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一种间歇过程的综合预测迭代学习控制方法

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摘要: 为了提高迭代学习控制方法在间歇过程轨迹跟踪问题中的收敛速度, 本文将批次间的比例型迭代学习控制与批次内的模型预测控制相结合, 提出了一种综合应用方法. 首先根据间歇过程的线性模型, 预测出比例型迭代学习控制的系统输出, 然后在批次内采用模型预测控制, 通过极小化一个二次型目标函数来获得控制增量. 该方法可使系统输出跟踪期望轨迹的速度比比例型迭代学习控制方法更快些. 最后通过仿真实例验证了该方法的有效性.

关键词: 迭代学习控制; 模型预测控制; 综合控制; 间歇过程
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An integrated predictive iterative learning control for batch process

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Abstract: In order to improve the convergence speed of iterative learning control (ILC), an integrated scheme for tracking problem of batch process is proposed by combining batch-to-batch P-type ILC and within-batch model predictive control (MPC). Based on a predefined batch-wise linear model of the process, the output of traditional P-type ILC can be predicted, and then MPC is induced to minimize a quadratic objective function within the current batch. The input is updated within the batch so that the output may approach the reference trajectory faster. An illustrative example is presented to demonstrate the performance of the proposed scheme.

Key words: iterative learning control; model predictive control; integrated control; batch process

1 Introduction

Iterative learning control (ILC), which was introduced by Arimoto[1] in 1984, has been widely studied and used in the repetitive industrial processes and batch processes[2]. By using the input and tracking error information of previous batches in the control law, an open-loop control scheme is established to track the reference trajectory. Various ILC methods have been proposed in these years[3], and the convergence conditions have also been proved to guarantee asymptotical convergence of the methods[3–4].

Compared with other control methods, the key characteri stic of ILC is that the convergence condition may not be related to the system matrices. However, people usually have some knowledge of the studied process, so that it is reasonable to combine ILC with other control methods, like model predictive control (MPC), in which the input of the system is adjusted according to the forecast outputs of the predefined dynamic model. Furthermore, the original ILC can be considered as a feedforward control scheme. When ILC is combined with other feedback control method, better performance of tracking control can be obtained[3]. Chin et al[5] presented a batch MPC (BMPC) technique by incorporating the real-time feedback control into ILC, and later it is extended to the quadratic BMPC (QBMPC)[6]. Based on the quadratic criterion-based ILC (Q-ILC)[7], Xiong et al[8] also used shrinking horizon model predictive control (SHMPC) in the current batch to improve the tracking performance. Even so, integration of two different control schemes should be done carefully due to inconsistent predictions of the future time, for example, it is indicated that SHMPC may ‘undo’ the ILC corrections[9].

In spite of these researches mentioned above, few studies show that the integration strategy with the special control structure contributes to the improved convergence speed of the algorithm. Moreover, from literatures[3–4], it can be found that convergence conditions in most of ILC methods are only sufficient, and the convergence speed is not usually concerned when these control laws are utilized. In fact, when the learning rate satisfies traditional convergence conditions, it cannot directly induce the optimal convergent track. Xu and Tan[4] defined a Q-factor to describe the convergence speed in the nonlinear ILC, but it has been shown that the index

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is not very effective as expected.

In this paper, in order to improve the convergence speed, an integrated predictive ILC (IPILC) method for batch process is proposed. In this method, the input within the current batch is re-adjusted by involving MPC, while the traditional Proportion type (P-type) ILC uses input from batch to batch. Based on the predefined model of the process and the ILC outputs of the last batch, MPC is implemented and the correction of input in the current batch is determined by minimizing a quadratic objective function. Effectiveness of this proposed method is also verified by simulation.

The rest of the paper is organized as follows: Section 2 describes the problem, and Section 3 introduces the IPILC method for linear time-invariant (LTI) system. A numerical simulation is given in Section 4. Finally the conclusions are drawn.

