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基于几何分析的分布参数系统的迭代学习控制

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摘要: 研究了一类不确定非线性分布参数系统的迭代学习控制问题. 基于几何分析方法, 给出了分布参数系统一种新的具有自适应因子的非线性迭代学习控制算法. 导出了新算法的收敛条件, 并利用广义λ范数从理论上证明了新算法的收敛性.

关键词: 迭代学习控制; 算法; 几何分析; 分布参数系统

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Iterative learning control of distributed parameter systems based on geometric analysis

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Abstract: Iterative learning control problem for uncertain nonlinear distributed parameter systems is discussed. Based on geometric analysis, a new nonlinear iterative learning control algorithm with adaptive factor is proposed for distributed parameter systems. Furthermore, the convergence conditions of the new algorithm are deduced, and the new algorithm convergence is proved by employing the generalized λ norm.

Key words: iterative learning control; algorithm; geometric analysis; distributed parameter systems

1 Introduction

In practical engineering, distributed parameter systems have extensive application, which are described by partial differential equation or partial differential integral equation. And there are many research results on the control problems for this kind of systems[1–4]. Iterative learning control algorithm was emphasized since it was proposed by Arimoto[5] in 1984. Now, many research results[6–10] relating to it have been produced, which makes it popular. Up to now, the research results applying iterative learning control to distributed parameter systems are not fruitful. Xie et al[11–12] discussed the iterative learning control problems for linear and nonlinear distributed parameter systems and proposed linear iterative learning control algorithms. Dai et al[13] discussed the iterative learning control problems for uncertain distributed parameter systems by employing geometric analysis method[14] and proposed a nonlinear iterative learning control algorithm by adjusting the first term on the right of the normal P-type algorithm.

Based on the geometric analysis method similar to that in [13–14], this paper studies the iterative learning control problem of an uncertain distributed parameter system, proposes a nonlinear iterative learning control algorithm with adaptive factor by adjusting the second term on the right of the normal P-type algorithm, and proves that the new algorithm is convergent by employing the norm defined literature [14].

2 Problem statement and new algorithm

Consider an uncertain distributed parameter systems in the following form:

$$\begin{cases} \dfrac{\partial Q(x,t)}{\partial t} = D\Delta Q(x,t) + A(t)Q(x,t) + \\ \qquad\qquad B(t)u(x,t), \\ y(x,t) = C(t)Q(x,t) + G(t)u(x,t), \\ (x,t) \in \Omega \times [0,T], \end{cases} \tag{1}$$

where $Q\in\mathbb{R}^n$, $u\in\mathbb{R}^m$, $y\in\mathbb{R}^l$, $D, A\in\mathbb{R}^{n\times n}$, $B\in\mathbb{R}^{n\times m}$, $G\in\mathbb{R}^{l\times m}$, and A, B, C are uncertain bounded matrices, D is a diagonal matrix, $D=\mathrm{diag}\{d_1, d_2, \cdots, d_n\}$, $0 < d \leqslant d_i < \infty (i = 1, 2, \cdots, n)$, d is given, $\Delta = \sum\limits_{i=1}^{q} \dfrac{\partial^2}{\partial x_i^2}$ is a Laplace operator on Ω, and Ω is an open bounded domain of $\mathbb{R}^q$ with smooth boundary $\partial\Omega$.

Assume that the initial-boundary condition of system (1) is

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$$\alpha Q(x,t)+\beta\frac{\partial Q(x,t)}{\partial\nu}=0,\ (x,t)\in\partial\Omega\times\mathbb{R}^+, \quad (2)$$

$$Q(x,0)=Q_0(x),\ x\in\Omega, \quad (3)$$

where $\alpha=\mathrm{diag}\{\alpha_1,\alpha_2,\cdots,\alpha_n\}$, $\beta=\mathrm{diag}\{\beta_1,\beta_2,\cdots,\beta_n\}$, $\alpha_i\geqslant 0$, $\beta_i>0$, $\dfrac{\partial}{\partial\nu}$ is the outward normal derivative on $\partial\Omega$.

