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寻找多智能体系统一致性的迭代学习方法

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摘要: 本文利用迭代学习的方法研究了带头结点的多智能体系统的一致性问题. 文中分别对单积分多智能体系统和一般的线性多智能体系统提出了迭代学习型的一致性算法. 该算法对每一个从节点所设计的分布迭代学习序列可以保证从节点能完全跟随上头结点. 假设头结点是全局可达的, 对于有向拓扑连接图, 给出了智能体达到完全一致的充分条件. 最后, 仿真实例说明了文中所给方法的有效性.

关键词: 多智能体系统; 迭代学习; 一致性算法
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Consensus seeking in multi-agent systems by the iterative learning control

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Abstract: Leader-following multi-agent consensus problems are studied by using the iterative learning control (ILC) approach. The consensus problems of single-integrator and the general linear multi-agent dynamics are considered by the developed scheme, respectively. ILC sequences of individual agents are developed such that they can ensure the follower agents can track the leader perfectly in the finite time interval. Assuming that the leader node is globally reachable, some sufficient conditions to guarantee the multi-agent consensus are derived for the directed communication topologies. Finally, simulation examples are given to illustrate the effectiveness of the proposed methods in this article.

Key words: multi-agent systems; iterative learning control; consensus algorithm

1 Introduction

In recent years, distributed coordination of multi-agent systems has attracted great attention due to several technological advances in the fields of communication and computation. Obviously, the consensus problem is an important topic in the study of multi-agent systems. The objective of consensus is to design distributed control laws to drive the group of agents to reach a common assessment or to agree upon certain quantities of interest. So far, most existing studies on the consensus problem focus on the case where agents are governed by first-order dynamics or second-order dynamics[1–4]. In [5], the authors mainly investigate the consensus problem for multi-agent systems modelled as a high-order integrator. And most of the existing consensus control algorithms for multi-agent systems are asymptotic consensus algorithms.

ILC has been widely employed for the tracking control of periodic dynamic systems. By generating a correct control signal from the previous control execution, it can achieve perfect tracking performance on the finite time interval[6]. The repetitive systems should have an initial resetting during every iteration. After resetting, the whole process keeps repeating itself until a desired convergence is achieved. Many ILC control approaches have been proposed, details please refer to [6].

Recently, ILC has been applied to the multi-agent formation control[7–10]. Ahn et al in [7] presented an ILC scheme to generate a sequence of control signals for multi-agent formation control and applied it to the nonlinear dynamic model. In [11], a high-order internal mode (HOIM) based iterative learning control (ILC) scheme for multi-agent system (MAS) formation was studied. The paper [12] has developed two coordination algorithms for distributed multi-agent formation using iterative learning control. In [13–14], the finite-time consensus problem of multi-agent systems with higher-order dynamics was studied, and a framework for effectively constructing distributed protocols which incorporate iterative learning control actions into output feedback was also presented. But it is worth pointing out that the iterative learning protocols in [13–14] only

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solve the finite-time consensus problem just at the desired terminal output. This observation motivates our present study.

In this paper, the main purpose is to study the ILC schemes on leader-following multi-agent consensus problems of first-order and a general linear multi-agent systems. In [13–14], a constant protocol was designed to track a constant reference. The contribution of this paper is to present the ILC algorithms so that the follower agents can track the leader perfectly in the finite time interval and the dynamic of the leader is time-varying. Moreover, we assume that the information of the leader is available to only a portion of follower agents in this paper and use the general consensus error which is available to any follower agent for control purposes. Then, sufficient conditions are obtained to guarantee the multi-agent consensus for the directed communication topologies.

The remainder of the paper is organized as follows. In Section 2, some preliminaries and the problem statement are briefly outlined. ILC schemes for multi-agent consensus problems are proposed in Section 3. In Section 4, the effectiveness of the consensus algorithms proposed in this paper is demonstrated by simulations. Finally, conclusions are drawn in Section 5.

