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一种新的复杂动态网络学习控制的自适应同步算法

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摘要: 提出了一种新的具有未知时变参数复杂动态网络同步的自适应学习控制方法. 运用重新参数化技术, 设计周期时变参数的自适应学习律、常参数的更新律以及自适应控制策略确保同步误差渐近收敛. 通过构造复合能量函数给出同步的一个充分条件. 最后给出一个复杂网络的例子验证所提方法的有效性.

关键词: 同步; 复杂动态网络; 时变耦合强度; 自适应学习控制

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New adaptive synchronization algorithm for complex dynamical networks via learning control approach

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Abstract: A new adaptive learning control approach for the synchronization of complex dynamical networks with unknown time-varying parameters is proposed. By using the reparameterization technique, the adaptive learning laws of periodically time-varying and constant parameters and an adaptive control strategy are designed to ensure the asymptotic convergence of the synchronization error. Then, a sufficient condition of the synchronization is given by constructing a composite energy function. Finally, an example of the complex network is used to verify the effectiveness of proposed approach.

Key words: synchronization; complex dynamical networks; time-varying coupling strength; adaptive learning control

1 Introduction

During the past decade, complex dynamical networks have been studied intensively in various disciplines, such as sociology, biology, mathematics and engineering[1–6]. In a complex network, each node represents an element with certain dynamical characteristics and information systems, while the edges represent the relationship or connection of these basic elements. In general, the complexity of dynamical behaviors in such a network is governed by the intrinsic dynamics at each node and diffusion due to the spatial coupling among nodes. Up to now, nonlinear dynamics of complex networks, as well as how topological structure of the networks influences its dynamical behaviors, has become a strategic subject of considerable interest in various fields, for example sociology, biology, mathematics and physics and others[1−42].

Recently, the controlled synchronization of complex dynamical networks has become a rather significant topic in both theoretical research and practical applications[7–9, 11–19, 24–42]. Among the existing results, some authors focused on pinning control on complex dynamical networks by applying fewer linear feedback controllers based on intuitive knowledge of network topologies[9], while other concerned with the same issue by introducing an adaptive feedback controller based on some mathematical analysis. In most of these studies, the controllers have been designed based exactly on the information of the networked systems, such as network topology, coupling strength, and output function of the isolated node. In [7, 28], Wang and Chen pinned a complex dynamical network to its equilibrium by using local injections. In [29], Li et al. introduced a linear state feedback controller to synchronize a complex network to a desired orbit. Ref. [30] shows several robust adaptive controllers for complex dynamical networks with unknown but bounded nonlinear couplings. In [31], Zhou et al. investigated adaptive synchronization of uncertain complex dynamical networks. In [32], Wang et al, investigated the problem of con-

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trolling a weighted complex dynamical network with coupling time-varying delay toward an assigned evolution, designed an adaptive controller for nodes of the controlled network. Ref. [33–34] investigated synchronization of a simple uniform dynamical network model, wherein the coupling strengths are the same for all connections. Ref. [35] extended the uniform model to the one with coupling delay, and derived several synchronization criteria in the form of linear matrix inequality. In [36], the authors considered periodic orbits synchronization of a time-varying dynamical network model. In [37], the author studied synchronization in an array of coupled nonlinear systems with delay and nonreciprocal time-varying coupling. Synchronization criteria depend on the nonlinear characteristic of uncoupled nodes in systems and elements of the coupling matrix in systems. In [38], Li et al, introduced a new robust adaptive synchronization approach for the global synchronization of complex dynamical networks. Although so many approaches have been developed in the field of complex networks, only a few of them are concerning time-varying coupling strength. On the other hand, there are many of real-world network systems such as biological networks, mobile communications networks, and social networks, whose structure will change over time, coupling parameter, and other system parameters. For example, the feed-forward loops are typical network motifs in many real world biological networks. To the best of our knowledge, up to now little work has been reported from the viewpoint of time-varying coupling strength to deal with complex networks.

Motivated by the above discussions, in this paper, we want to design an adaptive controller and an update law that ensures the states of each node to reach the desired manifold. Both the characteristics of the uncoupled nodes of the network and the coupling matrix are unknown, but only a periodic time-varying coupling strength is used. By introducing a periodic update law, an adaptive control is designed to ensure the asymptotic convergence of the synchronizing error.

The rest of this paper is organized as follows. The problem formulation and preliminary are given in Section 2. Section 3 gives an adaptive controller and adaptive gains. In Section 4, the convergence property of the proposed adaptive control method is given. In Section 5 illustrative examples are provided to verify the theoretical results. Finally, conclusions are given in Section 6.

2 Problem formulation and preliminaries

In this section, we introduce the network model considered in this paper and give some useful mathematical preliminaries.

