (41), we get

$$|\int_{-\tau}^{0} u(t+v)\mathrm{d}v| \leqslant \tau\bar{u}. \tag{71}$$

The neural network input parameter $\lambda_i(1 \leqslant i \leqslant n)$ in equations (14) stays in the compact set C_{λ_i} as

$$C_{\lambda_i} = \{\lambda_i \mid |y_\mathrm{r}| \leqslant \bar{y}_\mathrm{r},\ |\dot{y}_\mathrm{r}| \leqslant \bar{y}_\mathrm{r}, |z_i| \leqslant \frac{\sqrt{2\bar{V}_i}}{\gamma_{\varrho_i}} + \bar{\alpha}_{i-1},\ |\dot{z}_i| \leqslant \frac{\sqrt{2\bar{V}_i}}{\gamma_{\varrho_i}\iota_i}, \|x\| \leqslant \max\{\|x(0)\|,\ \bar{\xi} + \bar{\Omega}\|\theta\| + \bar{e}\}, \|\xi\| \leqslant \bar{\xi},\ |\frac{\partial g_1}{\partial x_j}| \leqslant \bar{p}_g,\ |\int_{-\tau}^{0} u(t+v)\mathrm{d}v| \leqslant \tau\bar{u}, 1 \leqslant j \leqslant i\}. \tag{72}$$

Remark 2 By equation (1), x and λ_i are in the compact set, respectively, at the initial time ($t = 0$). By Theorem 1 and Remark 1, x and λ_i are always in the compact set, respectively. Because x is in a compact set, the neural network approximation for unknown function $f_i(\bar{x}_i)$ is valid with approximation error ε_{f_i} bounded. Because λ_i is in a compact set, the neural network approximation for unknown function $q_i(\lambda_i)$ is valid with approximation error $\varepsilon_{w_i}(\lambda_i)$ bounded. Similar discussions can be found in the reference [25].

Remark 3 Although there are many items in the residual b_v, it can be arbitrarily small, if we choose proper design parameters, such as η_i, σ_i, etc. According to expressions (64)–(66) and (69), it is shown that the range of the compact set C_x is related to the design parameters such as k^*. Proper parameters lead to a small range of the compact set.

Remark 4 The introduction of filter and virtual observer has two advantages: 1) Combined with the novel variable definition, it converts the input delayed system to the non-delayed system. 2) It makes the controller design simple, without regarding the unknown items of the delay and the control gain function.

Remark 5 Although the states are measurable, but the derivation $\dot{x}_n$ is not available, since the function $f_n(x)$ and control gain $g(x)$ are all unknown (1). Therefore, we introduce a filter (3) and a virtual observer (4) in order to simplify the proof. From the definition of the filter (3), it implies that both ξ and $\dot{\xi}_n$ are available. From the definition of s_n in equation (10), we can get that by replacing x_n with ξ_n, both s_n and derivation $\dot{s}_n$ are available. In equation (34), this leads to the elimination of the item $g_1(x)u(t-\tau)$ in the proof. The result makes a great contribution to the solution of the problem.

4 Simulation example

Consider the nonlinear system with uncertain input delay as follows:

$$\begin{cases} \dot{x}_1 = 0.1x_1^3 + x_2 + 0.1\sin^2 x_2, \\ \dot{x}_2 = 0.3x_1x_2 + 0.2\sin(0.2t) + (2+ \sin x_1 + 0.05\sin x_2)u(t-\tau(t)), \\ y = x_1, \end{cases} \tag{73}$$

$\tau(t) = 0.3 + 0.1\sin t$, $\tau = 0.3$, $f_1(x_1) = 0.1x_2^3$, $f_2(x) = 0.3x_1x_2$, $g(x) = 2 + \sin x_1 + 0.05\sin x_2$, $g_1(x_1) = 2+\sin x_1$, $h_1(x,t) = 0.1\sin^2 x_1$, $h_2(x,t) = 0.2\sin(0.2t)$, $y_\mathrm{r}(t) = \sin(0.2t)\sin(0.5t)$, $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

