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离散区间线性切换正系统的镇定

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摘要: 这里同时考虑了快切换和慢切换两类切换情形, 即任意切换和平均驻留时间切换情形. 在这两种情形下, 均给出了系统状态反馈镇定控制器的存在条件, 所设计的控制器能够在原系统不必为正的情况下保证相应的闭环系统对于所有给定的区间不确定性不但为正而且渐近稳定. 由于所获得的控制器存在条件均被描述为了线性不等式的形式, 因此通过求解简单的线性规划问题很容易确定出所期望的控制器参数. 最后, 通过数值算例说明了本文所提出的镇定控制器设计方法的有效性.

关键词: 区间线性切换正系统; 镇定; 快切换; 慢切换; 线性规划
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Stabilization for discrete-time interval switched positive linear systems

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Abstract: In our investigation, both fast switching and slow switching—the arbitrary switching and the average dwell-time switching are considered, and the original system is not necessarily to be positive. In each switching case, we derive a sufficient condition for the existence of a set of state-feedback controllers guaranteeing the positivity and stability of the closed-loop system for all admissible interval uncertainties. All the obtained sufficient conditions are formulated in terms of linear inequalities, thus the desired controller parameters can be conveniently determined by solving simple linear programming problems. A numerical example is given to illustrate the effectiveness and the potential of the proposed approach.

Key words: interval switched positive linear systems; stabilization; fast switching; slow switching; linear programming

1 Introduction

As a typical class of hybrid systems, switched systems have received considerable attention in the past decades. By a switched system, it is usually composed of a family of subsystems and a rule orchestrating the switching between them[1]. The motivation for researching such systems comes from many facts, for example, there are a lot of practical systems operating in different modes with the variation of work environment[2], and in the control of some complex systems, a single controller cannot guarantee several conflicting performance requirements[3]. Owing to the multiplicity of switching rules, a switched system does not generally inherit the dynamics of all its subsystems, and the basic stability problem of such systems is not trivial at all[4–5]. To date, the stability issue of switched systems primarily concentrates on two topics, the stability under arbitrary switching and the stability under constrained switching[6]. The former topic is studied mainly based on common Lyapunov function approach[7], while multiple Lyapunov function approach[8] is proven to be more flexible and efficient in studying the latter topic. In the category of constrained switching, a concept of average dwell time (ADT) was proposed to characterize a class of typical restricted switching signals in [9], as an extension to the dwell time[10], and there an ADT-based approach for stability analysis of switched systems was also derived. This approach actually belongs to the multiple Lyapunov function methodology, and its peculiarity lies in that the used functions are allowed to increase to a certain bound at switching times whereas the stability is guaranteed at the expense of on average sufficiently slow switching. The arbitrary and ADT switchings are also viewed as fast and slow switchings, respectively.

Recently, switched positive linear systems (SPLSs) have been highlighted by several articles showing their applications in network congestion control[11], formation flying[12], and viral infection treatment[13]. A positive system means that any trajectory of the system starting from nonnegative states remains nonnegative for all nonnegative inputs[14]. The mathematical theory of such systems can be traced back to nonnegative matrix theory[15], which takes the famous Perron-Frobenius Theorem as its fundament. Since positive systems are defined on nonnegative cones rather than linear state spaces, some new problems are encountered in studying such systems, to name a few, all the poles of a controllable positive system cannot be placed arbitrarily due to the sign pattern constraint on its system matrix, and the conventionally required Lyapunov conditions are too conservative in proving the stability of positive systems. So far, it has been revealed that the existence

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relations between a linear copositive Lyapunov function, a quadratic Lyapunov function, and a diagonal quadratic Lyapunov function are equivalent for a stable positive linear system. And some useful results on stability analysis and control synthesis of positive systems have been obtained by means of quadratic and linear copositive Lyapunov functions in [16–18] and [19–21], respectively. Although the main properties of both switched systems and positive systems have been well understood, some basic problems on SPLSs remain open. In this regard, there are several papers reporting the stability of SPLSs under arbitrary switching based on common linear copositive Lyapunov function method, see, e.g., [22–26].

On the other hand, it is well known that uncertainties always exist in many practical systems due to some unpredictable factors, such as the aging of equipments, the changes of operating conditions, and the inabilities in measurements, so it is of great significance to research the stabilization problem of uncertain systems. Note that the stabilization issues of uncertain switched system[27–29] and uncertain positive systems[30–32] have been seriously studied in the literature. However, to the best of authors' knowledge, there are few results concerning the stabilization of uncertain SPLSs, which motivates the research of this paper.