Consider a dynamical network consisting of N identical nodes with diffusive couplings, in which each node is an n-dimensional dynamical system. The state equations of the networks are

$$\dot{x}_i(t) = f(x_i(t)) + \varphi_i(t)\sum_{j=1}^{N} a_{ij}\boldsymbol{\Gamma} x_j(t), \tag{1}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \cdots, x_{in}(t))^{\mathrm{T}} \in \mathbb{R}^n$ represents the state vector of the ith node, $f: \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$ is a smooth nonlinear vector-valued function, $\varphi_i(t) \geqslant 0$ represents the unknown time-varying coupling strength; $\boldsymbol{\Gamma} = (\gamma_{ij})_{n\times n} \in \mathbb{R}^{n\times n}$ is the constant inner-coupling matrix of the nodes; $\boldsymbol{A} = (a_{ij})_{N\times N}$ denotes the outer-coupling matrix of the network, in which a_{ij} is defined as follows: if there is a connection between node i and node $j(j \neq i)$, then $a_{ij} = a_{ji} = 1$, else $a_{ij} = a_{ji} = 0$, and the diagonal elements of matrix $\boldsymbol{A}$ are defined by

$$a_{ii} = -\sum_{j=1,\, j\neq i}^{N} a_{ij} < 0,\ i, j = 1, 2, \cdots, N.$$

The network (1) is said to have achieved asymptotic synchronization if

$$\lim_{t\to\infty} ||x_i(t) - s(t)|| = 0,\ i = 1, 2, \cdots, N, \tag{2}$$

where $||\cdot||$ stands for the Euclidean vector norm, synchronous evolution $s(t)$ is an arbitrary desired state, which is also an isolated node of the network (1) such that $\dot{s}(t) = f(s(t))$. We assume that for the system $\dot{s}(t) = f(s(t))$ exists stable equilibrium point, stable periodic orbit, or even chaotic attractor.

To obtain the main results, the following assumptions and lemmas are needed.

Assumption 1 In the network (1), suppose that there exists $l_i > 0$, satisfying

$$\begin{aligned}&[x_i(t) - s(t)]^{\mathrm{T}}[f(x_i(t)) - f(s(t))] \leqslant\\ &l_i[x_i(t) - s(t)]^{\mathrm{T}}[x_i(t) - s(t)],\end{aligned} \tag{3}$$

where $x_i(t)$ and $s(t)$ are time varying vectors.

Assumption 2 In the network (1), the time-varying coupling strength $\varphi_i(t)$ satisfy $\varphi_i(t) = \phi_i(t) + \theta_i$, $\phi_i(t)$ is unknown periodic time-varying parameter, that is to say $\phi_i(t) = \phi_i(t - T)$, $\phi_i(t)$ is unknown continuous function. θ_i is unknown time-invariant nonnegative parameter.

Remark 1 Generally, the different $\phi_i(t)$ has different period, we can choose T as the least common multiple of all $\phi_i(t)$. Since $\varphi_i(t) = \phi_i(t) + \theta_i$, obviously $\phi_i(t)$ is unknown continuous periodic function with a known period T, i.e., for any time instant $iT \leqslant t \leqslant (i + 1)T$ where $i \in \mathbb{N}$, we have $\phi(t) = \phi(t - T) = \cdots = \phi(t - iT)$. For convenience, we denote $\phi(t) = (\phi_1(t), \cdots, \phi_N(t))^{\mathrm{T}}$, $\theta = (\theta_1, \cdots, \theta_N)^{\mathrm{T}}$, obviously $\phi(t)$ is unknown continuous periodic function vector with a known period T.

Assumption 3 In the network (1), the inner coupling matrix $\boldsymbol{\Gamma}$ and coupling matrix $\boldsymbol{A}$ satisfy

$$||\boldsymbol{\Gamma}|| = \gamma, \tag{4}$$

$$|a_{ij}| \leqslant a, \ \forall i, j = 1, 2, \cdots, N, \tag{5}$$

where γ, a are positive constants.

Lemma 1 (Young's inequality) For any vectors $x, y \in \mathbb{R}^n$, and any $c > 0$, the following matrix inequality holds

$$x^{\mathrm{T}}y \leqslant cx^{\mathrm{T}}x + \frac{1}{4c}y^{\mathrm{T}}y. \tag{6}$$

Lemma 2 For a positive constant T, if $\forall t \geqslant 0$,

$$\lim_{t\to\infty}\int_t^{t+T} e^2(\tau)\mathrm{d}\tau = 0,$$

then

$$\lim_{t\to\infty} e(t) = 0.$$

Proof By integral mean value theorem, we have

$$\int_t^{t+T} e^2(\tau)\mathrm{d}\tau = Te^2(\xi_t), \ t \leqslant \xi_t \leqslant t + T,$$

yields

$$\lim_{t\to\infty}\int_t^{t+T} e^2(\tau)\mathrm{d}\tau = \lim_{t\to\infty} Te^2(\xi_t) = \lim_{\xi_t\to\infty} Te^2(\xi_t).$$

So, $\lim\limits_{t\to\infty} e^2(t) = 0$, we can obtain $\lim\limits_{t\to\infty} e(t) = 0$.

