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基于卡尔曼滤波的四旋翼飞行器姿态估计和控制算法研究

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摘要: 四旋翼飞行器作为无人机的一种, 由于其简单气动布局和复杂的动力学模型, 在控制领域获得了越来越多的学术关注；本文首先分析了微机电系统惯性测量单元(MEMS IMU)传感器的误差, 给出了基于自回归(auto-regressive, AR)噪声模型的卡尔曼滤波算法设计; 然后根据加速度计和陀螺仪长短周期测量的不同特性, 进一步对姿态数据做互补融合, 实验表明此算法可以实现良好的滤波效果; 基于上面的姿态估计, 本文又提出了一种双增益的PD控制算法对飞行器进行姿态控制; 最后将姿态估计算法和控制算法应用到实验平台中, 可以实现四旋翼在支架上的自主悬停等功能.

关键词: 四旋翼飞行器; 卡尔曼滤波; 姿态估计; 自回归(auto-regressive, AR)模型; 双增益PD控制器; 悬停控制
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Quadrotor aircraft attitude estimation and control based on Kalman filter

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Abstract: The quadrotor, as one type of unmanned aircraft vehicles, has gained increasing interests in the control community, partially due to its simple aerodynamics and complex dynamics. In this work, a quadrotor system has been constructed with commercial off-the-shelf products. The sensors of inertial measurement unit are micro-electro-mechanical system, whose errors can be analyzed in an auto regressive model. A new attitude estimation scheme based on Kalman filter is proposed, which conducts separate data fusion tasks in both short and long cycle. The proposed attitude sensing method has been validated using the experimental system. In addition, a double-gain proportional differential controller has been designed to regulate the attitude dynamics. A satisfactory control performance has been achieved in some test cases.

Key words: quadrotor aircraft; Kalman filter; attitude estimation; auto regressive model; double-gain PD controller; hover control

1 Introduction

In recent years, due to the technological advances in low cost micro-electro-mechanical system (MEMS) sensor, micro-controller unit (MCU) and brushless direct current (BLDC) motor, the architecture of quadrotor has gained great development. Quadrotor has become an increasingly significant research platform, partially due to its simple aerodynamics and complex dynamics. They have been employed from purely scientific research on civilian tasks to military[1], such as rescue in complex environments, inspection of civil engineering and exploration of disaster areas[2]. The relatively simple structure and ‘vertical takeoff and landing’ (VTOL) feature make them much more advantageous among unmanned aircraft vehicles (UAVs).

The quadrotor, as one nonlinear, multi-variable and strong coupling system, has six outputs and four inputs. Since the number of independent inputs is smaller than the number of degrees of freedom, this mini-aircraft belongs to the class of underactuated mechanical systems[3]. The outputs include three positions (roll, pitch, and yaw) and three attitudes (x, y and z), the inputs include four propeller thrusts. In order to achieve the target of automatic flight, it is necessary to design a closed loop negative feedback controller to hold attitude and position stable. Because the aircraft attitude and position keep the direct coupling relationship, only precise attitude control can make the whole attitude and position achieve pretty good control effect. In this work, we only concern ourselves with attitude control.

The attitude can be observed by numerical integration of angular velocity from MEMS gyroscopes or by three axis MEMS accelerometer (ACC). However, MEMS inertial measurement unit (IMU) sensors are mixed with large amount of system noise, drift errors and vibrations induced by the rotation of the rotors. Moreover, the attitude and position errors would increase over time in sample period. All these lead to great difficulties in quadrotor control and