2 Preliminaries and problem statement

In a multi-agent system, each agent can be considered as a node in a directed graph (digraph), and the information flow between neighboring agents can be regarded as a directed path between the neighboring nodes in the digraph. Thus, the interconnection topology of a multi-agent system is usually described as a digraph. A weighted digraph $G = (V, E, A)$ of order n consists of a set of vertices $V = \{V_1, \cdots, V_n\}$, a set of edges $E \subseteq V \times V$ and a weighted adjacency matrix $A = [a_{ij}] \in \mathbb{R}^{n\times n}$ with $a_{ij} \geqslant 0$. The node indexes belong to a finite index set $I = \{1, \cdots, n\}$. An edge of the weighted digraph G is denoted by $e_{ij} = (i, j) \in E$, i.e., e_{ij} is a directed edge from i to j. If there exists a path from node i to node j, then node j is said to be reachable from node i. A node i which is reachable from every other node of G is called a globally reachable node of G. We assume that the adjacency elements associated with the edges of the digraph are positive, i.e. $a_{ij} > 0 \Leftrightarrow e_{ij} \in E$. Moreover, we assume $a_{ii} = 0$ for all $i \in I$. The set of neighbors of node v_i is denoted by $N_i = \{j \in V : (i, j) \in E\}$. And $d_i = \sum\limits_{j=1}^{n} a_{ij}$, $D = \operatorname{diag}\{d_1, \cdots, d_n\}$ and the Laplacian matrix of digraph G is $L = D - A$.

For the leader-follower multi-agent systems with the topology graph $\bar{G}$, assume that the leader (labelled by 0) is represented by vertex v_0. The topology graph $\bar{G}$ with vertex set $\bar{V} = V \cup \{v_0\}$ is studied in this paper. The communication topology graph for the followers is denoted by G. The connection between the follower and the leader is directed. The connection weight between the follower and the leader is denoted by $b_i,\ i \in I$. If the ith follower can obtain the information from the leader, then $b_i > 0$, otherwise, $b_i = 0$. Let $B = \operatorname{diag}\{b_1, \cdots, b_n\}$.

The following lemma shows a relationship between H and the connectivity of graph $\bar{G}$[15].

Lemma 1 The eigenvalues of $H = L + B$ have positive real parts if and only if the leader node 0 is globally reachable in $\bar{G}$ (namely, Assumption 1 holds).

Consider the leader-follower consensus problem, $x_i(t)$ is the state of the ith agent. And the state of the leader is denoted by $x_0(t)$.

Define the consensus error of the multi-agent system

$$\delta_i(t, k) = x_i(t, k) - x_0(t), \tag{1}$$

which represents the disagreement of the leader and the agent i, where k represents the iterative times. The perfect consensus problem for the leader-following multi-agent system is to find an appropriate control input sequence $\{u_i(t, k), t_0 \leqslant t \leqslant T,\ i = 1, 2, \cdots, n\}$ such that the follower agents can track the leader $x_0(t)$, i.e., $\delta_i(t, k) \to 0$ as $k \to \infty$ for all $t \in [t_0, T]$, where T is a finite positive constant.

In this paper, the general consensus error for agent i is defined as

$$\begin{aligned} e_i(t, k) = &\sum_{j\in N_i} a_{ij}(x_j(t, k) - x_i(t, k)) + \\ &b_i(x_0(t) - x_i(t, k)). \end{aligned} \tag{2}$$

Remark 1 Compared with the general consensus error (2), the consensus error (1) is a global quantity that can not be computed locally at each node. While the error information represented by Eq.(2) is available to any agent i for control purposes.

3 ILC consensus algorithms for MAS

3.1 Single-integrator dynamics

In this section, the single-integrator MAS is considered, and the dynamic of ith agent at the kth iteration is given by

$$\dot{x}_i(t, k) = u_i(t, k), \tag{3}$$

where $x_i \in \mathbb{R}^m$, $u_i \in \mathbb{R}^m$ are the state and the input vectors of the ith agent, respectively.

The control signal of the ith agent at the $(k + 1)$-th iteration is updated as follows:

$$u_i(t, k + 1) = u_i(t, k) + \gamma_i \dot{e}_i(t, k), \tag{4}$$

where γ_i is a proper control gain.

The following assumptions are given in this paper.

Assumption 1 The leader (that is, node 0) is globally reachable in $\bar{G}$.

Assumption 2 The initial resetting condition is assumed for all agents, i.e., $x_i(t_0,k)=x_0(t_0)$ for $i=1,\cdots,n$ and $k=0,1,\cdots$; It follows that $\delta_i(t_0,k)=0$ for $k=0,1,\cdots$. It is easy to see that $e_i(t_0,k)=0$.