This paper investigates the stabilization problem of discrete-time SPLSs with interval uncertainties in both fast and slow switching cases. In each case, a sufficient condition for the existence of a set of state-feedback controllers guaranteeing the closed-loop positivity and stability under corresponding switching for all admissible uncertainties is derived. By fully exploiting the positivity nature of the closed-loop system, all the sufficient conditions are formulated in terms of linear inequalities rather than the conventional LMIs, and hence the desired controller gains can be conveniently determined by solving simple linear programming problems. The reminder of the paper is organized as follows. Section 2 introduces some necessary definitions and lemmas and formulates the problem to be treated. In Section 3, the stabilization problem of the underlying system is studied in fast and slow switching cases, respectively. Section 4 gives a numerical example to illustrate the validity of the developed controller design methods. Finally, the conclusion to the paper is drawn in Section 5.

Notation The superscript 'T' stands for matrix (vector) transposition and the inverse of matrix A^{T} is abbreviated as $A^{-\mathrm{T}}$. $\mathbb{R}^n$ represents the n-dimensional Euclidean space and for $x = [x_i] \in \mathbb{R}^n$, the notations $x \succeq 0$ $(x \succ 0)$ and $x \preceq 0$ $(x \prec 0)$ mean that $x_i \geqslant 0$ $(x_i > 0)$ and $x_i \leqslant 0$ $(x_i < 0)$ for $1 \leqslant i \leqslant n$, respectively, where x_i denotes the ith component of x. $\mathbb{R}^{n\times m}$ represents the space of $n \times m$ real matrices and for $A = [a_{ij}] \in \mathbb{R}^{n\times m}$, the notations $A \succeq 0$ $(A \succ 0)$ and $A \preceq 0$ $(A \prec 0)$ mean that $a_{ij} \geqslant 0$ $(a_{ij} > 0)$ and $a_{ij} \leqslant 0$ $(a_{ij} < 0)$ for $1 \leqslant i \leqslant n$, $1 \leqslant j \leqslant m$, respectively, where a_{ij} denotes the element in the (i, j) position of A. The notation $A \in [\check{A}, \hat{A}]$ means that $\check{A} \preceq A \preceq \hat{A}$. A real square matrix A is called Metzler if all of its off-diagonal entries are nonnegative, i.e., $a_{ij} \geqslant 0$, $i \neq j$; and it is said to be Hurwitz if the real part of any eigenvalue of A is less than zero. A continuous function $\alpha : [0, a) \to [0, \infty)$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$; if α is also unbounded, then it is said to be of class $\mathcal{K}_\infty$. Also a continuous function $\beta : [0, \infty) \times [0, \infty) \to [0, \infty)$ is said to belong to class $\mathcal{KL}$ if $\beta(\cdot, t)$ is of class $\mathcal{K}$ for each fixed $t \geqslant 0$ and $\beta(r, t)$ is decreasing to zero as $t \to \infty$ for each fixed $r \geqslant 0$. $\mathbb{N}$ $(\mathbb{N}_0)$ denotes the set of all positive (nonnegative) integers, $\mathbb{R}^n_+$ $(\mathbb{R}^n_{0,+})$ represents the set of all n-dimensional positive (nonnegative) vectors, and $\mathcal{N}$ corresponds to the set $\{1, 2, \cdots, n\}$.

2 Problem formulation and preliminaries

Consider a class of discrete-time switched linear systems given by

$$\begin{cases} x(k+1) = A_{\sigma(k)}x(k) + B_{\sigma(k)}u(k), \\ x(0) = x_0,\ k \in \mathbb{N}, \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state vector; $u(t) \in \mathbb{R}^m$ is the control input; σ is a piecewise constant function, called a switching signal, which takes its values in the finite set $\mathcal{P} = \{1, \cdots, N\}$; $N > 1$ is the number of subsystems; $A_p = [a^p_{ij}] \in \mathbb{R}^{n\times n}$ and $B_p = [b^p_{ij}] \in \mathbb{R}^{n\times m}$ are the system matrices of the pth subsystem. The switching signal $\sigma(k)$ can be either autonomous or controlled, and it may be dependent on k or $x(k)$, or both, or be generated by other logics. For a switching time sequence $0 \leqslant k_0 < k_1 < k_2 < \cdots$, when $k \in [k_i, k_{i+1})$, we say that the $\sigma(k_i)$th subsystem is active, and hence the trajectory of the system (1) is just the trajectory of the $\sigma(t_i)$th subsystem.

Definition 1[26] System (1) is said to be positive if, for any initial condition $x_0 \succeq 0$, any input $u(k) \succeq 0$, and any switching signal $\sigma(k)$, the corresponding trajectory $x(k) \succeq 0$ holds for all $k \in \mathbb{N}$.

Definition 2[1] The unforced system (1) $(u(k) = 0)$ is globally asymptotically stable (GAS) if there exists a class $\mathcal{KL}$ function β such that for a class of switching signals σ, the solution to the system with any finite initial condition x_0 satisfies $||x(k)|| \leqslant \beta(||x_0||, k)$ for all $k \in \mathbb{N}$.

Lemma 1[26] The system (1) is positive if and only if $A_p \succeq 0$ and $B_p \succeq 0$, $\forall p \in \mathcal{P}$.