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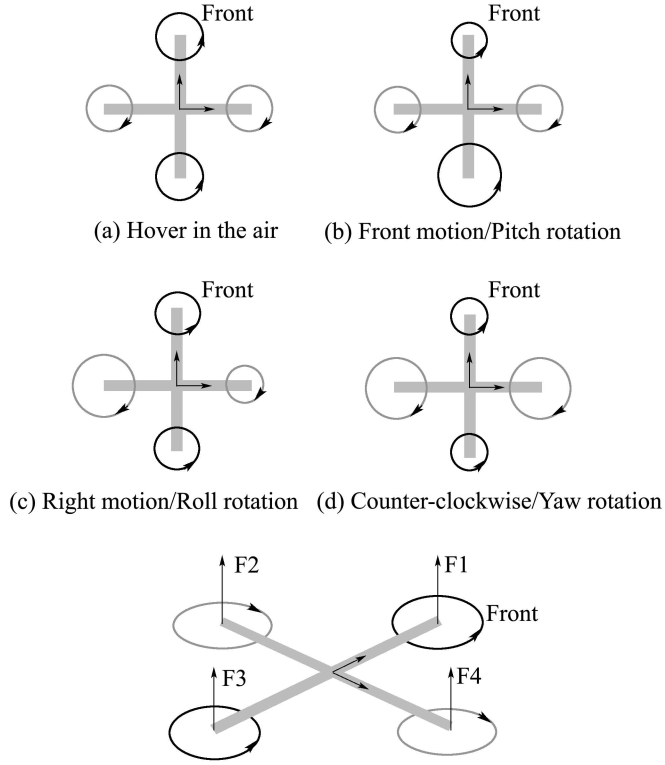
attitude estimation.

In this paper, an attitude estimation schedule based on Kalman filter and a double-gain PD controller are presented. First of all, we theoretically analyze the noise of MEMS IMU sensors, depict separately random process characteristics: random dirft, angular random walk and rate random walk, etc. On this basis, the Kalman filter model is built, and the raw signals of MEMS IMU are processed with the model. For higher estimation accuracy, we carry out further data fusion and compensation. As we known, the gyroscope measurement is more accuracy for short-period while the accelerometer measurement is more accuracy for long-period[4]. In our experiment, both 1 ms for short-period and 400 ms for long-period are set up to real-time fusing of post-filtered data. Next, a double-gain PD feedback controller is designed to stabilize quadrotor attitude. Although the non-linear control law can bring about a very good simulation result, its effect usually is not as good as PID control when the mathematical model of quadrotor can't be built and simplified effectively. Finally, we design a quadrotor experiment platform to check the estimation and controller result, the quadrotor can acheive much acurate practical estimation results, and can hover and VTOL on test-bench.

1.1 Principles of quadrotor dynamics

Generally, quadrotor layout mainly consists of a rigid cross airframe as illustrated in Fig.1, which is propelled by two pairs horizontal rotors that are attached to the frame end. For the purpose of balancing the whole helicopter spinning moments, two group propellors are equipped in opposite direction, one pair (front and back) turning clockwise and the other (left and right) with counter-clockwise. By changing the velocity of four propellers, we can acquire different attitude and position of the quadrotor.

Fig.1(a) describes the quadrotor hovering state. To achieve so, all propellors must attain the same spinning speed. With speed increasing, when lift from the four propellors is equal to quadrotor gravity, the quadrocopter can hover steadily in the air. To be able to fly forward/backward as shown in Fig.1(b), the back-propellor/front-propellor has to turn faster while the corresponding opposite propellor has to become slower, which we call 'Pitch'. The same principle also applies to the right/left motions as shown in Fig.1(c), and we call this 'Roll'. Fig.1(d) describes counter-clockwise yaw rotation, the front and back propellors will turn faster and the left and right propellors will slow down a little. Similarly, clockwise yaw rotation is by the same principle.