Let $e(t,k)=[e_1^{\mathrm{T}}(t,k)\ \cdots\ e_n^{\mathrm{T}}(t,k)]^{\mathrm{T}}$, $x(t,k)=[x_1^{\mathrm{T}}(t,k)\ \cdots\ x_n^{\mathrm{T}}(t,k)]^{\mathrm{T}}$ respectively.

From the definition of $e_i(t,k)$ and $\delta_i(t,k)$, then the error vector is given by

$$\begin{aligned}e(t,k)=&-((L+B)\otimes I_m)(x(t,k)-\mathbf{1}_n x_0)=\\&-(H\otimes I_m)\delta(t,k),\end{aligned}\tag{5}$$

where $\mathbf{1}_n=(1,1,\cdots,1)^{\mathrm{T}}\in\mathbb{R}^n$.

And the control vector is described as

$$u(t,k+1)=u(t,k)+(\mathit{\Gamma}\otimes I_m)\dot{e}(t,k),$$

where $\mathit{\Gamma}=\mathrm{diag}\{\gamma_1,\gamma_2,\cdots,\gamma_n\}$ is the control gain matrix.

From Eq.(5), we have

$$\begin{aligned}e(t,k+1)=&\ e(t,k)-(H\otimes I_m)\times\\&[\delta(t,k+1)-\delta(t,k)].\end{aligned}\tag{6}$$

The following state difference of the consensus errors between two successive iterations can be expressed as:

$$\begin{aligned}&\delta(t,k+1)-\delta(t,k)=x(t,k+1)-x(t,k)=\\&[x(t_0,k+1)-x(t_0,k)]+\\&\int_{t_0}^{t}[u(\tau,k+1)-u(\tau,k)]\mathrm{d}\tau=\\&\int_{t_0}^{t}[u(\tau,k+1)-u(\tau,k)]\mathrm{d}\tau=\\&\int_{t_0}^{t}(\mathit{\Gamma}\otimes I_m)\dot{e}(\tau,k)\mathrm{d}\tau=(\mathit{\Gamma}\otimes I_m)e(t,k).\end{aligned}\tag{7}$$

Substituting Eq.(7) into Eq.(6), we obtain

$$e(t,k+1)=(I-(H\mathit{\Gamma})\otimes I_m)e(t,k).\tag{8}$$

Definition 1 Given a vector function $h:[t_0,T]\mapsto\mathbb{R}^n$, its ∞-norm is defined by

$$\|h\|_\infty=\sup_{t\in[t_0,T]}\|h(t)\|.$$

Now, taking norms to both sides of Eq.(8), we can yield that

$$\|e(t,k+1)\|_\infty\leqslant\|I-(H\mathit{\Gamma})\otimes I_m\|\,\|e(t,k)\|_\infty.\tag{9}$$

Lemma 2 It follows from Eq.(5) that

$$||\delta(t,k)||_\infty\leqslant||e(t,k)||_\infty/\underline{\sigma}(H\otimes I_m),$$

where $\underline{\sigma}(H\otimes I_m)>0$ denotes the minimum singular value of $H\otimes I_m$. And $\lim\limits_{k\to\infty}e(t,k)=0$ can guarantee $\lim\limits_{k\to\infty}\delta(t,k)=0$.

Theorem 1 Under Assumptions 1–2, the ILC control law given in Eq.(4) ensures that the follower agents governed by Eq.(3) can track the leader perfectly if there exists a control gain matrix $\mathit{\Gamma}$ such that $\rho(I-(H\mathit{\Gamma})\otimes I_m)<1$ (ρ denotes the spectral radius) holds.

Proof It is easily verified from Eq.(9) that $\lim\limits_{k\to\infty}e(t,k)=0$ for all $t\in[t_0,T]$ and then from the Lemma 2, we obtain $\lim\limits_{k\to\infty}\delta(t,k)=0$ for all $t\in[t_0,T]$.

Remark 2 From the Lemma 1, the eigenvalues of H have positive real parts, that is to say H is a nonsingular matrix. And it is easy to design the matrix $\mathit{\Gamma}$ such that $\rho(I-(H\mathit{\Gamma})\otimes I_m)<1$ holds.

