

## 管式聚合反应器温度分布的动态建模与广义PI控制

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**摘要:** 针对阳离子聚合反应器的温度分布建模与控制问题, 提出了一种基于B样条神经网络的广义PI控制方法. 首先采用B样条复合网络建立分布函数的动态和静态模型, 并基于该模型, 将分布函数的跟踪问题等效为动态权值向量的时间域跟踪问题. 最后给出一种新型的广义PI控制方法, 实现对给定温度分布的跟踪控制. 同时, 为了更好地抑制未知干扰、参数摄动以及模型不匹配等问题, 模型权值状态、模型输出与实测温度分布所对应的权值误差都被引入到反馈控制回路, 因此能够大大增强系统的鲁棒性与抗干扰能力. 仿真结果表明该方法的可行性.

**关键词:** B样条网络; 分布参数系统; 阳离子聚合反应器; 广义PI控制

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## Dynamic modeling and generalized PI control for temperature distribution of the tubular polymerization

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**Abstract:** Model of temperature distribution in a tubular polymerization reaction is developed using a B-spline neural network, in which both dynamic and static network are applied to resolve the modeling of distribution function from a high dimensional data set. Based on this dynamic network model, a new-type generalized PI control algorithm has been studied. Then a control problem for distributed system is reduced to a tracking problem of nonlinear dynamic weights, which separates the time and the space effectively. In order to restrain unknown disturbances and parameter perturbation, not only the weights state of the network model are turn into feedback, but also the output error vector between the model and the real process is introduced at a certain percentage. This provides a feedback channel for the control, and therefore the robustness and anti-disturbance performance is largely enhanced. Simulation results demonstrate the effectiveness of the proposed method.

**Key words:** B-spline network; distributed parameter system; the cationic polymerization reactor; generalized PI control

### 1 Introduction

The molecular weight's distribution (MWD) of the polymer is one of the critical quality control variables for the industrial polymerization processes<sup>[1]</sup>, which is difficult to control directly for the lack of direct online measurement method<sup>[2]</sup>. Now the common methods are using the average molecular weight as quality index, such as the viscosity, weight-average molecular weight (Mw) and number-average molecular weight (Mn). These traditional indices can only reflect the average characteristics of production, but not provide all the quality information precisely.

A number of modeling and control strategies have been developed for the MWD control of polymerization processes. A generalized Kalman filter was combined with offline molecular weight analysis to estimate molecular chain length distribution of free-radical sol-

vent polymer, then a set-value sequence of reactor temperature was computed and MWD of polymerization was controlled by nonlinear programming<sup>[2]</sup>. Echevarria et al constructed a model-based nonlinear controller to control MWD in emulsion polymerization by measuring the content of unreacted monomer and chain transfer agent<sup>[3]</sup>. Vicente et al gave the optimal feeding curve calculation of the monomer and chain transfer agent, and the components and MWD of polymer is controlled by changing their concentration<sup>[4-5]</sup>. Alhamad et al<sup>[6-7]</sup> gave the detailed dynamic model of free-radical emulsion copolymerization, then the conversion, MWD, particle size distributions (PSD), Mw and Mn is effectively predicted. A multi-variable modeling prediction control strategy based on the reactor temperature and the flow of monomer, active agent, and initiator is proposed for the regulation and tracking of

MWD and PSD.

A model of MWD in a polymerization reaction was developed using a B-spline neural network in conjunction with a linear recurrent neural network for the control purpose<sup>[8]</sup>. An optimization control strategy was investigated under conditions with un-measurable noises and disturbances<sup>[9]</sup>, which provided a new method of modeling and control distributed parameter system. State estimation techniques have been implemented for monitoring on-line MWD and other time-varying model parameters or unknown conditions<sup>[10–11]</sup>. To track the molecular weight distribution in three-dimensional space, a new method is proposed by employing a generalized state-feedback controller<sup>[12]</sup>. By the hybrid dynamic recurrent support-vector-machine model, the distribution function is effectively divided into aspects of time and space. Then the tracking of three-dimensional distribution will transform into a state-vector tracking in the time domain, and the control difficulty will be greatly reduced.

The literature shows that most research is done by using chemical technology. Once the technological process is determined, studies on direct control of production quality in the view of process control are few. The following control indices are chosen most frequently as reactor temperature and other online measurable index functions, such as monomer conversion, average degree of polymerization. In the literatures about MWD control, most research is based on practical engineering with lack of quantitative relationship between control variables and distribution parameter variables.