In this paper, $\phi_i(t)$ is unknown periodic time-varying nonnegative parameter, it is hard to obtain the condition such that network (1) achieve asymptotically synchronized. But we use an adaptive controller under which the solutions of network (1) onto a desired evolution, i.e. achieve asymptotically synchronized.

3 Adaptive controller design

To achieve the control objective (2), we need an adaptive strategy to control nodes in the networks (1). Then the controlled networks is given by

$$\dot{x}_i(t)=f(x_i(t))+\varphi_i(t)\sum_{j=1}^{N} a_{ij}\boldsymbol{\Gamma}x_j(t)+u_i(t), \tag{7}$$

where $u_i(t) = (u_{i1}(t), u_{i2}(t), \cdots, x_{in}(t))^{\mathrm{T}} \in \mathbb{R}^n (i = 1, 2, \cdots, N)$ are the adaptive controllers to be designed.

We design controllers by

$$u_i(t) = -Na\gamma(\hat{\phi}_i(t) + \hat{\theta}_i(t))(x_i(t) - s(t)), \tag{8}$$

where $\gamma = ||\boldsymbol{\Gamma}||, \ a \geqslant |a_{ij}|$ are positive constants, N is the node number of the network (1), and $\hat{\phi}_i(t), \hat{\theta}_i(t)$ are estimations to $\phi_i(t), \theta_i$ respectively. For convenience, we denote

$$\tilde{\phi}(t) = \phi(t) - \hat{\phi}(t), \ \tilde{\theta}(t) = \theta - \hat{\theta}(t).$$

For writing convenience, we let $e_i(t) = x_i(t) - s(t), \ i = 1, 2, \cdots, N$ and

$$\begin{cases} S(t) = [s^{\mathrm{T}}(t) \ \ s^{\mathrm{T}}(t) \ \cdots \ s^{\mathrm{T}}(t)]^{\mathrm{T}} \in \mathbb{R}^{n\times N}, \\ e(t) = [e_1^{\mathrm{T}}(t) \ \ e_2^{\mathrm{T}}(t) \ \cdots \ e_N^{\mathrm{T}}(t)]^{\mathrm{T}} \in \mathbb{R}^{n\times N}. \end{cases} \tag{9}$$

Then we have the following dynamical error equations:

$$\dot{e}_i(t)=f(x_i)-f(s)+\varphi_i(t)\sum_{j=1}^{N} a_{ij}\boldsymbol{\Gamma}e_j+u_i(t), \tag{10}$$

and the closed loop dynamical systems

$$\dot{e}_i(t) = f(x_i(t)) - f(s(t)) + \varphi_i(t)\sum_{j=1}^{N} a_{ij}\boldsymbol{\Gamma}e_j(t) - Na\gamma(\hat{\phi}_i(t) + \hat{\theta}_i(t))e_i(t). \tag{11}$$

To guarantee negative feedback, the time-varying periodic adaptive gains and the time-invariance update law are designed as follows:

$$\hat{\phi}(t)=\begin{cases} \hat{\phi}(t-T)+Qe^{\mathrm{T}}e, & t \in [T, +\infty), \\ Q_0(t)e^{\mathrm{T}}(t)e(t), & t \in [0, T), \\ 0, & t \in [-T, 0), \end{cases} \tag{12}$$

$$\dot{\hat{\theta}}(t) = Re^{\mathrm{T}}(t)e(t), \tag{13}$$

where $Q=\mathrm{diag}\{q_1, \cdots, q_N\} > 0$ is a constant adaptation gain matrix, $Q_0(t)=\mathrm{diag}\{q_{10}(t), \cdots, q_{N0}(t)\} > 0$, each diagonal element $q_{i0}(t)$ is a continuous and strictly increasing function satisfying $q_{i0}(0) = 0$, $q_{i0}(T) = q_i$; $R = \mathrm{diag}\{r_1, \cdots, r_N\}$ is a positive definite diagonal matrix.

Obviously,

$$\hat{\phi}_i(t)=\begin{cases} \hat{\phi}_i(t-T)+q_ie_i^{\mathrm{T}}e_i, & t \in [T, +\infty), \\ q_{i0}(t)e_i^{\mathrm{T}}(t)e_i(t), & t \in [0, T), \\ 0, & t \in [-T, 0), \end{cases} \tag{14}$$

$$\dot{\hat{\theta}}_i(t) = r_ie_i^{\mathrm{T}}(t)e_i(t). \tag{15}$$

Remark 2 We choose the gain matrices in Eq.(12) is positive diagonal matrix, $q_{i0}(t)$ satisfies diagonal elements is strictly monotone increasing continuous function in the first period, but in the second period diagonal elements are constant which equal the value at the end of the first period. Such the gain matrices, ensure $\hat{\phi}_i(t)$ is continuous in the interval $[0, T)$, ensuring that the Lyapunov function is boundedness in a period.