3.2 General linear dynamics

Now, we focus on the following general linear dynamics of MAS. The linear dynamic of the ith agent at the kth iteration is described by

$$\dot{x}_i(t,k)=\bar{A}x_i(t,k)+\bar{B}u_i(t,k),\tag{10}$$

where $\bar{A}\in\mathbb{R}^{m\times m}$, $\bar{B}\in\mathbb{R}^{m\times m}$ are the coefficient matrixes. $x_i\in\mathbb{R}^m$, $u_i\in\mathbb{R}^m$ are the state and the input vectors of the ith agent, respectively.

The control signal of the ith agent at the $(k+1)$-th iteration is updated as follows:

$$u_i(t,k+1)=u_i(t,k)+\gamma_i\dot{e}_i(t,k),\tag{11}$$

where γ_i is a proper control gain to be designed.

Let $x(t,k)=[x_1^{\mathrm{T}}(t,k)\ \cdots\ x_n^{\mathrm{T}}(t,k)]^{\mathrm{T}}$, $u(t,k)=[u_1^{\mathrm{T}}(t,k)\ \cdots\ u_n^{\mathrm{T}}(t,k)]^{\mathrm{T}}$ respectively.

Thus, the vector form of $\dot{x}(t,k)$ and $u(t,k)$ are given by

$$\dot{x}(t,k)=(I_m\otimes\bar{A})x(t,k)+(I_m\otimes\bar{B})u(t,k),\tag{12}$$

$$u(t,k+1)=u(t,k)+(\mathit{\Gamma}\otimes I_m)\dot{e}(t,k).\tag{13}$$

The solution of the linear Eq.(12) is

$$\begin{aligned}x(t,k)=&\ \mathrm{e}^{(I_m\otimes\bar{A})(t-t_0)}x(t_0,k)+\\&\int_{t_0}^{t}\mathrm{e}^{(I_m\otimes\bar{A})(t-\tau)}(I_m\otimes\bar{B})u(\tau,k)\mathrm{d}\tau.\end{aligned}$$

Then, from the definition of $e_i(t,k)$ and $\delta_i(t,k)$, the error vector is given by

$$\begin{aligned}&e(t,k+1)=\\&e(t,k)-(H\otimes I_m)\times[\delta(t,k+1)-\delta(t,k)].\end{aligned}\tag{14}$$

The difference of the consensus errors between two successive iterations can be expressed as

$$\begin{aligned}&\delta(t,k+1)-\delta(t,k)=x(t,k+1)-x(t,k)=\\&\mathrm{e}^{(I_m\otimes\bar{A})(t-t_0)}[x(t_0,k+1)-x(t_0,k)]+\\&\int_{t_0}^{t}\mathrm{e}^{(I_m\otimes\bar{A})(t-\tau)}(I_m\otimes\bar{B})[u(\tau,k+1)-u(\tau,k)]\mathrm{d}\tau=\\&\int_{t_0}^{t}\mathrm{e}^{(I_m\otimes\bar{A})(t-\tau)}(I_m\otimes\bar{B})(\mathit{\Gamma}\otimes I_m)\dot{e}(\tau,k)\mathrm{d}\tau=\\&(I_m\otimes\bar{B})(\mathit{\Gamma}\otimes I_m)e(t,k)+\\&\int_{t_0}^{t}\mathrm{e}^{(I_m\otimes\bar{A})(t-\tau)}\times(I_m\otimes\bar{A})(I_m\otimes\\&\quad\bar{B})(\mathit{\Gamma}\otimes I_m)e(\tau,k)\mathrm{d}\tau.\end{aligned}\tag{15}$$

Substituting Eq.(15) into Eq.(14), we obtain

$$\begin{aligned}&e(t,k+1)=\\&e(t,k)-(H\otimes I_m)(I_m\otimes\bar{B})(\mathit{\Gamma}\otimes I_m)\times\end{aligned}$$

$$e(t,k) - (H\otimes I_m)\int_{t_0}^{t} \mathrm{e}^{(I_m\otimes\bar{A})(t-\tau)}\times$$
$$(I_m\otimes\bar{A})(I_m\otimes\bar{B})(\varGamma\otimes I_m)e(\tau,k)\mathrm{d}\tau.$$

From the property of Kronecker product

$$(H\otimes I_m)(I_m\otimes\bar{B})(\varGamma\otimes I_m)=(H\varGamma)\otimes\bar{B},$$
$$(I_m\otimes\bar{A})(I_m\otimes\bar{B})(\varGamma\otimes I_m)=\varGamma\otimes(\bar{A}\bar{B}).$$