Lemma 2[26] The unforced positive system (1) is GAS for all switching signals if and only if there exists a vector $d \in \mathbb{R}^n_+$ such that $(A_p - I)d \prec 0$, $\forall p \in \mathcal{P}$.

For the purpose of this paper, the definition of ADT is recalled as follows to characterize a typical class of constrained switching signals.

Definition 3[9] For a switching signal σ and any times $t_2 > t_1 > t_0$, let $N_\sigma(t_1, t_2)$ be the switching numbers of $\sigma(t)$ in the open interval (t_1, t_2). If $N_\sigma(t_1, t_2) \leqslant N_0 + (t_2 - t_1)/\tau_{\mathrm{a}}$ holds for given $N_0 > 0$ and $\tau_{\mathrm{a}} > 0$, then the constants τ_{a} and N_0 are called an average dwell time and a chatter bound, respectively.

Remark 1 Definition 3 means that if there exists a switching signal possessing the ADT property, with ADT τ_{a} and chatter bound N_0, then the average time between consecutive switches is at least $\tau_{\rm a}$ and the number of switches in any interval smaller than $\tau_{\rm a}$ is smaller than N_0. Therefore, a basic problem for switched systems under such switching is to determine the minimal $\tau_{\rm a}$ and the corresponding switching signals such that the system is stable. It has been analyzed in [1] that the ADT degenerates into the dwell time for $N_0 = 1$ and it corresponds to the arbitrary switching for $\tau_{\rm a} = 0$.

Lemma 3[2] Consider the switched system $x_{k+1} = f_{\sigma(k)}(x_k)$, $\sigma(k) \in \mathcal{P}$ and let $0 < \gamma < 1$, $\mu > 1$ be given constants. Suppose that there exist continuously differentiable functions $V_{\sigma(k)} : \mathbb{R}^n \to \mathbb{R}$, $\sigma(k) \in \mathcal{P}$, and two class $\mathcal{K}_\infty$ functions β_1 and β_2 such that

$$\forall\sigma(k) = p \in \mathcal{P},\ \beta_1(|x_k|) \leqslant V_p(x_k) \leqslant \beta_2(|x_k|),$$
$$\Delta V_p(x_k) \triangleq V_p(x_{k+1}) - V_p(x_k) \leqslant -\gamma V_p(x_k),$$
$$\forall(\sigma(k_l){=}p,\ \sigma(k_{l-1}){=}q){\in}\mathcal{P}{\times}\mathcal{P},\ V_p(x_{k_l}){\leqslant}\mu V_q(x_{k_l}),$$

then the system is GAS for any switching signal with ADT

$$\tau_{\rm a} > \tau_{\rm a}^* = -\frac{\ln\mu}{\ln(1-\alpha)}.$$

Lemma 4[19] A Metzler matrix A is Hurwitz if and only if there exists a vector $d \in \mathbb{R}^n_+$ such that $Ad \prec 0$.

Lemma 5[15] Let A and B be Metzler matrices and assume that A is Hurwitz and $B \preceq A$. Then, the following statements hold:

i) B is a Hurwitz matrix;

ii) $A^{-1} \preceq B^{-1} \preceq 0$;

iii) All the diagonal entries of A are negative.

This paper considers a class of discrete-time interval switched linear systems described by (1), where $A_p \in [\check{A}_p, \hat{A}_p]$ and $B_p \in [\check{B}_p, \hat{B}_p]$, with $\check{A}_p = [\check{a}^p_{ij}]$, $\hat{A}_p = [\hat{a}^p_{ij}]$, $\check{B}_p = [\check{b}^p_{ij}]$, and $\hat{B}_{pu} = [\hat{b}^p_{ij}]$. Our main objective is to design a set of state-feedback controllers

$$u(k) = G_{\sigma(k)}x(k), \tag{2}$$

such that the resultant closed-loop system given by

$$x(k) = A_{{\rm b}\sigma(k)}x(k),\ x(0) = x_0, \tag{3}$$

is not only positive but also GAS under the given interval uncertainties and the corresponding switching signals, where

$$A_{{\rm b}\sigma(k)} = A_{\sigma(k)} + B_{\sigma(k)}G_{\sigma(k)}. \tag{4}$$

3 Main results

In this section, the stabilization problem of the interval switched linear system (1) with positivity constraint is first studied in the fast switching case. Then, the corresponding results are extended to the slow switching case.

3.1 Fast switching case

We start with the following lemma.

Lemma 6 The unforced interval switched linear system (1) is positive and GAS for all switching signals if the matrices $\check{A}_p$ are nonnegative for all $p \in \mathcal{P}$ and there exists a vector $d \in \mathbb{R}^n_+$ such that

$$(\hat{A}_p - I)d \prec 0,\ \forall p \in \mathcal{P}. \tag{5}$$

Proof Since $\check{A}_p \preceq A_p \preceq \hat{A}_p$, it is easily known that $A_p \succeq 0$ and $(A_p - I)d \preceq (\hat{A}_p - I)d$. In view of (5), we obtain $(A_p - I)d \prec 0$. Hence, the unforced interval system (1) is positive and GAS under arbitrary switching according to Lemmas 1 and 2, which completes the proof.