However, the temperature distribution along a tubular polymerization reactor can show the extent and history of the polymerization reactions directly. As a result, MWD control problem can be substituted by the temperature distribution in some extent<sup>[13–16]</sup>. Investigation on the control methods of temperature distribution is of great importance for polymerization quality control. The model of temperature distribution in a tubular polymerization reaction was developed using a B-spline neural network<sup>[17]</sup>, in which both dynamic and static neural network were applied to resolve the modeling of distribution function from a high dimensional data set. Using the error set of the expected and the measured temperature distribution as control indexes, the optimal control strategy of extended integral square error (EISE) index based on the distribution model was investigated.

Recently, Wang<sup>[18]</sup> built the model of probability density function model for the stochastic systems using neural networks, which is applied to paper-making industry. Then based on this hybrid neural network model, a constrained PI strategy for the probability density functions is studied<sup>[19–20]</sup>. Moreover, the probability density function is similar to the reactor tempera-

ture distribution function in this paper. What's different is that the temperature distribution hasn't unitary constraint.

The cationic polymerization reactor considered here is a kind of distribution parameter systems, whose inputs and outputs are not only time-varying variables, but also parameter distributions. In order to provide realizable distribution control methods, recently B-spline expansions have been introduced to temperature distribution modeling and controller design problem in both theoretical studies and practical applications. A hybrid neural network has been used for modeling temperature distribution. Relationships between the model's input and output variables can be expressed as a topology polynomial, which is the mathematic basis of actual control strategy. The main design procedures included two steps.

The first step is to establish a dynamical hybrid neural network model for the input and measurable output temperature distribution, which is located in three dimension space. Using B-spline expansion technique, the relationships between the measurable output distribution and the constrained weights is given. Thus, the temperature distribution tracking problem in three dimensions can be transformed to a weight tracking control problem, in which the time and the space are separated effectively.

The second one is to use a new-type generalized PI control algorithm to solve weight tracking problem. The final control result is that the output temperature distribution of the reactor can track the given distribution. Considering the real situations of the industrial control, there are all kinks of unknown disturbance and model mismatch. Here, not only the state vectors of the networks model are fed into feedback, but also the temperature error vectors between the network model and the real object is introduced to feedback at a certain percentage. This control strategy can ensure that the temperature profiles in the tubular reactor asymptotically track the desired ones under the situations of disturbances.

## 2 Cationic polymerization tubular reactor

A cationic polymerization occurred in a laboratory tubular reactor with an effective length of 535 cm and the inner diameter of 1.27 cm, as shown in Fig.1. The monomer (Vinyl Butyl Ether, VBE, C<sub>6</sub>H<sub>12</sub>O) and initiator (Boron Trifluoride Diethyl Ether Complex) were stored in the reactant flumes respectively, and were fed into the mixed device through two metering pumps. Here, the monomer and initiator were mixed and dissolved in a solvent (Petroleum Ether, boiling point from 90° to 120°), and then fed into a horizontally mounted tubular reactor. There was a polymerization reactant that occurred in tubular reactor, and the heat given out by the reactant was measured real time by a series of

thermocouples mounted along the length of the tubular reactor. Polymeric product was fed into product flume.

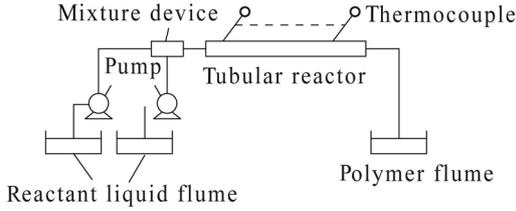


Fig. 1 The schematic diagrams of the tubular reactor

In the polymerization reaction, the history and the result of the polymerization reaction is determined by the initial concentration of the monomer and initiator, their concentration ratio, their flow rates and the total flow rate. It is known that the concentration change of the monomer and initiator will cause the temperature distribution to shift along the time axis<sup>[12–13]</sup>. This is mainly the axial move of the highest temperature. The feed ratio of the monomer and initiator affects the reaction extent mostly, which can be chosen as the manipulated variable  $u(k)$ , shown as

$$u(k) = \frac{[I^0]}{[M^0]} = \frac{F_i \times [I^{00}]/F}{F_m \times [M^{00}]/F} = \frac{F_i \times [I^{00}]}{F_m \times [M^{00}]}, \quad (1)$$

where  $F_i$  and  $F_m$  are the feed flow rate of the initiator and monomer respectively,  $[I^{00}]$  and  $[M^{00}]$  are the initial concentrations of the two materials,  $F = F_i + F_m$  is the total flow rate,  $[I^0]$  and  $[M^0]$  are the inlet concentrations of the initiator and monomer respectively.

The length of the tubular reactor is 535 cm, which is averaged into 40 segments. So a curve of the output distribution can be obtained from the 40 temperature points along the tubular reactor. Experiments were carried out in the experimental rig under the manipulated variable constraint  $u(k)$ . Good agreement between steady-state model and experimental results of the temperature distribution and molecular weight distribution was reached. Here a theoretical model developed in [12–13] was introduced instead of the above experiment process.