2 Problem description

In this study, we consider a single input single output (SISO) discrete linear time-invariant (LTI) process, which is operated over finite time duration in the batch process. It is assumed that the process consist of N sampling intervals and all batches run from the same initial conditions. The process output and input sequences are defined respectively as follows:

$$Y_k = [y_k(1)\ y_k(2)\ \cdots\ y_k(N)]^{\mathrm{T}}, \tag{1}$$

$$U_k = [u_k(0)\ u_k(1)\ \cdots\ u_k(N-1)]^{\mathrm{T}}, \tag{2}$$

where the subscript k means the batch index. A convenient description of the batch process is given by[8]

$$Y_k = GU_k + d_k, \tag{3}$$

where G is a model of the studied process and d_k denotes the collective effects of disturbances, bias errors and measurement noise. Here d_k is supposed to be bounded by a certain small positive constant such that[8]

$$|d_k| < B_{\mathrm{d}}. \tag{4}$$

Due to the causality, the structure of model G is restricted to the following lower-block triangular form

$$G = \begin{bmatrix} g_{1,0} & 0 & \cdots & 0 \\ g_{2,0} & g_{2,1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ g_{N,0} & g_{N,1} & \cdots & g_{N,N-1} \end{bmatrix} \in \mathbb{R}^{N\times N}. \tag{5}$$

And model G can be found by identification from process operating data or by linearizing a predescribed mechanical model. Developing mechanical model is often difficult and time-consuming, while diverse numerical methods can be used to identify the model, for example, partial least squares (PLS), the Kalman filtering, and so on[10].

The task of the proposed control method is for the process (3) to track a given reference trajectory, which is defined as

$$Y_{\mathrm{d}} = [y_{\mathrm{d}}(1)\ y_{\mathrm{d}}(2)\ \cdots\ y_{\mathrm{d}}(N)]^{\mathrm{T}}. \tag{6}$$

Define the tracking error as

$$e_k(t) = y_{\mathrm{d}}(t) - y_k(t),\ t \in [1, N]. \tag{7}$$

Notice that the model G can be partitioned as a block column matrix according to the time index[10]

$$G = [g_1^{\mathrm{T}}\ \ g_2^{\mathrm{T}}\ \ \cdots\ g_N^{\mathrm{T}}]^{\mathrm{T}}, \tag{8}$$

where

$$g_i = (\underbrace{g_{i,0}, g_{i,1}, \cdots, g_{i,i-1},}_{i} 0, \cdots, 0). \tag{9}$$

It can be easily obtained that

$$y_k(t) = g_t \cdot U_k(t-1) + d_k(t),\ t \in [1, N], \tag{10}$$

where $U_k(k-1)$ is defined as

$$U_k(t-1) = U_k \text{ with } u_k(t) = \cdots = u_k(N-1) = 0. \tag{11}$$

So the tracking error can be represented as

$$e_k(t) = y_{\mathrm{d}}(t) - g_t U_k(t-1) - d_k(t),\ \forall t \in [1, N]. \tag{12}$$

3 Integrated predictive ILC for batch process

In our previous work[8], the input within the current batch is updated based on the control profile determined by the ILC in the immediately completed batch. Here the same idea is used, and the following control law is given:

$$u_k(t) = u_k^{\mathrm{ILC}}(t) + u_k^{\mathrm{MPC}}(t), \tag{13}$$

where $u_k^{\mathrm{ILC}}(t)$ presents the input decided by ILC from batch to batch, and $u_k^{\mathrm{MPC}}(t)$ denotes the input decided by MPC within the batch, respectively.

The above control law to be determined is partitioned in two parts, and their details are presented as follows.

3.1 Batch-to-batch iterative learning control

The part of $u_k^{\mathrm{ILC}}(t)$ is decided by batch-to-batch traditional iterative learning control, which is defined as

$$u_k^{\mathrm{ILC}}(t) = u_{k-1}(t) + K^{\mathrm{ILC}} \cdot e_{k-1}(t+1). \tag{14}$$

It is noted that in this traditional ILC, the learning rate can be designed as normal P-type ILC[3] or Q-ILC[7] control law. According to the convergence condition, K^{ILC} should satisfy the following inequality[2]:

$$\|I - G \cdot K^{\mathrm{ILC}}\| < 1. \tag{15}$$

It is implied that all eigenvalues of the matrix should be inside the unit circle.

3.2 Integrated batch-to-batch control and within-batch control

Under the batch-to-batch ILC, the process output of the current batch only depends on the recipe which is already calculated before the beginning of the batch. However, the convergence condition in Eq.(15) is a sufficient condition, and the convergence speed is not indicated and it may be optimal even if K^{ILC} is chosen as $\|I - G \cdot K^{\mathrm{ILC}}\| = 0$.