(a) Hover in the air (b) Front motion/Pitch rotation

(c) Right motion/Roll rotation (d) Counter-clockwise/Yaw rotation

(e) Side view of quadrotor lift

Fig. 1 Quadrotor airframe

2 Attitude estimation schedule with Kalman filter

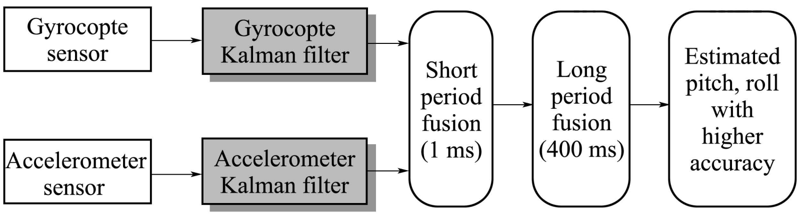


Fig. 2 Attitude estimation architecture based on Kalman filter

In practice, low-cost MEMS inertial sensors are often mixed with amounts of system noise and drift errors, and also, attitude and position errors also increate over time during sampling period. Real-time achieving accurate attitude (pitch, roll and yaw) is very important stage before applying control algorithm. To reduce the noise and integral errors, the estimation scheme based on Kalman filter is brought forward. Fig.2 describes the scheme architecture of attitude estimation and fusion based on Kalman filter, the following sections will analyze exhaustively the components in the figure, respectively.

2.1 System noise

According to the noise feature of MEMS, the noise error sources can be categorized into two groups, deterministic part and stochastic part[5]. Deterministic noise mainly include constant drift and vibration error, which can be real-time compensated and calibrated easily. Stochastic noise is not compensated easily because of its random feature. For MEMS IMU, gyroscope and accelerometer have different stochastic composition. The MEMS gyroscope noise typically consists of the following terms[5]: random dirft (bias instability): this is a stationary stochastic process which may be considered as a low-order zero-mean Gauss-Markov process; angular random walk (ARW): this is an angular error process which is due to white noise in angular rate; rate random walk (RRW): this is a rate error due to white noise in angular acceleration; Quantization error: this is an error representing the quantization noise. Similarly, the MEMS accelerometer are composed by the following items[5]: random dirft, velocity random walk, acceleration random walk, quantization error. For MEMS gyroscope:

$$\mathrm{GyroNoise} = x + (\mathrm{Bias} + \mathrm{ARW} + \mathrm{RRW} + \mathrm{Qe}). \tag{1}$$

Here, x is the deterministic noise; 'Bias' is bias instability; 'Qe' is Quantization error.

For above stochastic noises, their process and measurement variances can be analysized with 'Allen variance analysis method'[6]. Because of the noises statistical independence, the total variance can be described by the following expression:

$$\sigma^2_{\mathrm{all}} = \sigma^2_{\mathrm{bias}} + \sigma^2_{\mathrm{ARW}} + \sigma^2_{\mathrm{RRW}} + \sigma^2_{\mathrm{Qe}}. \tag{2}$$

Here, σ^2_{bias}, σ^2_{ARW}, σ^2_{RRW} and σ^2_{Qe} are variance of bias instability, ARW, RRW and Qe, respectively.

2.2 Auto-regressive modeling of random drift noise for IMU

Among all of random noises of MEMS IMU, random drift (bias instability) is a main factor affecting accuracy of IMU. It is necessary to build a random drift noise model. There are many ways used in modeling the drift signal: time series analysis, power spectral density analysis (PSD) and Allen variance analysis. In this work, we use auto-regressive (AR) model to analyse random drift, which is certified very effective to handling the colored feature of the whole system noise[7].

A scalar AR process of p-order is given by

$$x(n) = \sum_{k=1}^{p} a_k x(n-k) + e(n), \tag{3}$$

where $e(n)$ is assumed to be a sequence of independent and normal distributed random variables with zero expectation and a variance of σ_n^2[7]. This variable can be interpreted as the uncertainty of the prediction of the next signal value by regressing the previous observations with the AR coefficients[8]. AR(1) is more suitable and effective for most MEMS IMU, which is shown as following:

$$x_n = -a x_{n-1} + \varepsilon_n. \tag{4}$$

Here, $\varepsilon_n \sim N(0, \sigma_n^2)$.