### 3 Dynamic modeling with hybrid neural networks

According to the above analysis, it is well known that the model of output distribution (temperature distribution or MWD)  $f(u(k), l)$  is a function of manipulated variable  $u(k)$  and tubular position variable  $l = 1, 2, \dots, 40$ , as shown in Fig.2. Output distribution dynamic model is similar to the probability density function during polymerization reaction with tubular position as abscissa, whose curve shape is determined by the manipulated variable  $u(k)$ . In other words, controlled variable is not a single- or multi-variable on each moment  $k$ , but a distribution function. Even for single variable system, the relationship between input and output is built up in three dimension space.

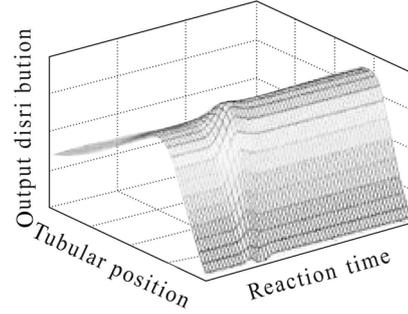
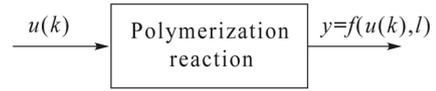


Fig. 2 Output distribution of polymerization process

Generally a dynamic model of the polymerization is built up with hybrid neural networks as shown in Fig.3. The hybrid network consists of recurrent neural network (RNN) and B-spline neural network (BSNN). The BSNN reflecting space characteristic can be used to describe the special algebraic relationship between the output distribution and position along the length of tubular reactor. The RNN reflecting time characteristic can be used to describe the dynamic relationship between the output distribution and the manipulated variable.

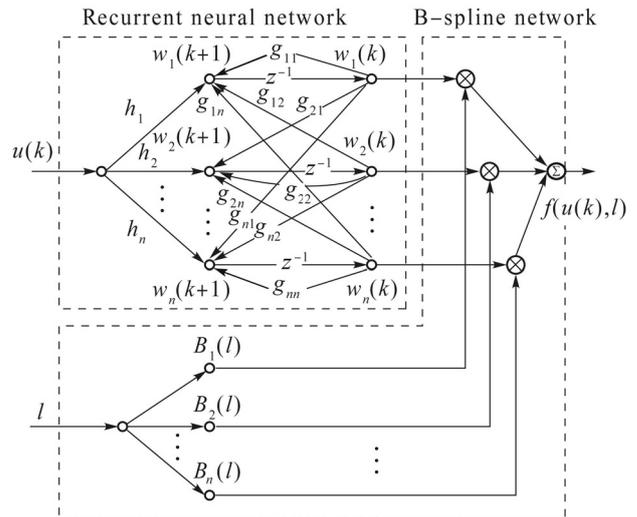


Fig. 3 The hybrid neural network

BSNN is a 3-layer network, in which the input is tubular position variable  $l$ , the hidden layer takes the B-spline function as its basis function, the outputs are weighted and summed as the inputs of the third layer. The input-output relationship of the single input BSNN can be represented as the linear combination of  $n$  multivariate basis functions

$$f(u(k), l) = \sum_{i=1}^n w_i(k) B_i(l) = W(k) B(l), \quad (2)$$

where  $W(k) = (w_1(k), w_2(k), \dots, w_n(k))^T \in \mathbb{R}^{n \times 1}$ ,  $B(l) = (B_1(l), B_2(l), \dots, B_n(l)) \in \mathbb{R}^{n \times 1}$ ,  $n$  is the number of the B-spline functions,  $w_i(k)$  represents the

weight connecting the  $i$ th basis function to the linear output node, which is the time varying output from RNN; each multivariate basis function  $B_i(l)$  is pre-specified from unvarying polynomial spline functions of order  $m$ , which are determined by trial and error considering the training time and precise.

It is shown in Fig.3 that RNN has feed-back connections from the third layer to the second layer. The input variable of RNN is the manipulated variable  $u(k)$ . RNN gives a dynamic relationship between the manipulated variable  $u(k)$  and output variable  $w_i(k)$ , shown as follows:

$$W(k + 1) = GW(k) + Hu(k), \quad (3)$$

where the system dynamic is determined by parameter matrices  $G \in \mathbb{R}^{n \times n}$ ,  $H \in \mathbb{R}^{n \times 1}$ .