Within the current batch, MPC is a suitable scheme to improve the tracking performance because the process outputs can be predicted based on a dynamic model. Unlike the SHMPC[10], it is assumed here that both prediction step and control step in the MPC law are set to be $m(m \leqslant N - t)$. And output prediction

$\hat{y}_k(t+i|t)$ is calculated as follows:

$$\hat{y}_k(t+i|t) = g_{t+i} \cdot U_k(t+i-1). \tag{16}$$

It is noted that at time t of the current kth batch, $U_k(t-1)$ was already known. Thus the following equation for the time transition of output within the batch is obtained:

$$\hat{Y}_k^{\mathrm{ILC}}\left({}^{t+1}_{t+m}|t\right) = G_{pt} \cdot U_k(t-1) + G_{mt} \cdot U_k^{\mathrm{ILC}}\left({}^{t}_{t+m-1}|t-1\right), \tag{17}$$

where

$$F_k\left({}^{t_1}_{t_2}|t\right) = (f_k(t_1|t), \cdots, f_k(t_2|t))^{\mathrm{T}}, \quad F \in \{Y, U, E\},\ f \in \{y, u, e\}, \tag{18}$$

$$G_{pt} = \begin{bmatrix} g_{t+1,0} & g_{t+1,1} & \cdots & g_{t+1,t-1} & 0 \cdots 0 \\ g_{t+2,0} & g_{t+2,1} & \cdots & g_{t+2,t-1} & 0 \cdots 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \quad \vdots \\ g_{t+m,0} & g_{t+m,1} & \cdots & g_{t+m,t-1} & 0 \cdots 0 \end{bmatrix}_{m \times N}, \tag{19}$$

$$G_{mt} = \begin{bmatrix} g_{t+1,t} & 0 & \cdots & 0 \\ g_{t+2,t} & g_{t+2,t+1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ g_{t+m,t} & g_{t+m,t+1} & \cdots & g_{t+m,t+m-1} \end{bmatrix}_{m \times m}. \tag{20}$$

After the re-adjustment of input in Eq.(14) is used, the system output can be estimated at time $t(t+m \leqslant N)$

$$\hat{Y}_k\left({}^{t+1}_{t+m}|t\right) = G_{pt} \cdot U_k(t-1) + G_{mt} \cdot \left(U_k^{\mathrm{ILC}}\left({}^{t}_{t+m-1}|t-1\right) + U_k^{\mathrm{MPC}}\left({}^{t}_{t+m-1}|t-1\right)\right). \tag{21}$$

When $t(t+m > N)$, the predict step m should be shrunk, and then G_{pt} and G_{mt} are changed as

$$G_{pt} = \begin{bmatrix} g_{t+1,0} & g_{t+1,1} & \cdots & g_{t+1,t-1} & 0 \cdots 0 \\ g_{t+2,0} & g_{t+2,1} & \cdots & g_{t+2,t-1} & 0 \cdots 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \quad \vdots \\ g_{N,0} & g_{N,1} & \cdots & g_{N,t-1} & 0 \cdots 0 \end{bmatrix}_{(N-t) \times N}, \tag{22}$$

$$G_{mt} = \begin{bmatrix} g_{t+1,t} & 0 & \cdots & 0 \\ g_{t+2,t} & g_{t+2,t+1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ g_{N,t} & g_{N,t+1} & \cdots & g_{N,N-1} \end{bmatrix}_{(N-t) \times (N-t)}. \tag{23}$$

The tracking error based on the model prediction is then estimated by

$$\hat{E}_k\left({}^{t+1}_{t+m}|t\right) = Y_{\mathrm{d}}\left({}^{t+1}_{t+m}|t\right) - G_{pt} \cdot U_k(t-1) - G_{mt}\left(U_k^{\mathrm{ILC}}\left({}^{t}_{t+m-1}|t-1\right) + U_k^{\mathrm{MPC}}\left({}^{t}_{t+m-1}|t-1\right)\right). \tag{24}$$

For the MPC method in the current batch, the following quadratic objective function is considered:

$$\min_{U_k^{\mathrm{MPC}}\left({}^{t}_{t+m-1}|t-1\right)} J_k\left({}^{t}_{t+m}|t\right) = \left(\hat{E}_k\left({}^{t+1}_{t+m}|t\right)\right)^{\mathrm{T}} Q \hat{E}_k\left({}^{t+1}_{t+m}|t\right) + \left(U_k^{\mathrm{MPC}}\left({}^{t}_{t+m-1}|t-1\right)\right)^{\mathrm{T}} R U_k^{\mathrm{MPC}}\left({}^{t}_{t+m-1}|t-1\right), \tag{25}$$

where Q and R are positive definitive matrices.