2.3 Kalman filter design based on AR modeling

The Kalman filter is not only an efficient autoregressive filter but also an optimal recursive mathematical processing method, which can predict and estimate the current system state under a series of incomplete and Gaussian noisy measurements for linear dynamic systems. The discrete-time Kalman filter model is defined as follows[9]:

The state model:

$$x_k = F_{k-1} x_{k-1} + B_{k-1} u_{k-1} + \Gamma_{k-1} w_{k-1}. \tag{5}$$

The observation model:

$$z_k = H_k x_k + v_k. \tag{6}$$

Here, x_k is called the true state value of the system; z_k is observation value at time k; u_{k-1} is the control vector; F_{k-1} is the state transition matrix which is applied to the previous state x_{k-1}; B_{k-1} is the control-input matrix which is applied to the control vector u_{k-1}; H_k is the observation matrix which maps the true state space into the observed space; w_{k-1} is the process noise which is assumed to be zero mean Gaussian white noises with covariances $Q_k, w_k \sim N(0, Q_k)$; v_k is the observation noise which is assumed to be zero mean Gaussian noises with covariances $R_k, v_k \sim N(0, R_k)$; the Kalman filter is most often conceptualized as two distinct phases: 'Predict' and 'Update' as shown in Fig.3. The predict phase uses a state estimate from previous timestep to produce a state estimate at the current timestep. In the update phase, current prediction is combined with current information to refine state estimate. This improved estimate is termed the a posteriori state estimate[10].

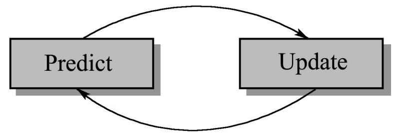


Fig. 3 'Predict' and 'Update' of Kalman filter

Predict phase:

$$\hat{x}_{k|k-1} = F_k \hat{x}_k^- + B_k u_{k-1}, \tag{7}$$

$$P_{k|k-1} = F_k P_{k-1|k-1} F_k^{\mathrm{T}} + Q_k. \tag{8}$$

Update phase:

$$K_k = P_{k|k-1} H_k^{\mathrm{T}} (H_k P_{k|k-1} H_k^{\mathrm{T}} + R_k)^-, \tag{9}$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k (z_k - H_k \hat{x}_{k|k-1}), \tag{10}$$

$$P_{k|k} = (I - K_k H_k) P_{k|k-1}. \tag{11}$$

The Kalman filter requires that process and measurement noises must be zero mean, constant variance and be unrelated to other white noise process. However, both process noise and measurement noise often are colored in reality. Before the Kalman filter estimates, colored noise must be processed to be suitable for Kalman filter mathematical model conditions. If the process noise is colored and the measurement noise be white, a method often used is augmenting the state vector, which is to make w_k a part of state vector[7]. Assume that the process noise w_k have the relation as follows:

$$w_k = L w_{k-1} + \xi_{k-1}. \tag{12}$$

Here ξ_k is zero mean white noise Gaussian process, L is the parameter of the AR model for random drift of gyroscope.

Augment the colored noise to state vector, and then, the state vector, process equation and measurement equation will be shown in [7, 9]:

The state model:

$$X_k = \bar{\Gamma}_{k-1} X_{k-1} + \bar{B}_{k-1} U_{k-1} + \bar{\Gamma}_{k-1} W_{k-1}. \quad (13)$$

The observation model:

$$z_k = \begin{bmatrix} H_k & 0 \end{bmatrix} X_k + v_k. \quad (14)$$

Here,

$$\bar{F}_{k-1} = \begin{bmatrix} F_{k-1} & \Gamma_{k-1} \\ 0 & L \end{bmatrix},\ X_k = \begin{bmatrix} x_k \\ w_k \end{bmatrix},\ \bar{B}_{k-1} = \begin{bmatrix} B_{k-1} \\ 0 \end{bmatrix},$$

$$U_{k-1} = [u_{k-1}],\ \bar{\Gamma}_{k-1} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ W_{k-1} = [\xi_{k-1}].$$

Through above analysis, Kalman filter state models with gyroscope and accelerometer can be given as following:

Gyroscope equation:

$$X_k = \begin{bmatrix} 1 & -\mathrm{d}t \\ 0 & 1 \end{bmatrix} X_{k-1} + \begin{bmatrix} \mathrm{d}t \\ 0 \end{bmatrix} U_{k-1} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} W_{k-1}, \quad (15)$$

$$z_k = [1\ \ 0] X_k + v_k. \quad (16)$$

Here,

$$X_k = \begin{bmatrix} \mathrm{PredictAngle} \\ \mathrm{RandomDrift} \end{bmatrix} = \begin{bmatrix} \theta \\ b_\theta \end{bmatrix},\ U_{k-1} = [\mathrm{AngularRate}],$$

W_{k-1} is process noise, $\mathrm{d}t$ is the sampling period, $z_k = [\mathrm{MessageAngle}]$, v_k is measurement noise.