Then Eq.(3) can be rewritten as

$$W(k + 1) = (zI - G)^{-1}Hu(k). \quad (4)$$

Substituting Eq.(4) into Eq.(2) gives

$$f(u(k), l) = B(l)(zI - G)^{-1}Hu(k). \quad (5)$$

The  $n$  outputs from the B-spline functions  $B_i(l)(i = 1, \dots, n)$  are multiplied with  $n$  outputs from the recurrent neural network  $w_i(k)$ , and the products are summed as the output of the hybrid network  $f(u(k), l)$ .

As a result of the following expansions:

$$(zI - G)^{-1}H \triangleq \frac{\sum_{i=0}^{n-1} D_i z^{-i}}{1 - \sum_{i=1}^n a_i z^{-i}}, \quad (6)$$

where  $a_i, D_i$  are undetermined coefficients. Substituting Eq.(6) into Eq.(5), it can be further obtained that

$$\begin{aligned} f(u(k), l) &\triangleq f_k(l) = \\ &a_1 f_{k-1}(l) + \dots + a_n f_{k-n}(l) + \\ &B(l)D_0 u(k) + \dots + B(l)D_{n-1} u(k-n+1) = \\ &\sum_{i=1}^n a_i f_{k-i}(l) + B(l) \sum_{j=0}^{n-1} D_j u(k-j) \triangleq \theta \varphi_k^T(l), \end{aligned} \quad (7)$$

where  $D_j = (d_{j,1}, \dots, d_{j,n-1}, d_{j,n}) \in \mathbb{R}^{n \times 1}$  and  $\theta = (a_1, \dots, a_n, D_0, \dots, D_{n-1})^T$  are unknown parameter, the observed vector is  $\varphi_k^T(x) = (f_{k-1}, \dots, f_{k-n}, B(x)u(k), \dots, B(x)u(k-n-1))$ . The standard recursive least squares algorithm<sup>[21]</sup> can be used to train the network to get parameter vector  $\theta$ . The characteristics and detailed hybrid network modeling approach is given<sup>[17]</sup>.

### 4 Generalized PI control for temperature distribution

#### 4.1 Generalized PI control algorithm

The temperature distribution model of polymerization reactor can be described by Eqs. (2) and (4). Here  $B_i(l)$  are pre-specified basis functions,  $W_i(k)(i =$

$1, \dots, n)$  are the weights of the expansion. It is noted that almost given temperature distribution  $g(l)$  can be expanded as Eq.(2) after selecting the appropriate set of basis functions  $B_i(l)$ , although there generally exists an error in such an approximation. This error is neglected here and we only focus on model Eq. (2). Then a control problem of the shape of temperature distribution  $f(u(k), l)$  is transferred into the tracking problem of nonlinear dynamic weight vector  $W(k)$ , which separates the time and the space effectively.

A desired temperature distribution  $g(l)$  can be given by  $g(l) = \hat{W}(K)B(l)$ , where  $\hat{W}(k)$  is the desired weight vector corresponding to Eq.(2) for the same  $B_i(l)(i = 1, \dots, n)$ . Therefore, the aim is to control the state variable vector  $W(k)$  to track the desired vector  $\hat{W} = (\hat{w}_1, \hat{w}_2, \dots, \hat{w}_n)^T \in \mathbb{R}^{n \times 1}$ . The information of error is used in the generalized state feedback controller, that is

$$E(k) = \hat{W} - W_m(k), \quad (8)$$

$$\Delta u(k) = K_p[E(k) - E(k-1)] + K_I E(k), \quad (9)$$

where  $E(k) = (e_1(k), e_2(k), \dots, e_n(k))^T$  is an error vector between the desired state vector  $\hat{W}$  and the model state vector  $W_m(k)$  at time  $k$ ,  $\Delta u(k) = u(k) - u(k-1)$  is input increment.

The control flow chart is shown in Fig.4, where P is the polymerization reactor and M is the dynamic model,  $E$  is the difference vector which can be expressed by Eq. (8).  $g(l)$  is the desired temperature distribution,  $f(l)$  is the actual output distribution and  $f_m(l)$  is model output distribution.  $B^{-1}$  is the Moore-Penrose generalized-inverse mapping operator which transforms the temperature distributions to the weight vector  $W$  according model relationship  $f(l) = W(k)B(l)$ . The predetermined matrix  $B(l)$  can be expressed as  $B(l) = F \times G$  by full rank factorization, where  $F \in \mathbb{R}^{1 \times r}$  and  $G \in \mathbb{R}^{r \times n}$  with rank  $r$ . Then an generalized-inverse is defined as  $B^{-1} = G^T(GG^T)^{-1}(F^T F)^{-1}F^T$ .