For the controll object described by system (1), the desired output we expect is $y_\mathrm{d}(x,t)$. Now, our target is to look for a corresponding desired input $u_\mathrm{d}(x,t)$, so that the actual output of system (1)

$$y^*(x,t)=C(t)Q_\mathrm{d}(x,t)+G(t)u_\mathrm{d}(x,t)$$

will be close to its desired output $y_\mathrm{d}(x,t)$. Since system (1) is uncertain, it is not easy to get the desired control. We will use iterative learning control method to gradually get the control sequence $\{u_k(x,t)\}$, so that

$$\lim_{k\to\infty}u_k(x,t)=u_\mathrm{d}(x,t).$$

The basic P-type iterative learning control algorithm to obtain the control sequence $\{u_k(x,t)\}$ is

$$u_{k+1}(x,t)=u_k(x,t)+\Gamma(t)e_k(x,t), \quad (4)$$

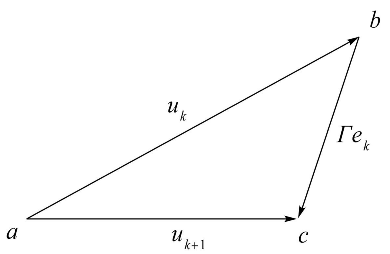
where $e_k(x,t)=y_\mathrm{d}(x,t)-y_k(x,t)$ is output error.

The basic idea of geometric analysis is: to make a sequence approach zero fast, the norm of the vector corresponding to the sequence only needs to become smaller and tend to zero fast. Regarding every term in Eq.(4) as a vector, we can obtain a graph of geometric analysis as Fig.1(a).

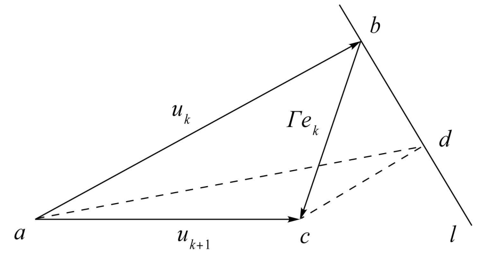
In order to make sequence $\{u_k(x,t)\}$ convergent fast, the norm $\|\Gamma(t)e_k(x,t)\|$ only needs to become smaller and tends to zero fast. For this reason, we draw a vertical line l of vector u_k through point b as Fig.1(b). When $\vec{cd}\ /\!\!/\ u_k$, similar to the analysis of literature [14], we can get an iterative learning algorithm as follows:

$$u_{k+1}(x,t)=u_k(x,t)+\Gamma(t)e_k(x,t)-\sigma\frac{u_k^\mathrm{T}(x,t)\Gamma(t)e_k(x,t)}{u_k^\mathrm{T}(x,t)u_k(x,t)}u_k(x,t), \quad (5)$$

where $t\in[0,T]$, $\sigma=a(1-\exp(-b\|e_k(x,t)\|))$, $(a,b)\in(0,1)\times[0,+\infty)$ are adjustable constant, determining the change condition of adaptive factor σ corresponding to error change. Because $(a,b)\in(0,1)\times[0,+\infty)$, $\sigma\in(0,1)$. Let $y_k(x,t)$ be the kth output of system (1) corresponding to the kth input $u_k(x,t)$, $\Gamma(t)$ is a learning gain matrix.



(a)



(b)

Fig. 1 Graph of geometric analysis

In the course of learning, we suppose the initial states of system (1) is in the following form:

$$Q_k(x,0)=\varphi_k(x),\ x\in\Omega,\ k=1,2,3,\cdots, \quad (6)$$

$$\|\varphi_{k+1}(x)-\varphi_k(x)\|^2_{L_2}\leqslant lr^k,\ r\in[0,1),\ l>0. \quad (7)$$

3 Convergence of the new algorithm

We take the norm

$$\|e_k\|_{(L^2,\lambda(\xi))}=\sup_{0\leqslant t\leqslant T}\{(\|e_k(\cdot,t)\|^2_{L^2}\mathrm{e}^{-\lambda t})\xi^k\}$$

in the following discussion.