Remark 3 The adaptation law is a difference-type pointwise integration over the period $[t - T, \, t]$, which takes the place of the differential-type adaptation law. The fundamental idea of the pointwise integration based periodic updating can be explained as follows. Since $\phi_i(t) = \phi_i(t - T)$, for any time instant $iT \leqslant t \leqslant (i + 1)T$, where $i = 1, 2, \cdots, N$, we have $\phi_i(t) = \phi_i(t - T) = \cdots = \phi_i(t - iT)$. This means that $\phi_i(t)$ can be treated as a 'constant' when it is sampled at any interval which is the integer multiple of period T. The periodic updating law (14) in the interval of one period T is a difference-type integrator for such a 'constant'. We can easily show that $\hat{\phi}_i(t)$ is continuous.

4 Convergence analysis

The convergence property of the proposed adaptive control method is summarized in the following theorem.

Theorem 1 Under Assumptions 1−3, the control law (8) with the periodic adaptive law (14) and update law (15) guarantees asymptotic synchronization of the controlled networks (7), while keeping all closed-loop signals are L_{T}^2 bounded.

Proof To facilitate the convergence analysis, let $V(t, e_i(t),\ \tilde{\phi}_i(t),\ \tilde{\theta}_i(t)) = V(t)$, we choose a Lyapunov-Krasovskii functional as follows:

$$V(t)=\begin{cases}\frac{1}{2}\sum\limits_{i=1}^{N}e_i^{\mathrm{T}}e_i+\frac{Na\gamma}{2}\sum\limits_{i=1}^{N}\int_0^t q_i^{-1}\tilde{\phi}_i^2(\tau)\mathrm{d}\tau+\\ \frac{Na\gamma}{2}\sum\limits_{i=1}^{N}r_i^{-1}(\tilde{\theta}_i(t)+L_i)^2,\ t\in[0,T),\\ \frac{1}{2}\sum\limits_{i=1}^{N}e_i^{\mathrm{T}}e_i+\frac{Na\gamma}{2}\sum\limits_{i=1}^{N}\int_{t-T}^t q_i^{-1}\tilde{\phi}_i^2(\tau)\mathrm{d}\tau+\\ \frac{Na\gamma}{2}\sum\limits_{i=1}^{N}r_i^{-1}(\tilde{\theta}_i(t)+L_i)^2,\ t\in[T,+\infty),\end{cases} \tag{16}$$

where L_i is sufficiently large positive constant which will be determined.

Firstly, we will prove that the finiteness of $V(t)$ in $[0,T)$. Secondly, we will prove the asymptotical convergence of $e_i(t)$.

Let us confirm the finiteness property of $V(t)$ for the first period $[0,T)$. From the system dynamics (7) and the proposed control laws (8)(14) and (15), it can be seen that the right-hand side of Eq.(7) is continuous with respect to all arguments. In light of the existence theorem of differential equation, Eq.(11) has a solution in an interval $[0,T_1)\subset[0,T)$, with $0<T\leqslant T_1$. Therefore, the boundedness of $V(t)$ over $[0,T_1)$ can be guaranteed and we need only focus on the interval $[T_1,\ T)$.

For any $t\in[T_1,T)$, the time derivative of $V(t)$ for $t\in[T_1,T]$ is given by

$$\dot{V}(t)=\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)\dot{e}_i(t)+\frac{Na\gamma}{2}\sum_{i=1}^{N}q_i^{-1}\tilde{\phi}_i^2(t)+ Na\gamma\sum_{i=1}^{N}r_i^{-1}(\tilde{\theta}_i(t)+L_i)\dot{\tilde{\theta}}_i(t),\ t\in[T_1,T). \tag{17}$$

Taking the first term on the right-hand side of $\dot{V}(t)$, Eq.(11) and Lemma 1, one obtains

$$\begin{aligned}&\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)\dot{e}_i(t)=\\&\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)[f(x_i(t))-f(s(t))+\\&\varphi_i(t)\sum_{j=1}^{N}a_{ij}\boldsymbol{\Gamma}e_j(t)-Na\gamma(\hat{\phi}_i(t)+\hat{\theta}_i(t))e_i(t)]\leqslant\\&\sum_{i=1}^{N}[l_ie_i^{\mathrm{T}}(t)e_i(t)+\varphi_i(t)(Na^2c+\frac{1}{4c}N\gamma^2)e_i^{\mathrm{T}}(t)e_i(t)-\\&Na\gamma(\hat{\phi}_i(t)+\hat{\theta}_i(t))e_i^{\mathrm{T}}(t)e_i(t)],\end{aligned}$$

where c is a positive constant. If we choose $c=\frac{1}{2}a^{-1}\gamma$, we have $Na^2c+\frac{1}{4c}N\gamma^2=Na\gamma$, then

$$\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)\dot{e}_i(t)\leqslant \sum_{i=1}^{N}[l_i+Na\gamma(\tilde{\phi}_i(t)+\tilde{\theta}_i(t))]e_i^{\mathrm{T}}(t)e_i(t). \tag{18}$$