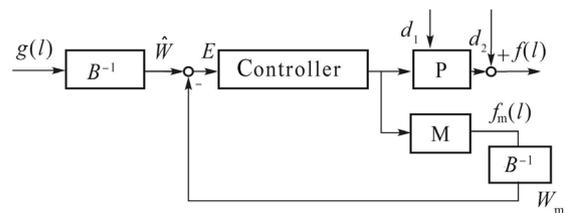


Fig. 4 Generalized state feedback control based on hybrid network model

The generalized PI control algorithm separates the time and the space effectively, and the temperature distribution tracking problem is reduced to a weight vector tracking problem. Considering the real situations of the industrial control, in which unknown disturbances and model mismatch always exist, not only the weight vectors of the network model are turned into feedback, but

also the temperature error vector between the network model and the real object is introduced into feedback at a certain percentage. Then an improved PI control algorithm is applied to control the temperature distribution along the polymerization reactor, which is detailed in next section.

Now the temperature distribution tracking control of error  $f(u(k), l) - g(l)$  has been reduced to the state feedback control tracking problem Eqs. (8) and (9) for discrete-time system (4). The control structure in Eq. (9) can simplify the formulation of the closed-loop dynamics and can also meet the control requirement. The objective is to find appropriate gain matrices of  $K_P$  and  $K_I$  such that the closed-loop system is stable and the tracking error  $E(k)$  converges to zero. Here the input variable is the feed ratio of the monomer and initiator, i.e  $u \in \mathbb{R}^{1 \times n}$ , then  $K_P \in \mathbb{R}^{1 \times n}$ ,  $K_I \in \mathbb{R}^{1 \times n}$ . Generally the parameter matrices  $K_P = [K_{Pj}]$  and  $K_I = [K_{Ij}] (j = 1, \dots, n)$  can be tuned by trial and error method.

**4.2 Online realization**

Based on the model (2) and (4), the control aim can be realized through the above strategy. But in practice, the presence of un-measurable disturbance, mismatch between the model and the practical temperature distribution will cause the control error increase, even lead to system unstable. Generally, the adaptive control strategy of retraining hybrid network can be introduced to decrease the model error partly. In fact, the control strategy (8) and (9) is open-loop, since only model error signal are introduced to feedback loop. Considering the real situations of unknown disturbances and model mismatch, the adaptive control with actual measured signal is adopted to control the temperature distribution. The control configuration is shown in Fig.5.

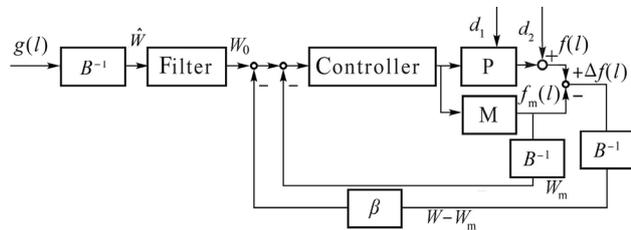


Fig. 5 the flow chart of closed-loop control

Here  $\Delta f(l)$  is the temperature distribution error between actual measurement  $f(l)$  and model output  $f_m(l)$ , which is introduced into feedback loop with coefficient  $\beta$ . When unknown disturbance or model mismatch appears, the error vector  $\Delta f(l)$  will become larger. Then it will be transformed to state vector error  $W - W_m$ . Feedback it into the closed-loop, the outputs of the polymerization reactor can track the desired temperature distributions with a perfect robustness.

The on-line adaptive control algorithm with modified error signal is given as

$$W_0(k + 1) = \alpha \hat{W} + (1 - \alpha)W_0(k), \tag{10}$$

$$E(k) = W_0 - W_m(k) - \beta(W(k) - W_m(k)), \tag{11}$$

$$\Delta u(k) = K_P[E(k) - E(k - 1)] + K_I E(k), \tag{12}$$

where  $0 < \alpha < 1$  is filter parameter,  $W(k)$  is the weight vector corresponding to the actual output distribution  $f(l)$ ,  $\Delta u(k) = u(k) - u(k - 1)$  is input increment. Eq.(10) is a soften filter, which it is shown that the model error factors  $W(k) - W_m(k)$  are fed into feedback channel simultaneously. In the adaptive control algorithm (10)–(12), the first item  $W_0 - W_m(k)$  aims at the set point tracking, and the second item  $W(k) - W_m(k)$  aims at decreasing the error between the measured values and model prediction caused by unknown disturbances or model mismatch.

**4.3 Robustness analysis of on-line control**

From the above analysis, it is known that the control problem of the shape of temperature distribution  $f(u(k), l)$  is transferred into the tracking problem of nonlinear dynamic weight vector  $W(k)$ , whose control structure is shown in Fig.5. For simply, this closed-loop control scheme for weight  $W(k)$  can be put into the internal model control (IMC) structure as shown in Fig.6.