Fig.4 shows the transform principle of accelerometer.

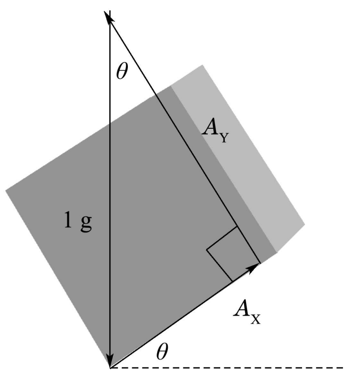


Fig. 4 Basic trigonometry

Attitude tilt angle can be obtained with the help of the acceleration from the accelerometer and the math function: $\arctan(\cdot)$[4]. Accelerometer equation:

$$X_k = \begin{bmatrix} 1 & -\mathrm{d}t \\ 0 & 1 \end{bmatrix} X_{k-1} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} W_{k-1}, \quad (17)$$

$$z_k = [1\ \ 0] X_k + v_k. \quad (18)$$

Here, $X_k = \begin{bmatrix} \mathrm{PredictAngle} \\ \mathrm{RandomDrift} \end{bmatrix} = \begin{bmatrix} \theta \\ b_\theta \end{bmatrix}$, W_{k-1} is process noise, $\mathrm{d}t$ is the sampling period, $z_k = [\mathrm{MessageAngle}]$, v_k is measurement noise.

Fig.5 describes the raw signal and Kalman filter signal of the acceleration.

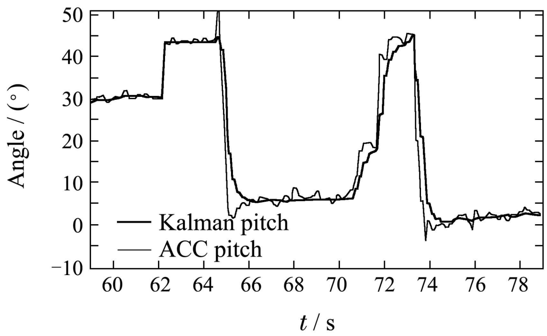


Fig. 5 Raw signal and Kalman filter signal

2.4 Data fusion and compensation

Although Kalman filter can eliminate a great majority of MEMS stochastic noises, the classic Kalman filter model requires sufficient prior knowledge from signal and noise statistics. In actual situations, it is difficult to meet the requirements. With the change of work environment, the noise characteristics would also correspondingly vary in the actual IMU system. In addition, due to the actual environment complexity, the progress noise and measurement noise would have random and time-variability statistical properties.

To improve the accuracy of attitude estimation, we propose further data fusion and compensation with long-period and short-period. According to the IMU engineering theory[4]: gyroscope measurement is more accuracy for short cycle while the accelerometer measurement is more accuracy for long cycle. Short cycle is configured using 1 ms while long cycle is 400 ms. The two cycles are used to real-time fuse the data after Kalman filter. The value which is re-compensated and re-fused, is a more accurate approximation to ture angle. Finally, we can get more precise estimate attitude.

Fig.6 shows the raw signal and Kalman filter signal with zero degree of reference value, it is apparent that noise variance has been reduced, but there still are some disturbances.