Lemma 1[13] If ρ, $\sigma\in(0,1)$, and for positive constants d, l, $\dfrac{dl\rho\sigma}{1-\rho}<1$ is satisfied, then there exist $\xi>1$ and appropriate large number $\lambda>0$, so that for the positive c, h, $F(\lambda,\xi)\in(0,1)$, where

$$F(\lambda,\xi)=\frac{\xi\rho}{1-\xi\rho}(\frac{\xi cdl(1+\sigma)^2}{\lambda-h}+\sigma\xi dl). \quad (8)$$

Lemma 2 For the appropriate large λ and $\xi>1$, we have the following estimation

$$\|\bar{Q}_k(\cdot,t)\|_{(L^2,\lambda(\xi))}\leqslant l(r\xi)^k+\frac{g}{\lambda-h}(1+\sigma)^2b_\Gamma\|e_k\|_{(L^2,\lambda(\xi))}, \quad (9)$$

where

$$\bar{Q}_k(x,t)=Q_{k+1}(x,t)-Q_k(x,t),$$
$$b_\Gamma=\max_{0\leqslant t\leqslant T}\|\Gamma(t)\|^2,\ g=\max_{0\leqslant t\leqslant T}(\lambda_{\max}(B(t))),$$
$$h=\max_{0\leqslant t\leqslant T}(\lambda_{\max}(A(t)+A^\mathrm{T}(t))+\lambda_{\max}(B(t))).$$

Proof Because

$$\frac{\partial}{\partial t}(Q_{k+1}(x,t)-Q_k(x,t))=D\Delta(Q_{k+1}(x,t)-Q_k(x,t))+A(t)(Q_{k+1}(x,t)-Q_k(x,t))+B(t)(u_{k+1}(x,t)-u_k(x,t)). \quad (10)$$

Using $(Q_{k+1}(x,t)-Q_k(x,t))^\mathrm{T}$ to left-multiply both sides of Eq.(10), then it becomes

$$\frac{\partial}{\partial t}(\bar{Q}_k^\mathrm{T}(x,t)\bar{Q}_k(x,t))=\bar{Q}_k^\mathrm{T}(x,t)D\Delta\bar{Q}_k(x,t)+\bar{Q}_k^\mathrm{T}(x,t)A(t)\bar{Q}_k(x,t)+\bar{Q}_k^\mathrm{T}(x,t)B(t)\bar{u}_k(x,t),$$

where $\bar{u}_k(x,t)=u_{k+1}(x,t)-u_k(x,t)$.

Integrating formula above about x on Ω, and employing Ostrogradsky-Gauss formula, we can get

$$\frac{\mathrm{d}}{\mathrm{d}t}(\|\bar{Q}_k(\cdot,t)\|_{L^2}^2)=$$
$$2\int_\Omega \bar{Q}_k^{\mathrm{T}}(x,t)D\Delta\bar{Q}_k(x,t)\mathrm{d}x+$$
$$\int_\Omega \bar{Q}_k^{\mathrm{T}}(x,t)(A^{\mathrm{T}}(t)+A(t))\bar{Q}_k(x,t)\mathrm{d}x+$$
$$2\int_\Omega \bar{Q}_k^{\mathrm{T}}(x,t)B(t)\bar{u}_k(x,t)\mathrm{d}x\leqslant$$
$$2\int_{\partial\Omega} \bar{Q}_k^{\mathrm{T}}(x,t)D\Delta\bar{Q}_k(x,t)\mathrm{d}S-$$
$$2\int_\Omega \nabla\bar{Q}_k^{\mathrm{T}}(x,t)D\nabla\bar{Q}_k(x,t)\mathrm{d}x+$$
$$\lambda_{\max}(A^{\mathrm{T}}(t)+A(t))\|\bar{Q}_k(\cdot,t)\|_{L^2}^2+$$
$$\lambda_{\max}(B(t))(\|\bar{Q}_k(\cdot,t)\|_{L^2}^2+\|\bar{u}_k(\cdot,t)\|_{L^2}^2).$$

Due to

$$\bar{Q}_k^{\mathrm{T}}(x,t)D\Delta\bar{Q}_k(x,t)|_{x\in\partial\Omega}=$$
$$\bar{Q}_k^{\mathrm{T}}(x,t)D\left(-\beta^{-1}\alpha\bar{Q}_k^{\mathrm{T}}(x,t)\right)|_{x\in\partial\Omega}\leqslant 0.$$