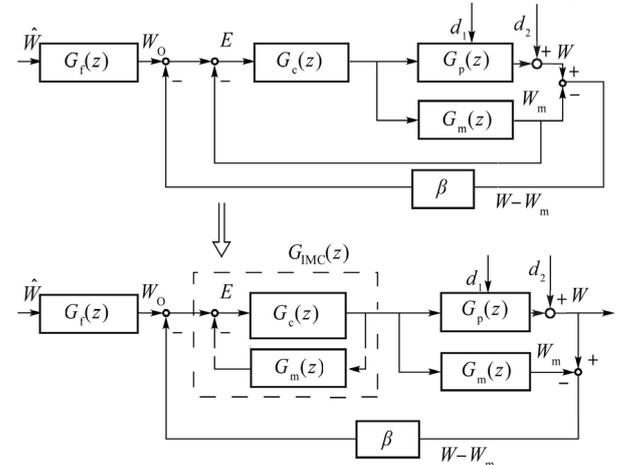


Fig. 6 IMC representation of on-line control

According the above analysis, it is known that

Filter:  $G_f(z) = \frac{\alpha}{z - (1 - \alpha)}$  from Eq.(10);

Controller:  $G_c(z)|_{1 \times n} = K_P + K_I \frac{z}{z - 1}$  from Eq.(12);

Model:  $G_m(z)|_{n \times 1} = (zI - G)^{-1}H$  from Eq.(6).

To have the same output for the both configurations in Fig.6, IMC controller  $G_{IMC}(z)$  is related to the class controller  $G_c(z)$  through the transformation

$$G_{IMC}(z) = \frac{G_c(z)}{1 + G_c(z)G_m(z)}. \tag{13}$$

The characteristic equation of closed-loop system is

$$1 + G_c(z)G_m(z) + \beta G_c(z)[G_p(z) - G_m(z)] = 0, \tag{14}$$

where  $G_p(z)$  is the actual plant impulse transfer function matrix and  $G_m(z)$  is the model of the plant. If the

uncertainties are given by  $G_p(z) = G_m(z) + \delta G(z)$ , where the  $\delta G(z)$  is the uncertainty matrix in  $G_p(z)$  such that  $|\delta G(z)| < |A(z)|$ . Then Eq.(14) can be rearranged as

$$1 + G_c(z)G_m(z) + \beta G_c(z)\delta G(z) = 0. \quad (15)$$

So the norm bound uncertainty region under the control configuration shown in Fig.6 is given as

$$|\delta G(z)| = \frac{|1 + G_c(z)G_m(z)|}{|\beta G_c(z)|} = \frac{|1 + [K_P + K_I \frac{z}{z-1}] [(zI - G)^{-1} H]|}{|\beta [K_P + K_I \frac{z}{z-1}]|}. \quad (16)$$

Further, for each subsystem model, we have

$$|\delta G_j(z)| = \frac{|1 + G_c(z)G_m(z)|}{|\beta G_{c_j}(z)|} = \frac{|1 + \sum_{j=1}^n [K_{P_j} + K_{I_j} \frac{z}{z-1}] [\frac{\sum_{i=0}^{n-1} d_{i,j} z^{-i}}{1 - \sum_{i=1}^n a_i z^{-1}}]|}{|\beta [K_{P_j} + K_{I_j} \frac{z}{z-1}]|}. \quad (17)$$

In order to maintain the closed-loop system robust stability, it should be satisfied that<sup>[22-23]</sup>

$$\frac{|1 + G_c(z)G_m(z)|}{|\beta G_c(z)|} > |A(z)|. \quad (18)$$

For low frequencies  $z \rightarrow 1$ , or  $\omega \rightarrow 0$  the norm bound uncertainty region  $|\delta G(z)|$  is given by the steady state gain of the model  $G_m(1)$ , integral gain  $K_I$  and feedback coefficient  $\beta$ ,

$$|\delta G_j(z)|_{\omega \rightarrow 0} = \frac{|\sum_{j=1}^n K_{I_j} [\frac{\sum_{i=0}^{n-1} d_{i,j}}{1 - \sum_{i=1}^n a_i}]|}{|\beta K_{I_j}|} = \frac{|\sum_{j=1}^n K_{I_j} G_{m_j}(1)|}{|\beta K_{I_j}|}. \quad (19)$$

Substituting Eq.(19) into inequality (18), the online-control system shown in Fig.6 is robust stable if and only if

$$|\sum_{j=1}^n K_{I_j} G_{m_j}(1)| > |\beta K_{I_j} A(1)|. \quad (20)$$

### 5 Simulation result

The online control procedure is shown in Fig.7. In the simulation process, random disturbances of the monomer flow rate  $F_m$  and measurement temperature  $T$  (amplitude  $\pm 5\%$ ) are added in every sampling time to simulate the real industry situations.