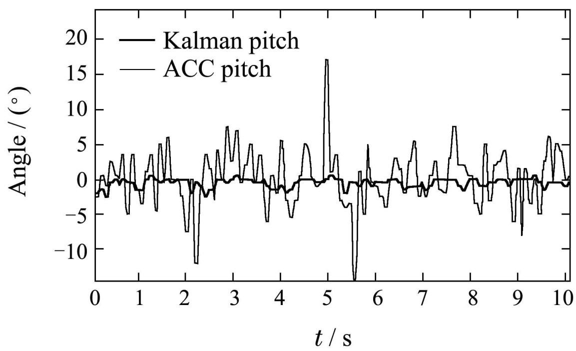


Fig. 6 Raw signal and signal after Kalman filter

Fig.7 shows the further compensation and fusion result after Kalman filter. By comparing, we can find more precise estimate attitude result from Fig.7.

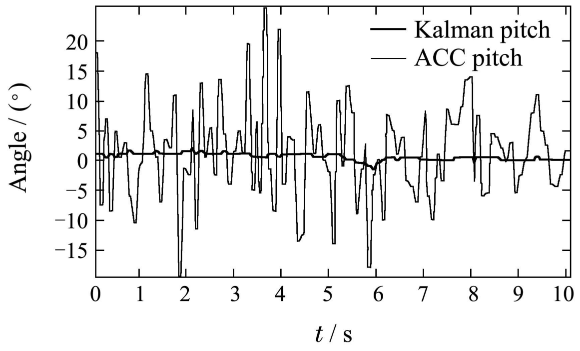


Fig. 7 Raw signal and fusion signal based on Kalman filter

3 Double-gain PD controller design

After further data fusion based on Kalman filter, a PD negative feedback controller is designed to stabilize the quadrotor attitude.

The control algorithm of the PID controller is given as follows:

$$u(t) = K_\mathrm{p}e(t) + K_\mathrm{i}\int_0^t e(\tau)\mathrm{d}\tau + K_\mathrm{d}\frac{\mathrm{d}e(t)}{\mathrm{d}t}. \quad (19)$$

Here, K_p is the proportional gain parameter; K_i is the integral gain; K_d is the derivative gain; t is instantaneous time; $e(t)$ is error (e.g. $e_\mathrm{Angle}(t)$ = Reference - KalmanAngle, $\mathrm{Reference}$ is the known reference value, $\mathrm{KalmanAngle}$ is the value after Kalman filter).

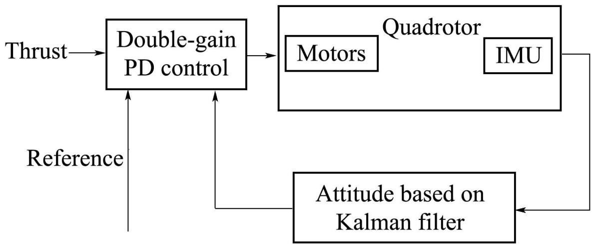


Fig. 8 Double-gain PD controller

Concerning the proportional gain, larger deviation can bring about the bigger control effect. However, the control effect sometimes is not enough strong when using single proportional gain. For instance, in our experiment, we find that when attitude angle (e.g., pitch angle) deviation is too large, the propellers can not provide enough lift to return to reference value quickly, this would cause quadrotor to shake severely and be out of balance. The target that the proportional gain can become larger with the deviation increasing is fulfilled. It can draw the quadrotor back to reference value quickly, and further strengthen corrective effort for the large deviations. When the deviation is small, we need very little corrective effort, it would enhance stability in the near of reference value.

In this work, we designed a double-gain PD controller for above control target. When attitude angle deviation is more than 20 degree, we set proportional gain as 1.5 while proportional gain is 0.8 for attitude angle deviation of less than 20 degree, as shown in Table1 and Table2. Fig.8 shows the double-gain PD controller.

Table 1 PD parameters for little deviation

Gain	K_p	K_d
Pitch	0.8	0.3
Roll	0.8	0.3

Table 2 PD parameters for large deviation

Gain	K_p	K_d
Pitch	1.5	0.2
Roll	1.5	0.2

4 Experimental platform and results

The experimental platform is called 'Quadrotor hover test-bench system', consisting of three major subsysterms: 'Mechanical test-bench', 'Quadrotor helicopter' and 'PC Ground Test Station'. Meanwhile, two wireless Xbee-Pro-ZB modules are separately assembled to 'Quadrotor helicopter' and 'Ground Test Station' for communication.