Choose parameters:

$$k = [3\ \ 2]^\mathrm{T},\ A_0 = \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix},\ a_1 = 1.2,\ a_2 = 1.3,$$
$$b_1 = 0.1,\ b_2 = 0.1,\ \gamma_1 = 1.2,\ \gamma_2 = 1.2,$$
$$\sigma_1 = 0.05,\ \sigma_2 = 0.05.$$

Construct a virtual observer $\hat{x}$ for system (73) as

$$\begin{cases} \dot{\xi} = A_0\xi + Ky + Bg_1(x_1)u(t-0.3), \\ \dot{\Omega} = A_0\Omega + \phi(x), \\ \hat{x} = \xi + \Omega\theta, \end{cases}$$

where

$$\Omega = \begin{bmatrix} \Omega_1 \\ \Omega_2 \end{bmatrix},\ \Omega_i \in \mathbb{R}^{1\times 5},\ \xi = \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix}.$$

Define a one-order filter as

$$\iota_2\dot{z}_2 = -z_2 + \alpha_1, \tag{74}$$

where ι_2 is a positive design parameter. The virtual controls α_1 is defined in the following. Define dynamic surfaces as

$$\begin{cases} s_1 = x_1 - y_\mathrm{r}, \\ s_2 = \xi_2 - z_2 + g_1(x)\int_{-0.3}^{0} u(t+v)\mathrm{d}v. \end{cases} \tag{75}$$

Define the virtual controls

$$\alpha_i = -c_i^2 s_i - \hat{W}_i^\mathrm{T} S_i(\lambda_i),\ 1 \leqslant i \leqslant 2, \tag{76}$$

where c_i are design parameters,

$$\begin{cases} \lambda_1 = [x_1 \ \ y_\mathrm{r} \ \ \dot{y}_\mathrm{r}]^\mathrm{T}, \\ \lambda_2 = [x_1 \ x_2 \ z_2 \ \dot{z}_2 \ \xi_1 \ \xi_2 \ \dfrac{\partial g_1}{\partial x_1} \ \displaystyle\int_{-0.3}^{0} u(t+v)\mathrm{d}v]^\mathrm{T}. \end{cases} \tag{77}$$

Define assistant functions

$$\begin{cases} q_1(\lambda_1) = \phi_1(x_1)\theta - \dot{y}_\mathrm{r} + (\eta_1^2+1)s_1, \\ q_2(\lambda_n) = k_2(x_1-\xi_1) - \dot{z}_2 + (\eta_2^2+0.5)s_2 + \\ \qquad \dfrac{\partial g_1}{\partial x_1}\displaystyle\int_{-0.3}^{0} u(t+v)\mathrm{d}v[x_2+\phi_1(x)\theta] + \\ \qquad \gamma^2\Big(\dfrac{\partial g_1}{\partial x_1}\displaystyle\int_{-0.3}^{0} u(t+v)\mathrm{d}v\Big)^2 s_2. \end{cases} \tag{78}$$

Select control law as $u(t) = -g_1^{-1}(x)\alpha_2$ and adaptive law as

$$\dot{\hat{W}}_i = \Gamma_i(S_i(\lambda_i)s_i - \sigma_i^2\hat{W}_i), \ 1 \leqslant i \leqslant 2, \tag{79}$$

where

$$S_1 \in \mathbb{R}^{1\times 27}, \ S_2 \in \mathbb{R}^{1\times 6561},$$

$$S_{1,i} = \mathrm{e}^{-\sum\limits_{j=1}^{3}(\lambda_{1,j}-\mu_{i,j})^2}, \ 1 \leqslant i \leqslant 27,$$

$$S_{2,i} = \mathrm{e}^{-\sum\limits_{j=1}^{8}(\lambda_{2,j}-\nu_{i,j})^2}, \ 1 \leqslant i \leqslant 6561,$$

$$\mu_{i,j} \in \{-1,0,1\}, \ \nu_{i,j} \in \{-1,0,1\}.$$

Select initial values

$$\hat{W}_1(0) = 0, \ \hat{W}_2(0) = 0, \ \xi(0) = 0,$$

$$x(t) = [2\cos t \ \ -\cos^3 t]^\mathrm{T}, \ t \in [-0.3, 0].$$

The results of the control scheme is shown in Figs.1–4.