Using the parametric updating law (15) we have

$$Na\gamma\sum_{i=1}^{N}r_i^{-1}(\tilde{\theta}_i(t)+L_i)\dot{\tilde{\theta}}_i(t)= -Na\gamma\sum_{i=1}^{N}r_i^{-1}(\tilde{\theta}_i(t)+L_i)e_i^{\mathrm{T}}(t)e_i(t). \tag{19}$$

Let us focus on the second term on the right-hand side of Eq.(17). Since $q_{i0}(t)$ is a continuous function and strictly increasing in $[0,\ T)$, $q_i^{-1}\leqslant q_{i0}^{-1}(t)<\infty$ is ensured in the interval $[T_1,\ T)$, and

$$\begin{aligned}&\frac{Na\gamma}{2}\sum_{i=1}^{N}q_i^{-1}\tilde{\phi}_i^2(t)\leqslant\frac{Na\gamma}{2}\sum_{i=1}^{N}q_{i0}^{-1}(t)\tilde{\phi}_i^2(t)\leqslant\\&\frac{Na\gamma}{2}\sum_{i=1}^{N}q_{i0}^{-1}(t)[\phi_i^2(t)+2\hat{\phi}_i^2(t)-2\phi_i(t)\hat{\phi}_i(t)]=\\&\frac{Na\gamma}{2}\sum_{i=1}^{N}q_{i0}^{-1}(t)\phi_i^2(t)-Na\gamma\sum_{i=1}^{N}\tilde{\phi}_i(t)e_i^{\mathrm{T}}e_i.\end{aligned} \tag{20}$$

Substituting Eqs.(18)–(20) into Eq.(17) yields

$$\dot{V}(t)\leqslant\sum_{i=1}^{N}(l_i-Na\gamma L_i)e_i^{\mathrm{T}}(t)e_i(t)+ \frac{Na\gamma}{2}\sum_{i=1}^{N}q_{i0}^{-1}(t)\phi_i^2(t). \tag{21}$$

It is obvious that there exist sufficiently large positive constants L_i such that $l_i-Na\gamma L_i<0$. So we have

$$\dot{V}(t)\leqslant\frac{Na\gamma}{2}\sum_{i=1}^{N}q_{i0}^{-1}(t)\phi_i^2(t). \tag{22}$$

Since $\phi_i(t)$ is periodic, it is bounded. The boundedness of $\phi_i(t)$ leads to the boundedness of $\dot{V}(t)$, as $V(T_1)$ is bounded, $\forall t\in[T_1,T)$. The finiteness of $V(t)$ is obvious.

Let us compute the difference of $V(t)$ over one period for $t\in[T,\infty)$, which is

$$\begin{aligned}&\Delta V(t)=V(t)-V(t-T)=\\&\frac{1}{2}\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)e_i(t)-\frac{1}{2}\sum_{i=1}^{N}e_i^{\mathrm{T}}(t-T)e_i(t-T)+\\&\frac{Na\gamma}{2}\sum_{i=1}^{N}\int_{t-T}^{t}[q_i^{-1}\tilde{\phi}_i^2(\tau)-q_i^{-1}\tilde{\phi}_i^2(\tau-T)]\mathrm{d}\tau+\\&\frac{Na\gamma}{2}\sum_{i=1}^{N}r_i^{-1}(\tilde{\theta}_i(t)+L_i)^2-\\&\frac{Na\gamma}{2}\sum_{i=1}^{N}r_i^{-1}(\tilde{\theta}_i(t-T)+L_i)^2.\end{aligned} \tag{23}$$

Looking into the first two terms on the right-hand side of $\Delta V(t)$, and using the error dynamics (10), using the

Assumptions 1–3 and Lemma 1, we have

$$\frac{1}{2}\sum_{i=1}^{N} e_i^{\mathrm{T}}(t)e_i(t) - \frac{1}{2}\sum_{i=1}^{N} e_i^{\mathrm{T}}(t-T)e_i(t-T) \leqslant \sum_{i=1}^{N}\int_{t-T}^{t}[l_i + Na\gamma(\tilde{\phi}_i + \tilde{\theta}_i)]e_i^{\mathrm{T}}(\tau)e_i(\tau)\mathrm{d}\tau. \quad (24)$$

Using the algebraic relation

$$(a-b)^{\mathrm{T}}\boldsymbol{H}(a-b) - (a-c)^{\mathrm{T}}\boldsymbol{H}(a-c) = (c-b)^{\mathrm{T}}\boldsymbol{H}[2(a-b)+(b-c)],$$

where $\boldsymbol{H}$ is a positive definite symmetric matrix, and substituting the parameter updating law (14), we have

$$\frac{Na\gamma}{2}\sum_{i=1}^{N}\int_{t-T}^{t}[q_i^{-1}\tilde{\phi}_i^2(\tau) - q_i^{-1}\tilde{\phi}_i^2(\tau-T)]\mathrm{d}\tau = -Na\gamma\sum_{i=1}^{N}\int_{t-T}^{t}[\tilde{\phi}_i(\tau) + \frac{1}{2}q_i e_i^{\mathrm{T}}e_i]e_i^{\mathrm{T}}e_i\mathrm{d}\tau. \quad (25)$$