4.1 Mechanical test-bench

The test-bench stand is used to support the whole quadrotor helicopter for simulating the actual flight. It can perform three attitude motions (pitch, roll and yaw) and vertical movement of the quadrotor. When the whole test-bench has been assembled, it is free for the yaw motion, up to 70 degree roll/pitch rotation and 80 cm vertical motion.

4.2 Quadrotor helicopter hardware

The quadrotor mainly includes fight controller board, motor driver boards and four BLDC motors. As is shown in Fig.9, the light grey parts describes interactive relationship of the quadrotor components. Fight controller board is the carrier of controller algorithm, which mainly comprises MCU, IMU sensors and some peripheral circuit. We implement the controllers algorithm in C language with avr studio IDE. Each of BLDC motor is controlled and driven by pulse width modulation (PWM) signals and six MOSFETs from motor driver board. Using inter-integrated circuit (I2C) interface, the motor driver board can real-time communicate with flight controller board. With C programming, we have achieved the following functions: real-time collect gyroscope, accelerometer and other sensors data by the analog to digital converter (ADC) interface; real-time process raw signal value with Kalman filter and further fusion to estimate the attitude; transmit data to 'Ground Test Station' using Xbee-Pro-ZB; receive and parse protocol frame from 'ground test station' and handle command parameters.

4.3 Ground test station

The 'ground test station' is an upper computer (PC machine). The dark grey region in Fig.9 describes the structure, AADC is the main software which is coded with C++ language, mainly consisting of two blocks: a graphical user interface (GUI) block and a communication block. The communication block is used to transmit, receive and parse sensor data from the quadrotor. The GUI block can real-time draw IMU signal curve data. 'ground test station' fuctions have been achieved as follows: Real-time monitor the quadrotor status include attitude and motor information by Xbee serial interface by Xbee; real-time download the quadrotor parameters and commands into the flight control chip by Xbee; dynamically real-time show the sensor data and parameters with curve for further data analysis.

4.4 Experimental results

When the lift from the four propellers (corresponding every motor PWM value is 100) equals the whole weight of the quadrotor, the quadrotor would hover on the test-bench.

Figs.10 and 11 show pitch and roll angle curves, respectively. Here, the balance position is zero degree. Although there are several disturbances, the attitude (pitch or roll) can still return to reference value quickly and can maintain the value around zero degree ultimately.