Then

$$e(t,k+1)=$$
$$(I-(H\varGamma)\otimes\bar{B})e(t,k)-(H\otimes I_m)\times$$
$$\int_{t_0}^{t}\mathrm{e}^{(I_m\otimes\bar{A})(t-\tau)}(\varGamma\otimes(\bar{A}\bar{B}))e(\tau,k)\mathrm{d}\tau. \quad (16)$$

For the further derivation, we use the following notations: $c_1\triangleq||H\otimes I_m||$, $c_2\triangleq||\varGamma\otimes\bar{A}\bar{B}||$, $c_3\triangleq\sup\limits_{t\in[t_0,T]}||\mathrm{e}^{(I_m\otimes\bar{A})t}||$, $c\triangleq c_1c_2c_3$.

Now, taking norms to two sides of Eq.(16), we can yield that

$$||e(t,k+1)||\leqslant||I-(H\varGamma)\otimes\bar{B}||||e(t,k)||+$$
$$c\int_{t_0}^{t}||e(\tau,k)||\mathrm{d}\tau. \quad (17)$$

From Eq.(17), and taking λ-norm, we have

$$||e(t,k+1)||_\lambda\leqslant$$
$$||I-(H\varGamma\otimes\bar{B})||||e(t,k)||_\lambda+$$
$$c\frac{1-\mathrm{e}^{-\lambda T}}{\lambda}||e(t,k)||_\lambda=$$
$$(||I-(H\varGamma)\otimes\bar{B}||+c\frac{1-\mathrm{e}^{-\lambda T}}{\lambda})||e(t,k)||_\lambda. \quad (18)$$

Theorem 2 Under Assumptions 1–2, the ILC control law given in Eq.(11) ensures that the follower agents governed by Eq.(10) can track the leader perfectly if there exists a control gain matrix $\varGamma$ such that $\rho(I-(H\varGamma)\otimes\bar{B})<1$ holds.

Proof It is easy verified from Eq.(18) that if

$$\rho(||I-(H\varGamma\otimes\bar{B})||+c\frac{1-\mathrm{e}^{-\lambda T}}{\lambda})<1,$$

the ILC control law given in Eq.(11) ensures the follower agents governed by Eq.(10) can track the leader perfectly. And note that $\rho(I-(H\varGamma\otimes\bar{B}))<1$ can guarantee

$$\rho(||I-(H\varGamma\otimes\bar{B})||+c\frac{1-\mathrm{e}^{-\lambda T}}{\lambda})<1$$

by selecting λ sufficiently large. Then, the theorem is proved.

4 Simulations

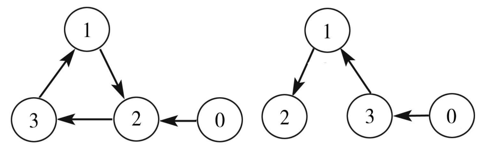
Example 1 Single-integrator MAS.

We consider three agents of the single-integrator dynamics $\dot{x}_i(t)=u_i(t),\ i=1,2,3$; and the communication graphs of the three agents and the leader are shown in Fig.1(a). It is easy to verify the leader node 0 is globally reachable in $\bar{G}$.

The trajectory of the leader node is $x_0(t)=\sin t$ and $t\in[0,10]$.

The weighted adjacency matrix is

$$A=\begin{bmatrix}0&0&0.5\\0.6&0&0\\0&0.1&0\end{bmatrix},\ B=\mathrm{diag}\{0,0.8,0\}.$$



(a) (b)

Fig. 1 Communication graphs of the three agents and the leader

Then we choose the control gains matrix $\varGamma=\mathrm{diag}\{2,1,3\}$. It is easily verify that $\rho(I-H\varGamma)=0.8646<1$.

The results of using iterative consensus protocol (4) with the iteration $k=80$ are shown in Fig.2.

Clearly, from Fig.2 we can infer that the perfect consensus will have been achieved because of the evolution of the norm of consensus errors. Moreover, the controls of the three agents are bounded at each iteration.