And then

$$\frac{\mathrm{d}}{\mathrm{d}t}(\|\bar{Q}_k(\cdot,t)\|_{L^2}^2)\leqslant$$
$$-2d\|\nabla\bar{Q}_k(\cdot,t)\|_{L^2}^2+\lambda_{\max}(A^{\mathrm{T}}(t)+$$
$$A(t))\|\bar{Q}_k(\cdot,t)\|_{L^2}^2+$$
$$\lambda_{\max}(B(t))(\|\bar{Q}_k(\cdot,t)\|_{L^2}^2+\|\bar{u}_k(\cdot,t)\|_{L^2}^2)\leqslant$$
$$-2d\|\nabla\bar{Q}_k(\cdot,t)\|_{L^2}^2+h\|\bar{Q}_k(\cdot,t)\|_{L^2}^2+g\|\bar{u}_k(\cdot,t)\|_{L^2}^2\leqslant$$
$$h\|\bar{Q}_k(\cdot,t)\|_{L^2}^2+g\|\bar{u}_k(\cdot,t)\|_{L^2}^2. \tag{11}$$

Integrating (11) about t, and employing Bellman-Gronwall inequality, we can get

$$\|\bar{Q}_k(\cdot,t)\|_{L^2}^2\leqslant\|\bar{Q}_k(\cdot,0)\|_{L^2}^2\mathrm{e}^{ht}+ g\int_0^t \mathrm{e}^{h(t-s)}\|\bar{u}_k(\cdot,s)\|_{L^2}^2\mathrm{d}s.$$

Choosing appropriate large positive number λ, so that $\lambda>h$, multiply both sides of the formula above by $\mathrm{e}^{-\lambda t}\xi^k$, then it becomes

$$\|\bar{Q}_k(\cdot,t)\|_{L^2}^2\mathrm{e}^{-\lambda t}\xi^k\leqslant$$
$$\|\bar{Q}_k(\cdot,t)\|_{L^2}^2\mathrm{e}^{-(\lambda-h)t}\xi^k+$$
$$g\int_0^t \mathrm{e}^{-(\lambda-h)(t-s)}\|\bar{u}_k(\cdot,s)\|_{L^2}^2\mathrm{e}^{-\lambda s}\xi^k\mathrm{d}s\leqslant$$
$$\|\bar{Q}_k(\cdot,0)\|_{L^2}^2\xi^k+$$
$$g\|u_{k+1}-u_k\|_{(L^2,\lambda(\xi))}\int_0^t \mathrm{e}^{-(\lambda-h)(t-s)}\mathrm{d}s\leqslant$$
$$l(r\xi)^k+\frac{g}{\lambda-h}\|u_{k+1}-u_k\|_{(L^2,\lambda(\xi))}.$$

From Eq.(5) it follows that

$$\|u_{k+1}(x,t)-u_k(x,t)\|\leqslant(1+\sigma)\|\Gamma(t)e_k(x,t)\|,$$

then

$$\|u_{k+1}-u_k\|_{(L^2,\lambda(\xi))}\leqslant(1+\sigma)^2 b_\Gamma\|e_k\|_{(L^2,\lambda(\xi))},$$

so

$$\|\bar{Q}_k(\cdot,t)\|_{(L^2,\lambda(\xi))}\leqslant$$
$$l(r\xi)^k+\frac{g}{\lambda-h}(1+\sigma)^2 b_\Gamma\|e_k\|_{(L^2,\lambda(\xi))}. \tag{12}$$

The following are the main results of this paper.

Theorem 1 If the parameter σ and learning gain matrix $\Gamma(t)$ of the algorithm (5) satisfy

1) $\|I-G(t)\Gamma(t)\|^2\leqslant\rho, 3\rho+r<1$; 2) $\dfrac{9\rho\sigma b_G b_\Gamma}{1-3\rho}<1$. Then, algorithm defined by Eq.(5) is uniformly convergent on $[0,T]$ and

$$\lim_{k\to\infty}\|e_k(x,t)\|_{L^2}=0,\ \forall t\in[0,T],$$

where $b_G=\max\limits_{0\leqslant t\leqslant T}\|G(t)\|^2$.