The trial and error tuning method is employed to decide all the elements in  $K_P$  and  $K_I$ , and  $n = 12$ , where

$$K_P = [6 \ 10 \ 13 \ 15 \ 17 \ 16 \ 12 \ 9 \ 6 \ 4 \ 3 \ 0],$$

$$K_I = [5 \ 4 \ 3 \ 2 \ 1 \ 7 \ 6 \ 4 \ 3 \ 2 \ 2 \ 0].$$

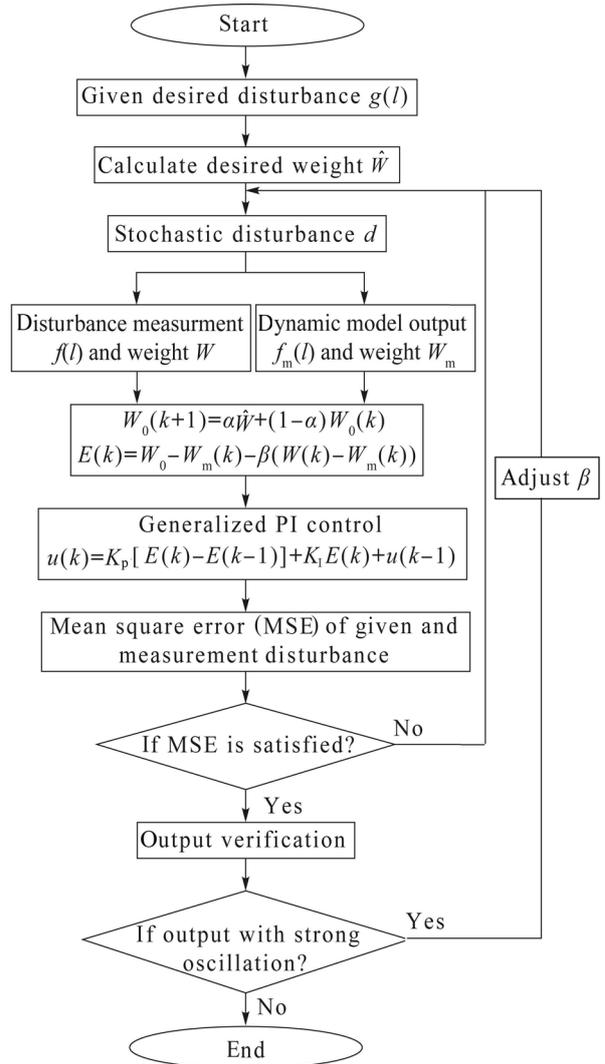


Fig. 7 On-line control flow chat of generalized PI

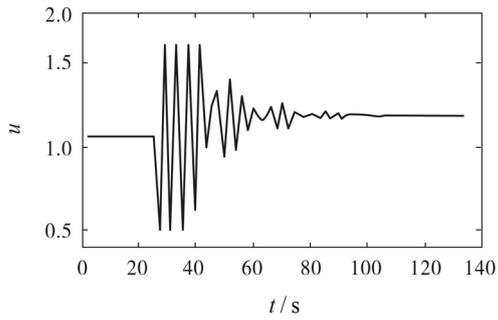
In the proposed generalized PI control,  $\beta$  is the feedback coefficient of the state vector  $W - W_m$ , and  $\beta \in (0, 1)$ . Fig.8 shows the input signal  $u(k)$  under different coefficient  $\beta$ , whose value is 1.06 at first, and is very close to the set point 1.2 at the 60th second. If  $\beta$  is too big, the feedback effect is too strong and it will lead to larger overshoot and oscillation. If  $\beta$  is too small, the feedback effect is too weak to restrain the unknown disturbances and model mismatch on time. Here it is selected as  $\beta = 0.2$ .

The control responds are shown in Fig.9. Fig.9(a) shows the mean square error (MSE) of  $E(n)$ , which converges to 0 at the 60th second. Here the mean square error of the weights is defined as

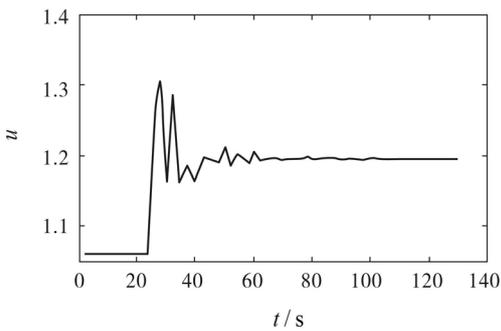
$$MSE_W = \sum_{i=1}^n e_i^2(k)/n.$$

Although the tracking error was relatively bigger at the very beginning, it decreased quickly and the system tended to be stable (see Fig.9(b)). It can be seen that the controlled output can track the desired one perfectly and a desired tracking performance has been achieved. The whole process is demonstrated in Fig.9(c) by a 3-D