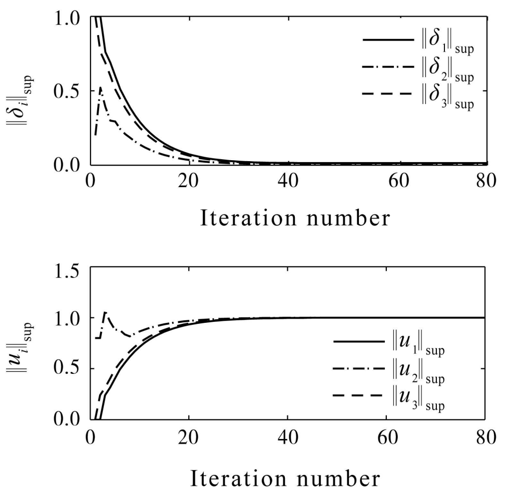


Fig. 2 Simulation results with iteration $k=80$

Example 2 General linear MAS.

We consider three agents of the following linear dynamics

$$\dot{x}_i(t)=\bar{A}x_i(t)+\bar{B}u_i(t),\ i=1,2,3,$$

where the coefficient matrixes

$$\bar{A}=\begin{bmatrix}0.35&0.1\\-0.1&0.2\end{bmatrix},\ \bar{B}=\begin{bmatrix}0.85&0\\0&0.85\end{bmatrix}.$$

The communication graphs of the three agents and the leader are shown in Fig.1(b). It is also easy to verify the leader node 0 is globally reachable in $\bar{G}$.

The weighted adjacency matrix

$$A = \begin{bmatrix} 0 & 0 & 0.8 \\ 0.65 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ B = \mathrm{diag}\{0, 0, 1.5\}.$$

Suppose that the trajectory of the leader node is $x_0(t) = [\sin t \ \ \cos t]^{\mathrm{T}}$ and $t \in [0, 8]$.

Then we choose the control gain matrix $\varGamma = \mathrm{diag}\{2, 1.5, 0.68\}$. It is easily verify that $\rho(I - (H\varGamma) \otimes \bar{B}) = 0.36 < 1$.

The results of using iterative consensus protocol (11) with the iteration $k = 60$ are shown in Fig.3.

Obviously, the algorithm has run 60 cycles and the simulation results in Fig.3 show that the three agent can track the leader perfectly and the controls of the three agents are bounded at each iteration.

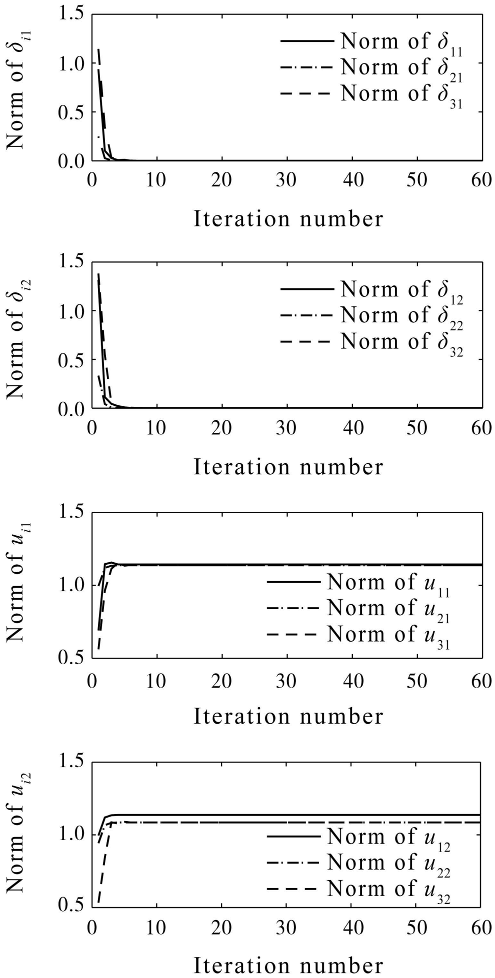


Fig. 3 Responses of the three agents under the protocol (11)

5 Conclusions

This paper has considered the ILC scheme for multi-agent consensus problems. ILC consensus algorithm for the single-integrator and the general linear multi-agent dynamics have given respectively. And we have derived some sufficient conditions to guarantee the multi-agent consensus in the given finite interval $[t_0, T]$ for the directed communication topologies. The following agents can reach the leader if there exists control gain matrix $\varGamma$ satisfying the conditions of the Theorems. The simulation results show that the consensus algorithms proposed in this paper are effective. How to determine an optimal learning gain matrix $\varGamma$ so as to improve the convergence performance and the robustness problem of the resetting errors will be studied further in the near future.

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