Proof From system (1) and algorithm (5) it follows that

$$e_{k+1}(x,t)=e_k(x,t)-y_{k+1}(x,t)+y_k(x,t)=$$
$$e_k(x,t)-G(t)(u_{k+1}(x,t)-u_k(x,t))-$$
$$C(t)(Q_{k+1}(x,t)-Q_k(x,t))=$$
$$(I-G(t)\Gamma(t))e_k(x,t)-$$
$$C(t)(Q_{k+1}(x,t)-Q_k(x,t))+$$
$$\sigma G(t)\frac{u_k^{\mathrm{T}}(x,t)\Gamma(t)e_k(x,t)}{u_k^{\mathrm{T}}(x,t)u_k(x,t)}u_k(x,t)=$$
$$\bar{e}_k(x,t)+\bar{C}_k(x,t)+\bar{G}_k(x,t),$$

where

$$\bar{e}_k(x,t)=(I-G(t)\Gamma(t))e_k(x,t),$$
$$\bar{C}_k(x,t)=-C(t)(Q_{k+1}(x,t)-Q_k(x,t)),$$
$$\bar{G}_k(x,t)=\sigma G(t)\frac{u_k^{\mathrm{T}}(x,t)\Gamma(t)e_k(x,t)}{u_k^{\mathrm{T}}(x,t)u_k(x,t)}u_k(x,t).$$

And then

$$\bar{e}_{k+1}^{\mathrm{T}}(x,t)\bar{e}_{k+1}(x,t)=$$
$$(\bar{e}_k^{\mathrm{T}}(x,t)+\bar{C}_k^{\mathrm{T}}(x,t)+\bar{G}_k^{\mathrm{T}}(x,t))\cdot$$
$$(\bar{e}_k(x,t)+\bar{C}_k(x,t)+\bar{G}_k(x,t))\leqslant$$
$$3(\bar{e}_k^{\mathrm{T}}(x,t)\bar{e}_k(x,t)+\bar{C}_k^{\mathrm{T}}(x,t)\bar{C}_k(x,t)+$$
$$\bar{G}_k^{\mathrm{T}}(x,t)\bar{G}_k(x,t))=$$
$$3(\|\bar{e}_k(x,t)\|^2+\|\bar{C}_k(x,t)\|^2+\|\bar{G}_k(x,t)\|^2)\leqslant$$
$$3\|I-G(t)\Gamma(t)\|^2\|e_k(x,t)\|^2+$$
$$3\|C(t)\|^2\|\bar{Q}_k(x,t)\|^2+3\sigma\|G(t)\|^2\|\Gamma(t)e_k(x,t)\|^2\leqslant$$
$$3\rho\|e_k(x,t)\|^2+3b_C\|\bar{Q}_k(x,t)\|^2+$$
$$3\sigma b_G b_\Gamma\|e_k(x,t)\|^2, \tag{13}$$

where $b_C=\max\limits_{0\leqslant t\leqslant T}\{\|C(t)\|^2\}$.

Integrating (13) for $x\in\Omega$, we get

$$\|e_{k+1}(\cdot,t)\|_{L^2}^2\leqslant$$
$$3\rho\|e_k(x,t)\|_{L^2}^2+3b_C\|\bar{Q}_k(x,t)\|_{L^2}^2+$$
$$3\sigma b_G b_\Gamma\|e_k(x,t)\|_{L^2}^2.$$

And then we have

$$\|e_k(\cdot,t)\|_{L^2}^2\leqslant$$
$$(3\rho)^k\|e_0(\cdot,t)\|_{L^2}^2+\sum_{i=1}^{k-1}(3\rho)^{k-i-1}(3b_C\|\bar{Q}_i(\cdot,t)\|_{L^2}^2+$$
$$3\sigma b_G b_\Gamma\|e_i(\cdot,t)\|^2).$$