And since

$$\frac{Na\gamma}{2}\sum_{i=1}^{N} r_i^{-1}(\tilde{\theta}_i(t) + L_i)^2 - \frac{Na\gamma}{2}\sum_{i=1}^{N} r_i^{-1}(\tilde{\theta}_i(t-T) + L_i)^2 = -Na\gamma\sum_{i=1}^{N}\int_{t-T}^{t}(\tilde{\theta}_i(\tau) + L_i)e_i^{\mathrm{T}}(\tau)e_i(\tau)\mathrm{d}\tau. \quad (26)$$

Substituting Eqs.(24)–(26) into Eq.(23), we can attain

$$\Delta V(t) \leqslant -\sum_{i=1}^{N}\int_{t-T}^{t}(Na\gamma L_i - l_i)e_i^{\mathrm{T}}(\tau)e_i(\tau)\mathrm{d}\tau. \quad (27)$$

We have

$$\Delta V(t) < 0. \quad (28)$$

Applying Eq.(23) repeatedly for any $t \in [lT, (l+1)T]$, $l = 1, 2, \cdots$, and denoting $t_0 = t - lT$, we have

$$V(t) = V(t_0) + \sum_{j=0}^{l-1}\Delta V(t - jT). \quad (29)$$

Since $t_0 \in [0, T)$, according to Eq.(29), we obtain

$$\lim_{t\to\infty} V(t) < \max_{t_0\in[0,T)} V(t_0) - \lim_{l\to\infty}\sum_{j=0}^{l-1}\sum_{i=1}^{N}\int_{t-(j+1)T}^{t-jT}(Na\gamma L_i - l_i)e_i^{\mathrm{T}}(\tau)e_i(\tau)\mathrm{d}\tau.$$

Consider the positiveness of $V(t)$, and $V(t_0)$ bounded in the interval $[0, T)$, according to the convergence theorem of the sum of series, the error $e_i(t)$ converges to zero asymptotically in L_{T}^2 norm. That is to say, we have

$$\lim_{t\to\infty}\int_{t-T}^{t} e_i^{\mathrm{T}}(\tau)e_i(\tau)\mathrm{d}\tau = \lim_{t\to\infty}\int_{t-T}^{t} e_i^2(\tau)\mathrm{d}\tau = 0.$$

By Lemma 2, we can obtain the asymptotic synchronization of the controlled networks (7).

Finally, we prove all the closed-loop signals are bounded in L_{T}^2 norm. $\forall t \in [T, \infty)$, the derivative of $V(t)$ is

$$\dot{V}(t) = \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)\dot{e}_i(t) + \frac{Na\gamma}{2}\sum_{i=1}^{N} q_i^{-1}\tilde{\phi}_i^2(t) - \frac{Na\gamma}{2}\sum_{i=1}^{N} q_i^{-1}\tilde{\phi}_i^2(t-T) + Na\gamma\sum_{i=1}^{N} r_i^{-1}(\tilde{\theta}_i + L_i)\dot{\tilde{\theta}}_i \leqslant -Na\gamma\sum_{i=1}^{N}\tilde{\phi}_i e_i^{\mathrm{T}}e_i - \sum_{i=1}^{N}(Na\gamma L_i - l_i)e_i^{\mathrm{T}}e_i. \quad (30)$$

By Eq.(30), one can obtain

$$V(t) \leqslant V(T) - Na\gamma\sum_{i=1}^{N}\int_{T}^{t}\tilde{\phi}_i(\tau)e_i^{\mathrm{T}}(\tau)e_i(\tau)\mathrm{d}\tau - \sum_{i=1}^{N}\int_{T}^{t}(Na\gamma L_i - l_i)e_i^{\mathrm{T}}(\tau)e_i(\tau), \quad (31)$$

where

$$V(t) = \frac{1}{2}\sum_{i=1}^{N} e_i^{\mathrm{T}}(t)e_i(t) + \frac{Na\gamma}{2}\sum_{i=1}^{N}\int_{t-T}^{t} q_i^{-1}\tilde{\phi}_i^2(\tau)\mathrm{d}\tau + \frac{Na\gamma}{2}\sum_{i=1}^{N} r_i^{-1}(\tilde{\theta}_i(t) + L_i)^2,\ t \in [T, +\infty).$$

From the boundedness of $\tilde{\phi}_i(t)$ and $V(t)$, we can conclude e_i, $\int_{t-T}^{t}\tilde{\phi}_i^2(\tau)\mathrm{d}\tau$, $\tilde{\theta}_i(t)$ are all bounded. This further implies the L_{T}^2 boundedness of $\hat{\phi}_i(t)$ and $\hat{\theta}_i(t)$, thereafter the L_{T}^2 boundedness of the control input $u_i(t)$. This completes the proof.

Remark 4 Note that matries Q and R have no effect on $\dot{V}(t)$, thus one can change the matrix arbitrarily during control without worrying about the control robustness.