Next, the main result in the fast switching case is obtained in the following theorem.

Theorem 1 Consider the interval switched linear system (1). If there exist a diagonal matrix $D = \mathrm{diag}\{d_1, d_2, \cdots, d_n\} \succ 0$ and matrices $Z_p = [z^p_{ij}] \in \mathbb{R}^{m\times n}$ satisfying $\forall p \in \mathcal{P}$,

$$\check{a}^p_{ij}d_j + \sum_{k=1}^{m}\min(\check{b}^p_{ik}z^p_{kj}, \hat{b}^p_{ik}z^p_{kj}) \geqslant 0,\ \forall 1 \leqslant i, j \leqslant n, \tag{6}$$

$$\sum_{j=1}^{n}\hat{a}^p_{ij}d_j + \sum_{j=1}^{n}\sum_{k=1}^{m}\max(\check{b}^p_{ik}z^p_{kj}, \hat{b}^p_{ik}z^p_{kj}) < d_i,\ \forall 1 \leqslant i \leqslant n, \tag{7}$$

then there exists a set of controllers (2) such that the closed-loop system (3) is not only positive but also GAS for all switching signals. Moreover, if the above conditions hold, an admissible controller is given by

$$G_p = Z_pD^{-1}. \tag{8}$$

Proof Taking account of (4) and letting $A_{{\rm b}p} = [a^p_{{\rm b}ij}]$, we have

$$a^p_{{\rm b}ij} = a^p_{ij} + \sum_{k=1}^{m} b^p_{ik}g^p_{kj} \geqslant \check{a}^p_{ij} + \sum_{k=1}^{m}\min(\check{b}^p_{ik}g^p_{kj}, \hat{b}^p_{ik}g^p_{kj}). \tag{9}$$

Due to the fact that $d^p_j > 0$ for $1 \leqslant j \leqslant n$, the inequality (9) is equivalent to

$$a^p_{{\rm b}ij}d_j \geqslant \check{a}^p_{ij}d_j + \sum_{k=1}^{m}\min(\check{b}^p_{ik}g^p_{kj}d_j, \hat{b}^p_{ik}g^p_{kj}d_j). \tag{10}$$

Since the equality (8) implies $z^p_{ij} = g^p_{ij}d_j$, we can rewrite (10) as

$$a^p_{{\rm b}ij}d_j \geqslant \check{a}^p_{ij}d_j + \sum_{k=1}^{m}\min(\check{b}^p_{ik}z^p_{ij}, \hat{b}^p_{ik}z^p_{ij}). \tag{11}$$

In virtue of (6), one gets from (11) that $a^p_{{\rm b}ij}d_j \geqslant 0$, which implies that $a^p_{{\rm b}ij} \geqslant 0$, i.e., $A_{{\rm b}p} \succeq 0$. Thus, the closed-loop system (3) is positive.

Next, let us analyze the stability of the positive system (3). Defining $d = [d_1\ \ d_2\ \ \cdots\ \ d_n]^{\rm T}$, one obtains that

$$\begin{aligned}[A_{{\rm b}p}d]_i = &\sum_{j=1}^{n}[(a^p_{ij} + \sum_{k=1}^{m} b^p_{ik}g^p_{kj})d_j] = \\ &\sum_{j=1}^{n} a^p_{ij}d_j + \sum_{j=1}^{n}\sum_{k=1}^{m} b^p_{ik}g^p_{kj}d_j = \\ &\sum_{j=1}^{n} a^p_{ij}d_j + \sum_{j=1}^{n}\sum_{k=1}^{m} b^p_{ik}z^p_{kj} \leqslant \\ &\sum_{j=1}^{n} \hat{a}^p_{ij}d_j + \sum_{j=1}^{n}\sum_{k=1}^{m} \max(\check{b}^p_{ik}z^p_{kj}, \hat{b}^p_{ik}z^p_{kj}).\end{aligned} \tag{12}$$

Therefore, it follows from (7) and (12) that $(A_{{\rm b}p} - I)d_p \prec 0$. Thus, by Lemma 6, we conclude that the positive closed-loop system is GAS for all switching signals. This completes the proof.

3.2 Slow switching case

The main result in the slow switching case is derived in this subsection, beginning with the following lemma.