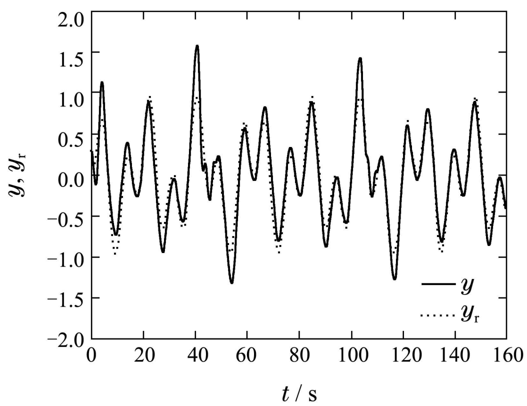

Fig. 1 Output $y(t)$ and reference signal $y_\mathrm{r}(t)$

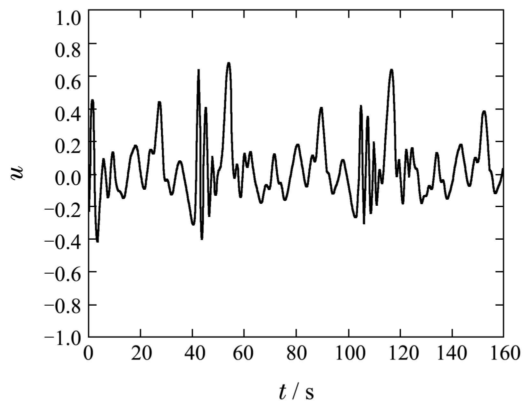

Fig. 2 Control input $u(t)$

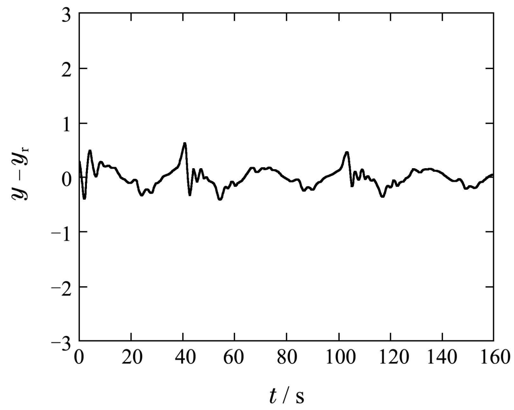

Fig. 3 Tracking error $e(t)$

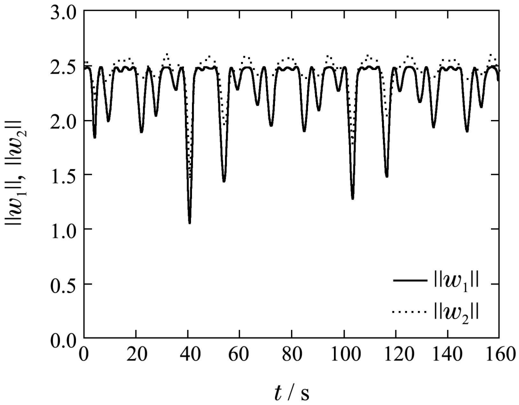

Fig. 4 Norm of the neural network weight

5 Conclusion

In this paper, an adaptive dynamic surface control scheme is proposed for the tracking control problem in order to improve the traditional backstepping approach. Unknown continuous functions are approximated by neural networks online. By introducing a novel dynamic surface variable including a integral term of control input, the input delay is eliminated in the last subsystem. The SGUUB of closed-loop system is guaranteed by Lyapunov approach. Finally, the simulation results demonstrate the effectiveness of the scheme.

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