5 Simulation examples

To demonstrate the theoretical result obtained in Section 3, the Chua's chaotic circuit is used as a dynamical node of the network.

The Chua's chaotic circuit is described by

$$\begin{cases} \dot{x}_1 = p(-x_1 + x_2 - g(x_1)), \\ \dot{x}_2 = x_1 - x_2 + x_3, \\ \dot{x}_3 = -qx_2, \end{cases} \quad (32)$$

where $g(x_1) = m_0 x_1 + \frac{1}{2}(m_1 - m_0)(|x_1 + 1| - |x_1 - 1|)$ with $p = 10$, $q = 14.7$, $m_0 = -0.68$ and $m_1 = -1.27$. Fig.1 depicts the chaotic trajectory of Eq.(32) with initial value $s(0) = (-0.2, 0.2, 0.5)^{\mathrm{T}}$.

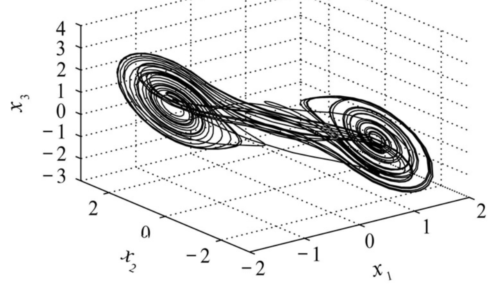

Fig. 1 The desired orbit, where $s(0) = (-0.2, 0.2, 0.5)^{\mathrm{T}}$

We take the system (32) as identical nodes of network, which is given by

$$\begin{pmatrix}\dot{x}_{i1}\\ \dot{x}_{i2}\\ \dot{x}_{i3}\end{pmatrix} = \begin{pmatrix} p(-x_{i1}+x_{i2}-g(x_{i1}))\\ x_{i1}-x_{i2}+x_{i3}\\ -qx_{i2}\end{pmatrix} + \varphi_i(t)\sum_{j=1}^{N} a_{ij}\boldsymbol{\Gamma} x_j(t) + u_i(t), \qquad (33)$$

where

$$\boldsymbol{A} = \begin{pmatrix} -1 & 1 & 0 & 0 & 0 & 0\\ 1 & -2 & 1 & 0 & 0 & 0\\ 0 & 1 & -3 & 2 & 0 & 0\\ 0 & 0 & 2 & -4 & 1 & 1\\ 0 & 0 & 0 & 1 & -5 & 4\\ 0 & 0 & 0 & 1 & 4 & -5 \end{pmatrix},\ \boldsymbol{\Gamma} = \begin{pmatrix} 1\,0\,0\\ 0\,1\,0\\ 0\,0\,1\end{pmatrix}.$$

For simplicity, the parameters are selected as follows: The parameters are selected as follows:

$$N=6,\ T=6,\ \phi_1(t)=0.2\sin\frac{2\pi t}{3},$$
$$\phi_2(t)=2\cos\pi t,\ \phi_3(t)=-\sin\frac{2\pi t}{3},$$
$$\phi_4(t)=\cos\pi t,\ \phi_5(t)=-2\sin\frac{2\pi t}{3},$$
$$\phi_6(t)=2\sin\frac{2\pi t}{3},\ \theta=(2,10,8,3,2,5)^{\mathrm{T}}.$$

The problem can not be solved by the method in Ref. [38]; however, our proposed synchronization strategy can solve it. In the following simulations, we choose

$$Q= \mathrm{diag}\{0.001, 0.003, 0.002, 0.005, 0.001, 0.002\},$$
$$Q_0=\frac{t}{6}Q,\ R=0.0005I_6.$$

The initially estimated value of the unknown parameter $\phi(t)$ is

$$\hat{\phi}(0)=(0,0,0,0,0,0)^{\mathrm{T}},$$

the initial states are chosen as

$$x_1(0)=(2,0,-1)^{\mathrm{T}},\ x_2(0)=(3,1,-2)^{\mathrm{T}},$$
$$x_3(0)=(-3,-2,1)^{\mathrm{T}},\ x_4(0)=(-1,0,-2)^{\mathrm{T}},$$
$$x_5(0)=(-1,1,0)^{\mathrm{T}},\ x_6(0)=(3,-2,0.8)^{\mathrm{T}},$$

respectively. According to Theorem 1, the weighted network (31) can be synchronized by applying the adaptive controllers (8)(14)–(15). Fig.2 shows the error evolutions under the designed controllers. We clearly see that the states of the network (33) asymptotically synchronizes with the states of the desired orbit (32). Furthermore, Fig.3 depicts the time evolution of the controllers, and Fig.4 shows the evolution of the estimated time-varying parameters. Fig.5 displays the evolution of the estimated constant parameters. Obviously, Figs.3–5 show that the control signals and estimated parameters are all bounded.