Lemma 7 Consider the unforced interval switched linear system (1) and let a constant $0 < \gamma < 1$ be given. If the matrices $\check{A}_p$ are nonnegative for all $p \in \mathcal{P}$ and there exist vectors $\lambda_p = [\lambda_1^p\ \ \lambda_2^p\ \ \cdots\ \ \lambda_n^p]^{\mathrm{T}} \in \mathbb{R}_+^n$ such that

$$\lambda_p^{\mathrm{T}}(\hat{A}_p - \gamma I) \preceq 0,\ \forall p \in \mathcal{P}, \tag{13}$$

then the system is positive and GAS for any switching signal with ADT $\tau_{\mathrm{a}} > \tau_{\mathrm{a}}^* = -\ln\mu/\ln\gamma$, where

$$\mu = \max_{(p,q,k)\in\mathcal{P}\times\mathcal{P}\times\mathcal{N}} \frac{\lambda_k^p}{\lambda_k^q}. \tag{14}$$

Proof It is obvious that the system (1) is positive in virtue of the fact $\check{A}_p \succeq 0$ for all $p \in \mathcal{P}$.

Choose the multiple Lyapunov function candidates as $V_p(x(k)) = \lambda_p^{\mathrm{T}} x(k)$. Then, computing the derivative of V_p along the pth subsystem of the system (1) and applying (13) yields

$$\begin{aligned}&\Delta V_p(x(k)) = \lambda_p^{\mathrm{T}} x(k+1) - \lambda_p^{\mathrm{T}} x(k) = \\ &\lambda_p^{\mathrm{T}}(A_p - I)x(k) \preceq \lambda_p^{\mathrm{T}}(\hat{A}_p - I)x(k) \preceq \\ &-(1-\gamma)V(x(k)).\end{aligned} \tag{15}$$

Furthermore, if we select the norm of vector $x \in \mathbb{R}^n$ as $\|x\| = \sum\limits_{i=1}^{n}|x_i|$ and let $\bar{\beta}_1 = \min\limits_{(p,k)\in\mathcal{P}\times\mathcal{N}} \lambda_k^p$, $\bar{\beta}_2 = \max\limits_{(p,k)\in\mathcal{P}\times\mathcal{N}} \lambda_k^p$, $\mu = \max\limits_{(p,q,k)\in\mathcal{P}\times\mathcal{P}\times\mathcal{N}} \lambda_k^p/\lambda_k^q$, then it gives

$$\begin{aligned}&\bar{\beta}_1\|x(k)\| \leqslant V_p(x(k)) \leqslant \bar{\beta}_2\|x(k)\|, \\ &\forall x \in \mathbb{R}_{0,+}^n,\ p \in \mathcal{P},\end{aligned} \tag{16}$$

$$\begin{aligned}&V_p(x(k)) \leqslant \mu V_q(x(k)), \\ &\forall x \in \mathbb{R}_{0,+}^n,\ (p,q) \in \mathcal{P}\times\mathcal{P},\ p \neq q.\end{aligned} \tag{17}$$

Therefore, taking account of (15), (16) and (17) and according to Lemma 3, it is concluded that the positive system (1) is GAS for any switching signal with ADT $\tau_{\mathrm{a}} > \tau_{\mathrm{a}}^* = -\ln\mu/\ln\gamma$ if μ is given by (14). So, the proposed lemma is proved.

Theorem 2 Consider the interval switched linear system (1) and let a scalar $0 < \gamma < 1$ be given. If there exist a diagonal matrices $D_p = \mathrm{diag}\{d_1^p, d_2^p, \cdots, d_n^p\} \succ 0$ and matrices $Z_p = [z_{ij}^p] \in \mathbb{R}^{m\times n}$ satisfying $\forall p \in \mathcal{P}$,

$$\begin{aligned}&\check{\pi}_{ij}^p \triangleq \check{a}_{ij}^p d_j^p + \sum_{k=1}^{m}\min(\check{b}_{ik}^p z_{kj}^p, \hat{b}_{ik}^p z_{kj}^p) \geqslant 0, \\ &\qquad \forall 1 \leqslant i, j \leqslant n,\end{aligned} \tag{18}$$

$$\begin{aligned}&\sum_{j=1}^{n}\hat{\pi}_{ij}^p \triangleq \sum_{j=1}^{n}\hat{a}_{ij}^p d_j^p + \sum_{j=1}^{n}\sum_{k=1}^{m}\max(\check{b}_{ik}^p z_{kj}^p, \hat{b}_{ik}^p z_{kj}^p) < \gamma d_i^p, \\ &\qquad \forall 1 \leqslant i \leqslant n,\end{aligned} \tag{19}$$

then there exists a set of controllers (2) such that the closed-loop system (3) is not only positive but also GAS for any switching signal with ADT $\tau_{\mathrm{a}} > \tau_{\mathrm{a}}^* = -\ln\mu/\ln\gamma$, where

$$\mu = \max\nolimits_{(p,q,i)\in\mathcal{P}\times\mathcal{P}\times\mathcal{N}} \frac{\sum\limits_{k=1}^{m} d_k^p \hat{h}_{ki}^{-p} \check{h}_{kk}^p}{\sum\limits_{k=1}^{m}\sum\limits_{j=1,j\neq k}^{n} d_k^q \hat{h}_{ki}^{-q} \check{h}_{kj}^q}, \tag{20}$$

with $\check{h}_{kk}^p = \check{\pi}_{kk}^p - \gamma d_k^p$, $[\hat{h}_{ki}^{-p}] = [\hat{h}_{ki}^p]^{-1}$, and

$$\hat{h}_{ki}^p = \begin{cases}\hat{\pi}_{ki}^p, & \text{if } i \neq k, \\ \hat{\pi}_{kk}^p - \gamma d_k^p, & \text{if } i = k.\end{cases}$$