Multiplying both sides of formula above using $\xi^k\mathrm{e}^{-\lambda t}(\xi>1)$ and employing Eq.(12), then it becomes

$$(\|e_k(\cdot,t)\|_{L^2}^2\xi^k)\mathrm{e}^{-\lambda t}\leqslant$$

$$
\begin{aligned}
&(3\rho\xi)^k\|e_0(\cdot,t)\|_{L^2}^2\mathrm{e}^{-\lambda t}+\\
&\sum_{i=1}^{k-1}(3\xi\rho)^{k-i-1}\xi(3b_Cl(\xi r)^i+\\
&\frac{3b_Cg}{\lambda-h}(1+\sigma)^2b_\Gamma\|e_i\|_{(L^2,\lambda(\xi))}+\\
&3\sigma b_Gb_\Gamma\|e_i\|_{(L^2,\lambda(\xi))})\leqslant\\
&\|e_0\|_{(L^2,\lambda)}+\sum_{i=1}^{k-1}(3\xi\rho)^{k-i-1}\xi(3b_Cl(\xi r)^i+\\
&\frac{3b_Cg}{\lambda-h}(1+\sigma)^2b_\Gamma\|e_i\|_{(L^2,\lambda(\xi))}+\\
&3\sigma b_Gb_\Gamma\|e_i\|_{(L^2,\lambda(\xi))}).
\end{aligned}\tag{14}
$$

So, if λ is sufficiently large, then from Lemma 1, we have $\xi_1>1$, making $F(\lambda,\xi_1)<1$, and $\xi_1(3\rho+r)<1$. Replacing the previous ξ with ξ_1, then $F(\lambda,\xi_1)=F(\lambda,\xi)$, where

$$F(\xi,\lambda)=\frac{3\xi\rho}{1-3\xi\rho}(\frac{3\xi gb_Cb_\Gamma(1+\sigma)^2}{\lambda-h}+3\xi\sigma b_Gb_\Gamma).$$

From Eq.(14), we get

$$
\begin{aligned}
&(\|e_k(\cdot,t)\|_{L^2}^2\xi^k)\mathrm{e}^{-\lambda t}\leqslant\\
&\|e_0\|_{(L^2,\lambda)}+\sum_{i=1}^{k-1}(3\xi\rho)^{k-i-1}3\xi b_Cl(\xi r)^i+\\
&F(\xi,\lambda)\sup_{1\leqslant i\leqslant k}\{\|e_i\|_{(L^2,\lambda(\xi))}\},
\end{aligned}
$$

then

$$
\begin{aligned}
&\|e_i\|_{(L^2,\lambda(\xi))}\leqslant\\
&\|e_0\|_{(L^2,\lambda)}+3b_Cl\xi(\xi(3\rho+r))^{k-1}+\\
&F(\xi,\lambda)\sup_{1\leqslant i\leqslant k}\{\|e_i\|_{L^2,\lambda(\xi)}\}\leqslant\\
&\|e_0\|_{(L^2,\lambda)}+3b_Cl\xi+F(\xi,\lambda)\sup_{1\leqslant i\leqslant k}\{\|e_i\|_{(L^2,\lambda(\xi))}\}.
\end{aligned}
$$

Because $F(\lambda,\xi)\in(0,1)$, and

$$\sup_{1\leqslant i\leqslant k}\{\|e_i\|_{L^2,\lambda(\xi)}\}\leqslant\frac{\|e_0\|_{(L^2,\lambda)}+3b_Clr^k}{1-F(\xi,\lambda)}.$$

And then

$$
\begin{aligned}
\|e_k(\cdot,t)\|_{L^2}^2&\leqslant\xi^{-k}\mathrm{e}^{\lambda t}\sup_{1\leqslant i\leqslant k}\{\|e_i\|_{(L^2,\lambda(\xi))}\}\leqslant\\
&\xi^{-k}\mathrm{e}^{\lambda T}\frac{\|e_0\|_{(L^2,\lambda)}+3b_C\xi}{1-F(\xi,\lambda)},
\end{aligned}
$$

since $\xi>1$, then

$$\lim_{k\to\infty}\|e_k(\cdot,t)\|_{L^2}^2=0,\ \forall t\in[0,T].$$

The uniform convergence of $\{u_k(x,t)\}$ can be proved from the previous process.

4 Conclusions

This paper has discussed the iterative learning control problem for distributed parameter systems, which has extensive application in practical engineering. Using the method of geometric analysis, we have obtained a new nonlinear iterative learning control algorithm of distributed parameter systems. And the research we do is not only an exploration for the iterative learning control applied to distributed parameter systems, but also an enrichment for the iterative learning control method based on geometric analysis.

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