The unconstrained case is considered here, then an analytical solution to the objective function (25) can be obtained as

$$U_k^{\mathrm{MPC}}\left({}^{t}_{t+m-1}|t-1\right) = P \cdot G_{mt} Q \hat{E}_k^{\mathrm{ILC}}\left({}^{t+1}_{t+m}|t\right), \tag{26}$$

where

$$\begin{cases} P = [G_{mt}^{\mathrm{T}} Q G_{mt} + R]^{-1}, \\ \hat{E}_k^{\mathrm{ILC}}\left({}^{t+1}_{t+m}|t\right) = Y_{\mathrm{d}}\left({}^{t+1}_{t+m}|t\right) - G_{pt} U_k(t-1) - \\ \qquad G_{mt} \cdot U_k^{\mathrm{ILC}}\left({}^{t}_{t+m-1}|t-1\right). \end{cases} \tag{27}$$

At each time t within the current batch, the objective function (25) is solved, the first element $u_k^{\mathrm{MPC}}(t)$ of input (26) is implemented to the process.

3.3 Summary of the algorithm

The procedure of the proposed integrated control scheme is as follows:

Step 1 Base on the historical batch operation data and appropriate method, obtain the dynamic model G. According to convergence condition (15), the ILC learning rate K^{ILC} is selected. Set $k = 1$.

Step 2 Before the beginning of a new batch, update the ILC control input $u_k^{\mathrm{ILC}}(t)$ according to equation (14). Set time step $t = 1$.

Step 3 At time t in the kth batch, obtain the estimated output according to Eq.(17), and calculate the re-adjusted input $u_k^{\mathrm{MPC}}(t)$ according to Eq.(26). Implement the $u_k(t)$ according to Eq.(13).

Step 4 If $t < N$, set $t = t + 1$ and go back to Step 3, else set $k = k + 1$ and go to Step 2.

3.4 Analysis of the algorithm

The convergence of the traditional ILC algorithm is presented and proved directly in previous works[10]. In this paper, the process outputs are predicted based on the predetermined model of the process. Then by involving the MPC method within the batch, a re-adjusted input can approach the reference input much faster. The most important advantage of the proposed method is that the algorithm can improve the convergence speed and tracking performance while the convergence condition is not changed.

However, the convergence speed and tracking performance are still affected by the matrices Q and R in the quadratic objective function in MPC. A larger eigenvalue of Q and a smaller one of R may cause faster convergence which is more sensitive to disturbances. A smaller eigenvalue of Q and a larger eigenvalue of R depict the opposite. The robustness are still worthy of study[11].

The tracking performance also depends on the accuracy of model G[10]. And it is also shown that perfect tracking will be obtained when the term d_k in the process (3) is a repetitive disturbance for all batch index k[8].

4 Numerical simulation

Consider the following linear time-invariant discrete system to illustrate the IPILC algorithm

$$y_k(t) = 0.8u_k(t) + 0.5u_k(t-1) - 0.3u_k(t-2) + d_k(t), \qquad (28)$$

where the initial condition are $u_k(0) = 0$ and $y_k(0) = 0$. The initial input sequence of IPILC is set as $u_1(t-1) = 0$ for all $t = 1, 2, \cdots, 21$. The disturbance d_k is uniformly distributed between $[-0.05, 0.05]$. Reference trajectory is set as $y_\mathrm{d}(t) = 0.1(t-1)$ for all $t = 1, 2, \cdots, 21$.

The P-type ILC is used while the learning rate is chosen as K^{ILC}, which satisfies the convergence condition in Eq.(15). After some simulations, the positive matrices Q and R are determined to be set as $Q = R = I$. The prediction step in MPC is set as $m = 3$.

The sum-square-error (SSE) of tracking error is used to illustrate the tracking performance, which is defined as

$$\mathrm{Err} = \sum_{t=1}^{N} [y_\mathrm{d}(t) - y_k(t)]^2. \qquad (29)$$

Fig.1 shows the tracking performance of the system output at different time steps and batches when the IPILC is used. It can be seen in Figure 1 that outputs can quickly converge to reference profile under the IPILC method.