Moreover, if the above conditions hold, an admissible controller is given by

$$G_p = Z_p D_p^{-1}. \tag{21}$$

Proof In a similar vein to the proof of Theorem 1, letting $d_p = [d_1^p\ \ d_2^p\ \ \cdots\ \ d_n^p]^{\mathrm{T}}$, the inequalities in (18) and (19) are respectively equivalent to

$$A_p + B_p G_p \succeq 0, \tag{22}$$

$$(A_p + B_p G_p - \gamma I)d_p \prec 0. \tag{23}$$

Thus, in view of (4), we get from (22) and (23) that $A_{\mathrm{b}p}$ are nonnegative matrices and $A_{\mathrm{b}p} - \gamma I$ are Hurwitz-Metzler matrices for all $p \in \mathcal{P}$ by Lemma 4. So, it follows from Lemma 5 that $(A_{\mathrm{b}p} - \gamma I)^{-1} \preceq 0$, and hence we conclude that there exists a vector $\lambda_p = (A_{\mathrm{b}p} - \gamma I)^{-\mathrm{T}}(A_{\mathrm{b}p} - \gamma I)d_p \succ 0$ such that $\lambda_p^{\mathrm{T}}(A_{\mathrm{b}p} - \gamma I) \prec 0$. Next, by invoking (21) and defining $H_p = [h_{ij}^p] = A_p D_p + B_p Z_p - \gamma D_p$, one has

$$\begin{aligned}\lambda_p = {} & (A_p + B_p G_p - \gamma I)^{-\mathrm{T}}(A_p + B_p G_p - \gamma I)d_p = \\ & H_p^{-\mathrm{T}} D_p H_p D_p^{-1} d_p,\end{aligned} \tag{24}$$

and h_{ij}^p is calculated as

$$h_{ij}^p = \begin{cases} a_{ij}^p d_j^p + \sum\limits_{k=1}^{m} b_{ik}^p z_{kj}^p, & \text{if } j \neq i, \\ a_{ii}^p d_i^p + \sum\limits_{k=1}^{m} b_{ik}^p z_{ki}^p - \gamma d_i^p, & \text{if } j = i.\end{cases} \tag{25}$$

Taking into account the fact that $A_p = [a_{ij}^p] \in [\check{A}_p, \hat{A}_p]$, $B_p = [b_{ij}^p] \in [\check{B}_p, \hat{B}_p]$ and the definitions of $\check{\pi}_{ij}^p$ and $\hat{\pi}_{ij}^p$, we obtain from (25) that

$$[\check{h}_{ij}^p] = \check{H}_p \preceq H_p \preceq \hat{H}_p = [\hat{h}_{ij}^p], \tag{26}$$

with

$$\check{h}_{ij}^p = \begin{cases}\check{\pi}_{ij}^p, & \text{if } j \neq i, \\ \check{\pi}_{ii}^p - \gamma d_i^p, & \text{if } j = i,\end{cases} \quad \hat{h}_{ij}^p = \begin{cases}\hat{\pi}_{ij}^p, & \text{if } j \neq i, \\ \hat{\pi}_{ii}^p - \gamma d_i^p, & \text{if } j = i.\end{cases}$$

Moreover, it follows from (18) and (19) that $\check{h}_{ij}^p = \check{\pi}_{ij}^p \geqslant 0$, $\forall j \neq i$, $\sum\limits_{j=1}^{n}\hat{h}_{ij}^p = \sum\limits_{j=1}^{n}\hat{\pi}_{ij}^p - \gamma d_i^p < 0$, $\forall 1 \leqslant i \leqslant n$, which imply that $\check{H}_p$, H_p, and $\hat{H}_p$ are Hurwitz-Metzler matrices for all $p \in \mathcal{P}$ by using Geršgorin's Theorem and Lemma 5. Therefore, together with (26), one has

$$[\hat{h}_{ij}^{-p}] = \hat{H}_p^{-1} \preceq H_p^{-1} = [h_{ij}^{-p}] \preceq \check{H}_p^{-1} = [\check{h}_{ij}^{-p}] \preceq 0. \tag{27}$$