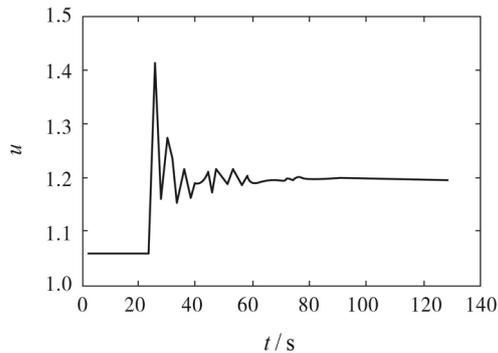
mesh plot, in which the shape of output is shown along the time.



(a)  $\beta = 0.8$

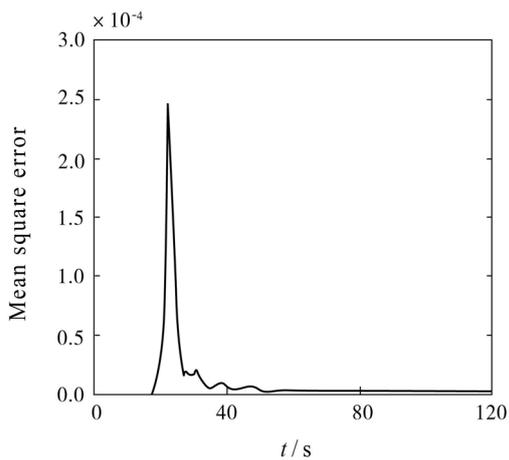


(b)  $\beta = 0.2$

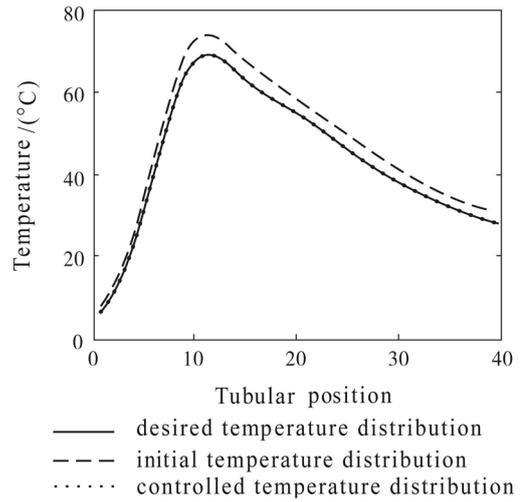


(c)  $\beta = 0.1$

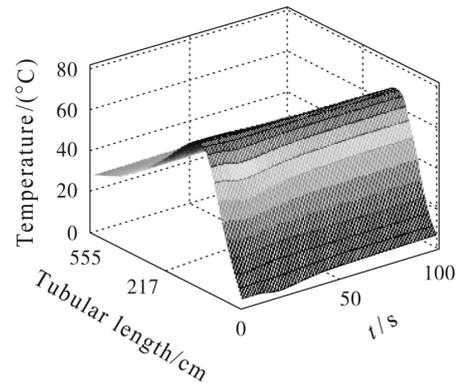
Fig. 8 Controller output under different  $\beta$



(a) MSE of the weights



(b) output tracking



(c) dynamic output at respective time  $k$

Fig. 9 System performance under on-line control

## 6 Conclusions

The molecular weight's distribution of the polymer is very important for the polymerization reactions, which is difficult to acquire on-line. However, the temperature distribution of the cationic tubular reactor can be used instead, which shows the extent of the reactions in the cationic polymerization reactor directly. Therefore, the control of molecular weight's distribution can be replaced by that of temperature distribution.

A hybrid neural network is used to model the temperature distribution with higher precision. The hybrid network consists of a recurrent neural network (RNN) and a B-spline neural network (BSNN). Then the analytical relationship between the measurable output distribution and the constrained weights is given using B-spline expansion technique. Thus, the temperature distribution tracking problem in three dimensions can be transformed to a weight vector tracking control problem. The difficulty of control is reduced greatly and many traditional control methods can be extended to solve this problem, such as PID and state feedback.

A generalized PI control algorithm is proposed based on the dynamic model, in which the model weight

vector is introduced as feedback signal directly. However, this is a kind of open-loop PI control which just depends on system model, and leads to poor robustness. Then considering the real situations of the industrial control, just as unknown disturbances and model mismatch, not only the state vectors of the network model are fed into feedback, but also the temperature distribution error between the network model and the measured output is introduced at a certain percentage  $\beta$ . This kind of adaptive control strategy can improve the system robustness obviously. The simulation shows that under the situations of unknown disturbance and model mismatch, the output of the polymerization reactor can converge to the desired temperature distribution rapidly.

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