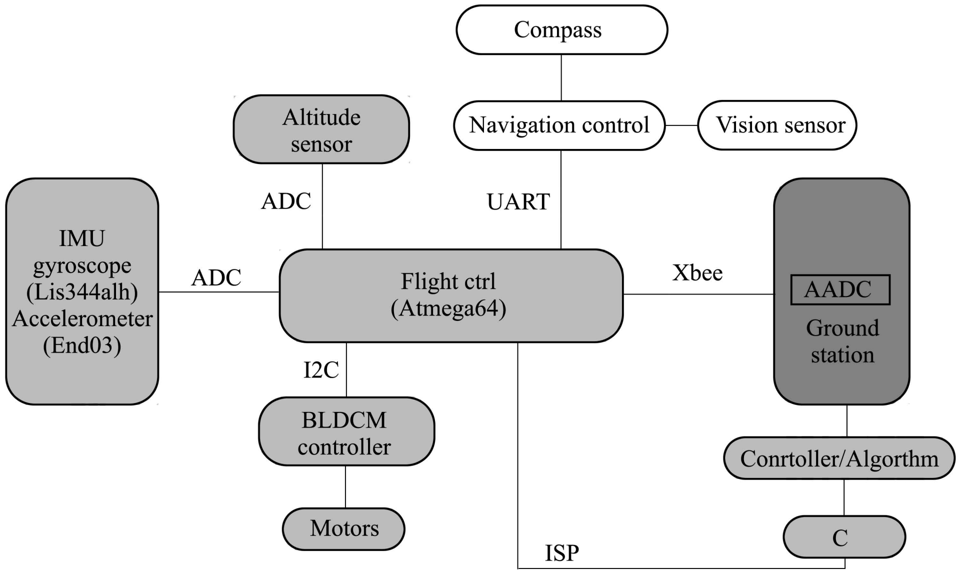


Fig. 9 Quadrotor system

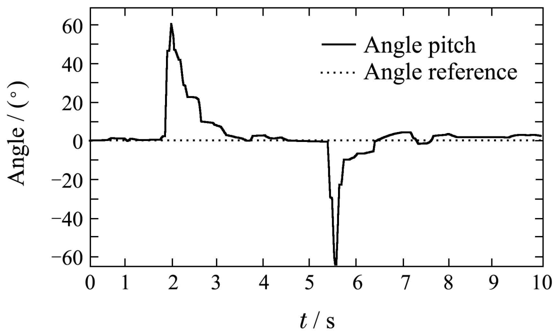
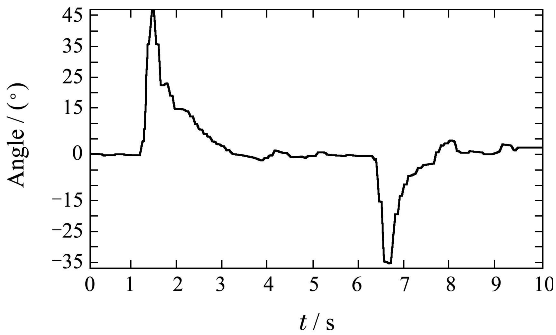
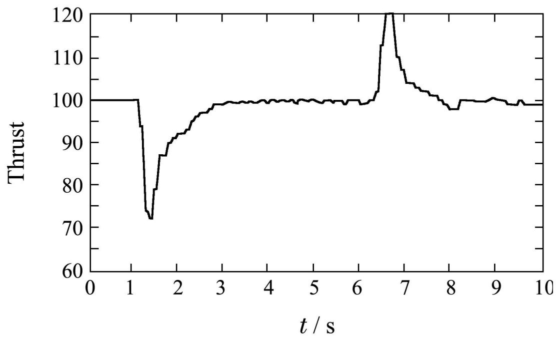


Fig. 10 Double-gain PID pitch hold at reference is zero degree



(a) Pitch angle



(b) Motor PWM

Fig. 12 Pitch angle and motor PWM value response after disturbance

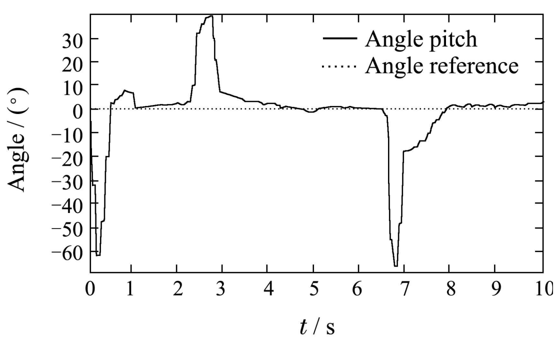


Fig. 11 Double-gain PID roll hold at reference is zero degree

Fig.12 shows pitch angle and the corresponding motor thrust curve.

5 Conclusion and ongoing work

In this paper, an attitude estimation scheme based on Kalman filter, and also the double-gain PD controller are presented. Firstly, in order to build the Kalman filter model of MEMS IMU, we theoretically analyze their random process characteristics. On this basis, the Kalman filter model is built, and the raw signals of MEMS IMU are processed with the model. For higher estimation accuracy, we carry out further data fusion and compensation. Secondly, the double-gain PD feedback controller is designed to stabilize quadrotor attitude at a sample rate of 400 Hz. Finally, the quadrotor can hover and VTOL on test-bench.

Although hovering on test-bench platform and attitude stablization control are successful, experiment and controller results still need to be improved. Future work contains hovering in the air, tracking control based on vision as shown in the light region of Fig.9.

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