Let $\lambda_p = [\lambda_1^p\ \ \lambda_2^p\ \ \cdots\ \ \lambda_n^p]^{\mathrm{T}}$. Then, (24) leads to

$$\begin{aligned}&\lambda_i^p = \sum_{j=1}^{n}\sum_{k=1}^{m} d_k^p h_{ki}^{-p} h_{kj}^p = \\ &\sum_{k=1}^{m} d_k^p h_{ki}^{-p} h_{kk}^p + \sum_{k=1}^{m}\sum_{j=1,j\neq k}^{n} d_k^p h_{ki}^{-p} h_{kj}^p.\end{aligned} \tag{28}$$

From Lemma 5, it is also known that $h_{kk}^p < 0$ for $k \in \mathcal{N}$ since H_p are Hurwitz-Metzler matrices for all $p \in \mathcal{P}$. Thus, combining (27) and (28), we get

$$\begin{aligned}&\lambda_i^p \leqslant \sum_{k=1}^{m} d_k^p \hat{h}_{ki}^{-p} \check{h}_{kk}^p + \sum_{k=1}^{m}\sum_{j=1,j\neq k}^{n} d_k^p \check{h}_{ki}^{-p} \check{h}_{kj}^p \leqslant \\ &\sum_{k=1}^{m} d_k^p \hat{h}_{ki}^{-p} \check{h}_{kk}^p,\end{aligned}$$

$$\lambda_i^p \geqslant \sum_{k=1}^{m} d_k^p \check{h}_{ki}^{-p} \hat{h}_{kk}^p + \sum_{k=1}^{m} \sum_{j=1,j\neq k}^{n} d_k^p \hat{h}_{ki}^{-p} \hat{h}_{kj}^p \geqslant \sum_{k=1}^{m} \sum_{j=1,j\neq k}^{n} d_k^p \hat{h}_{ki}^{-p} \hat{h}_{kj}^p.$$

So, we conclude that the positive closed-loop system (3) is GAS for any switching signal with ADT satisfying $\tau_{\rm a} > \tau_{\rm a}^* = -\ln\mu/\ln\gamma$ and μ given by (20) according to Lemma 7. This completes the proof.

4 Numerical example

In this section, a numerical example is presented to illustrate the validity of the developed approaches.

Example 1 Consider the discrete-time interval switched linear system (1) consisting of two subsystems described by

$$\left[\begin{array}{c|c}\check{A}_1 & \check{B}_1\\ \hline \hat{A}_1 & \hat{B}_1\end{array}\right] = \left[\begin{array}{cc|cc} 0.85 & 0.64 & 0.32 & 0.20\\ -0.42 & 0.95 & -0.35 & -0.40\\ \hline 0.87 & 0.76 & 0.34 & 0.23\\ -0.40 & 1.00 & -0.32 & -0.36\end{array}\right],$$

$$\left[\begin{array}{c|c}\check{A}_2 & \check{B}_2\\ \hline \hat{A}_2 & \hat{B}_2\end{array}\right] = \left[\begin{array}{cc|cc} -0.82 & 0.50 & 0.24 & -0.50\\ 0.44 & -0.68 & -0.40 & 0.25\\ \hline -0.80 & 0.54 & 0.25 & -0.48\\ 0.45 & -0.66 & -0.37 & 0.28\end{array}\right].$$

It can be verified that all of the matrices $\check{A}_1$, $\check{A}_2$, $\hat{A}_1$, and $\hat{A}_2$ are neither nonnegative nor stable. The objective here is to design a set of state-feedback controllers (2) such that the closed-loop system (3) is not only positive but also asymptotically stable under fast and slow switchings, respectively.

By applying Theorem 1, the inequalities (6) and (7) are feasible and we obtain a family of solutions as $D = \mathrm{diag}\{5.0076, 3.7011\}$ and

$$Z_1 = \begin{bmatrix} -8.7360 & -5.7066\\ -1.9959 & 10.3198\end{bmatrix},\ Z_2 = \begin{bmatrix} -1.8302 & -8.7427\\ -9.7538 & -1.7067\end{bmatrix}.$$

Then, according to (8) and (4), we have

$$G_1 = \begin{bmatrix} -1.7446 & -1.5419\\ -0.3986 & 2.7883\end{bmatrix},\ G_2 = \begin{bmatrix} -0.3655 & -2.3622\\ -1.9478 & -0.4611\end{bmatrix},$$

$$\check{A}_{\rm b1} = \begin{bmatrix} 0.1652 & 0.6734\\ 0.2817 & 0.3281\end{bmatrix},\ \check{A}_{\rm b2} = \begin{bmatrix} 0.0236 & 0.1308\\ 0.0298 & 0.0649\end{bmatrix},$$

$$\hat{A}_{\rm b1} = \begin{bmatrix} 0.2320 & 0.9079\\ 0.3700 & 0.5359\end{bmatrix},\ \hat{A}_{\rm b2} = \begin{bmatrix} 0.0862 & 0.2036\\ 0.1092 & 0.1696\end{bmatrix}.$$