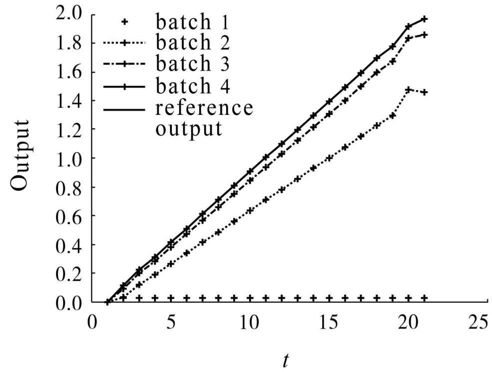

Fig. 1 Tracking performance of the system output at different time steps and batches under IPILC

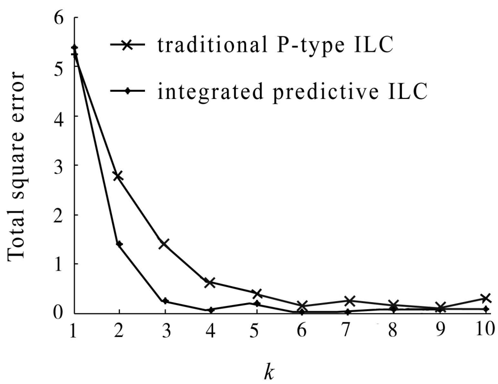

Fig. 2 Tracking performance of IPILC compared with the traditional ILC

Fig.2 shows the SSE of tracking error under IPILC compared with the traditional ILC, in which the learning rate is also set. It can be also found that the convergence speed of the proposed IPILC algorithm is faster than the traditional ILC.

5 Conclusions

By involving within-batch MPC into traditional batch-to-batch P-type ILC, the input can be updated within the batch so that the output may approach the reference trajectory faster. The proposed scheme is illustrated on a linear time-invariant system, and simulation results demonstrate that the convergence speed of tracking performance is improved.

References:

[1] ARIMOTO S, KAWAMURA S, MIYAZAKI F. Bettering operation of robotics by learning [J]. *Journal of Robotic System*, 1984, 12(2): 123 – 140.

[2] LEE J H, LEE K S. Iterative learning control applied to batch processes: an overview [J]. *Control Engineering Practice*, 2007, 15(10): 1037 – 1046.

[3] WANG Y Q, GAO F, DOYLE F J. Survey on iterative learning control, repetitive control, and run-to-run control [J]. *Journal of Process Control*, 2009, 19(10): 1589 – 1600.

[4] XU Jianxin, TAN Ying. Robust optimal design and convergence properties analysis of iterative learning control approaches [J]. *Automatica*, 2002, 38(11):1867 – 1880.

[5] CHIN I S, LEE K S, LEE J H. A technique for integrated quality control, profile control, and constraint handling for batch processes [J]. *Industrial and Engineering Chemistry Research*, 2000, 39(3):693 – 705.

[6] LEE K S, LEE J H. Iterative learning control-based batch process control technique for integrated control of end product properties and transient profiles of process variables [J]. *Journal of Process Control*, 2003, 13(7): 607 – 621.

[7] LEE J H, LEE K S, KIM W C. Model-based iterative learning control with quadratic criterion for time-varying linear systems [J]. *Automatica*, 2000, 36(5): 641 – 657.

[8] XIONG Zhihua, ZHANG Jie. Tracking control for batch processes through integrating batch-to-batch iterative learning control and within-batch on-line control [J]. *Industrial and Engineering Chemistry Research*, 2005, 44(11): 3983 – 3992.

[9] LEE K S, LEE J H, YANG D R, et al. Integrated run-to-run and on-line model-based control of particle size distribution for a semi-batch precipitation reactor [J]. *Computers and Chemical Engineering*, 2002, 26(7/8): 1117 – 1131.

[10] XIONG Zhihua, ZHANG Jie. Product quality trajectory tracking in batch processes using iterative learning control based on time-varying perturbation models [J]. *Industrial and Engineering Chemistry Research*, 2003, 42(26): 6802 – 6814.

[11] FANG Zhong, HAN Zhengzhi, CHEN Pengnian. Recent developments of iterative learning control [J]. *Control Theroy & Applications*, 2002, 19(2): 161 – 166.
(方忠, 韩正之, 陈彭年. 迭代学习控制新进展 [J]. 控制理论与应用, 2002, 19(2): 161 – 166.)

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