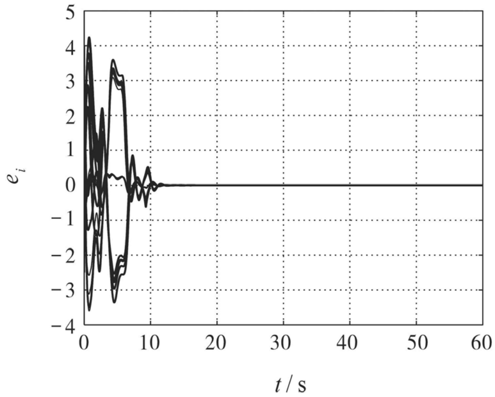

Fig. 2 The evaluations of synchronization errors $(i=1,2,\cdots,6)$

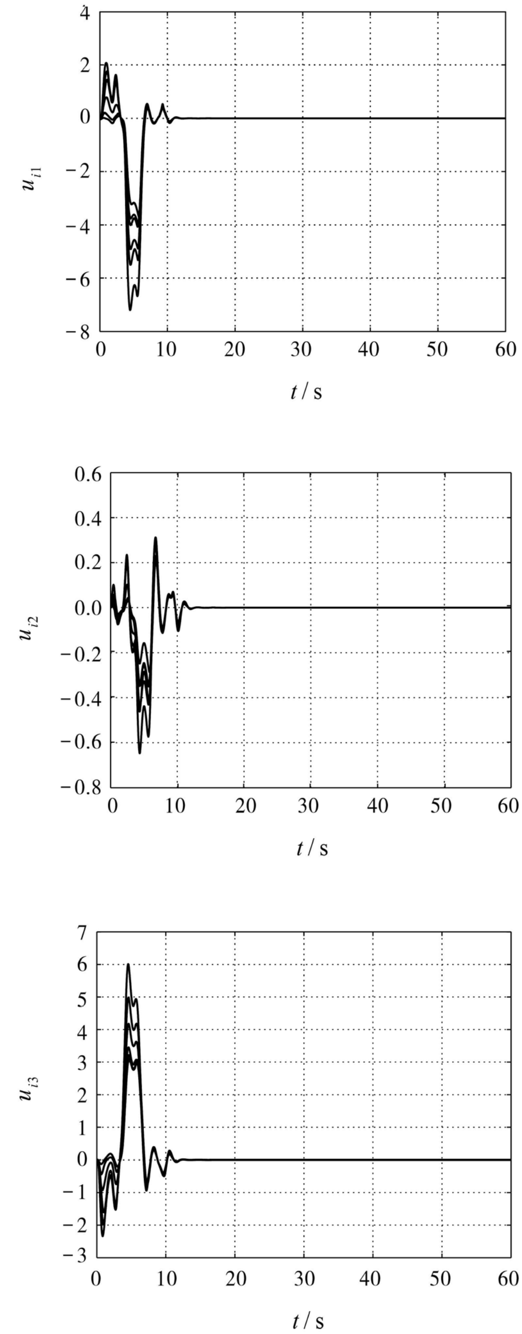

Fig. 3 The curve of control inputs $u_i(i=1,2,\cdots,6)$

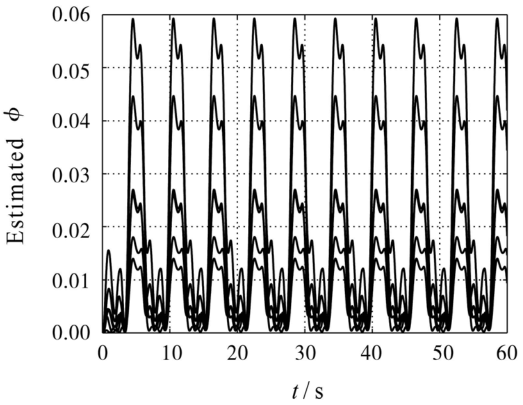

Fig. 4 The evaluations of parameters ϕ_i

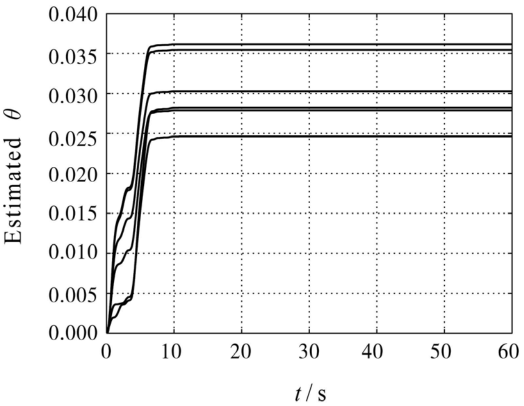

Fig. 5 The evaluations of parameters θ_i

6 Conclusions

In this paper, we present a new adaptive synchronization approach for the synchronization of time-varying complex dynamical networks. Both the characteristics of the uncoupled nodes of the network and the coupling matrix are unknown, and only a periodic time-varying coupling strength is used in this paper. We design an adaptive controller and hybrid differential-periodic adaptation law that ensures the asymptotic convergence of the synchronizing error. Simulations also verify our theoretical results.

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