By applying Theorem 2, the inequalities (18) and (19) are feasible when choosing $\gamma = 0.6$, and we obtain a family of solutions as

$$D_1 = \mathrm{diag}\{3.3409, 1.2069\}, D_2 = \mathrm{diag}\{3.7477, 3.8392\},$$

$$Z_1 = \begin{bmatrix} -9.3385 & -2.4564\\ 3.9506 & 4.7639\end{bmatrix},\ Z_2 = \begin{bmatrix} -2.5192 & -8.4009\\ -8.3960 & -0.9473\end{bmatrix}.$$

Hence, it follows from (21) and (4) that

$$G_1 = \begin{bmatrix} -2.7952 & -2.0353\\ 1.1825 & 3.9473\end{bmatrix},\ G_2 = \begin{bmatrix} -0.6722 & -2.1882\\ -2.2403 & -0.2468\end{bmatrix},$$

$$\check{A}_{\rm b1} = \begin{bmatrix} 0.1361 & 0.7374\\ 0.0015 & 0.0224\end{bmatrix},\ \check{A}_{\rm b2} = \begin{bmatrix} 0.0873 & 0.0714\\ 0.0614 & 0.0606\end{bmatrix},$$

$$\hat{A}_{\rm b1} = \begin{bmatrix} 0.2475 & 1.0166\\ 0.1526 & 0.2913\end{bmatrix},\ \hat{A}_{\rm b2} = \begin{bmatrix} 0.1588 & 0.1382\\ 0.1588 & 0.1536\end{bmatrix}.$$

According to equality (20), we can also get the minima ADT $\tau_{\rm a}^* = 4.8958$.

It can be seen that the closed-loop system is positive in each switching case. Choosing the initial condition $x_0 = [5.5\ \ 9.0]^{\rm T}$ and the system matrices $A_1 = (\check{A}_1 + \hat{A}_1)/2 + (\check{A}_1 - \hat{A}_1)/2\cos(2k)$, $B_1 = (\check{B}_1 + \hat{B}_1)/2 + (\check{B}_1 - \hat{B}_1)/2\cos(2k)$, $A_2 = (\check{A}_2 + \hat{A}_2)/2 + (\check{A}_2 - \hat{A}_2)/2\cos(2k)$, $B_1 = (\check{B}_2 + \hat{B}_2)/2 + (\check{B}_2 - \hat{B}_2)/2\cos(2k)$, we can obtain the state response of the closed-loop system under a randomly generated switching signal $\sigma(k)$, as shown in Fig. 1. Also, the state response of the closed-loop system under a randomly generated switching signal $\sigma(k)$ with ADT $\tau_{\rm a} = 5$ and $N_0 = 2$ is given in Fig. 2. It is clear that the closed-loop system is asymptotically stable under both fast and slow switchings.

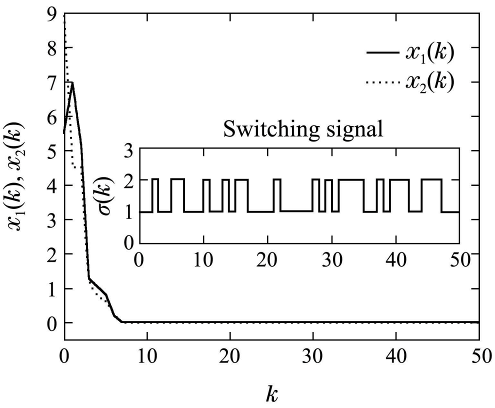

Fig. 1 Closed-loop state response under fast switching

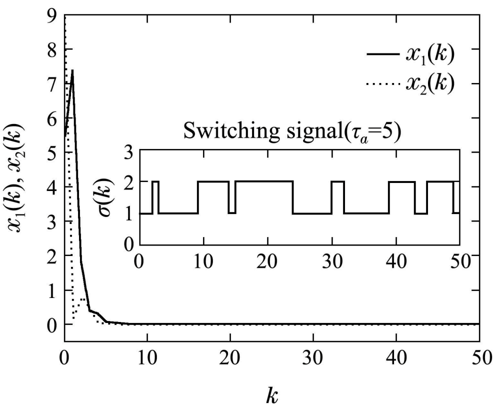

Fig. 2 Closed-loop state response under slow switching

5 Conclusion

In this paper, the stabilization issue of discrete-time switched positive linear systems with interval uncertainties is addressed. Both fast and slow switching cases, i.e., the arbitrary and ADT switching cases, are considered. In each case, a sufficient controller existence condition is obtained. And the developed state-feedback controller can guarantee that the closed-loop system is not only positive but also asymptotically stable for all admissible interval uncertainties without the positivity requirement to the underlying system. All the derived sufficient conditions are formulated as linear inequalities, and hence the desired controller parameters can be easily obtained by solving the corresponding linear programming problems. Finally, an illustrative example is given to show the validity of the theoretical results. It is expected that the achieved results will be extended to the underlying system with more general polytopic uncertainties in the future.

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