具有初始状态不确定性的非线性系统脉冲补偿迭代学习控制

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摘要: 针对于具有初始状态不确定性的非线性时不变系统, 采用矩形脉冲信号补偿传统的比例微分型一阶和二阶迭代学习控制律. 在Lebesgue-p范数度量跟踪误差意义下, 利用卷积的推广的Young不等式分析学习控制律的跟踪性能. 分析表明, 在适当选取比例学习增益, 微分学习增益和非线性状态函数的Lipschitz常数以保证收敛因子小于1的前提下, 渐近跟踪误差是由初始状态不确定性引起的, 而且可通过调节补偿因子予以消减. 数值仿真验证了补偿策略的有效性和理论分析的正确性.

关键词: 迭代学习控制; 非线性系统; 脉冲补偿; 初始状态不确定性; Lebesgue-p范数

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Pulse compensated iterative learning control to nonlinear systems with initial state uncertainty

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Abstract: A type of rectangular pulse is adopted to compensate for conventional proportional-derivative-type first-order and second-order iterative learning controllers of nonlinear time-invariant systems with initial state uncertainty. The tracking error is measured in the form of Lebesgue-p norm and the tracking performance is analyzed by the technique of generalized Young inequality of convolution integral. The analysis shows that the asymptotical tracking error is incurred by the initial state uncertainty and can be eliminated by tuning the compensation gain in the presuppose that the proportional and derivative learning gains together with the Lipschitz constant of the nonlinear state function are properly chosen to guarantee that convergence factor is less than one. Numerical simulations exhibit the validity of the theoretical derivation and the effectiveness of the compensation strategy.

Key words: iterative learning control; nonlinear systems; pulse compensation; initial state uncertainty; Lebesgue-p norm

1 Introduction

As studied for robotic systems by Arimoto[1], target trajectory tracking is one of the important topics. In this regard, iterative learning control (ILC) has become a popular strategy in intelligent control community. The basic mode of the ILC scheme is that the system operates on a fixed finite time interval to track a unique desired trajectory. It is the multi-operation feature that has made the ILC mechanism feasible to make use of the observed tracking error of the current operation to upgrade its input so as to generate a control input for the next operation. Owing to its satisfactory tracking performance by using less prior system knowledge, ILC has been widely applied to repetitive operations including robot manipulations, batch industrial processes and so on[2–5].

For most of the existing ILC investigations, a basic postulate is that the initial state of the system at each cycle is to reset at the desired state[6–8]. But, in the real world, it is difficult to always reset the iteration-wise initial state precisely at the desired one due to unavoidable noise produced by instrument sensitivity limitation or unidentified disturbance. As such, an early study[9] has reported that a small mismatch of the initial state might deteriorate the learning process. To handle the deterioration, the proportional tracking error has been introduced into the error derivative-type (D-type) iterative learning rule, formed as a proportional-derivational-type (PD-type) rule, in order to make the tracking error smaller[10]. This implies that the PD-type scheme is better than the D-type scheme in terms of improving the learning performance. Further, for the initial state uncertainties in a mean form, an average operator-based PD-type iterative learning control strategy has been developed to drive the system to follow a desired trajectory as close as possible, in which

the asymptotical tracking discrepancy is estimated by the system parameters and the original initial state mismatch[11]. Meanwhile, pieces of investigations have focused on the robustness of the ILC schemes concerning initial state shifts issue for nonlinear and time-discrete systems[12–14]. Those tracking ILC strategies for the initial state uncertainty issue are, however, ordinarily error-based forms and performance feature analysis. An ideal method is to compensate for the existing ILC scheme by single impulse signal[15], but this is not practically executable. Another active method has been to rectify the P-type ILC scheme by a sequence of iteration-varying polynomial functions of time variable[16]. The investigation exhibits that the rectifying action may alleviate the tracking error incurred by the initial state uncertainty effectively but mildly perhaps because that the updating ILC law has no proportional error modification and rectifying action is mild.

Recall that, regarding to the tracking error which is a time-varying function over the fixed operation period, its lambda-norm is defined as the supremum value of the weighted tracking error function by an exponential function of minus time variable multiplying a positive parameter lambda. Due to the weighting mode of the lambda-norm, which is mostly adopted for the performance analysis in the above-mentioned literatures, the sufficient largeness of the parameter lambda, which is required to guarantee the convergence, may extremely suppress the tracking error function. This also may cause the neglect of the fact that the system state dynamics and the proportional learning gain of the ILC rule does influence the convergence[17]. Besides, the supremum norm evaluates the point-wise maximum without considering the operation interval length. In this circumstance, even though the iteration index is so large that ensures the tracking error in lambda-norm seems very small, the iteration-wise control input generated by so-called convergent ILC scheme may possibly drive the system not to track the desired trajectory within a practical engineering error tolerance.

To avoid the above-mentioned phenomenon, Lebesgue-p norm is regarded as a good measure technique since it concerns all the tracking error scales in an integration form over the whole operation time interval. Motivated by the dissatisfaction of the mild error elimination of the rectifying action-based D-type ILC scheme and mentioned drawbacks of the lambda-norm, this paper is to develop a pulse-compensated ILC scheme for nonlinear systems with initial state uncertainty and analyze its tracking performance by accessing the tracking error in the form of Lebesgue-p norm.

The remaining of the paper is organized as follows. Section 2 addresses the constraint to the initial state uncertainty and its property in the sense of Lebesgue-p norm and then corresponding ILC schemes. In Section 3 the tracking performance is derived and further discussions are remarked. The validity and the effectiveness are simulated in Section 4. Finally, Section 5 concludes the paper.

2 Initial state uncertainty and ILC strategies

Consider a class of single-input-single-output nonlinear systems as follows:

$$\begin{cases} \dot{x}(t) = f(x(t), t) + Bu(t), \\ y(t) = Cx(t), \\ x(0) = x_0, \ t \in [0, T_0], \end{cases} \quad (1)$$

where $x(t) \in \mathbb{R}^n$ denotes n-dimensional state vector, $u(t) \in \mathbb{R}$ and $y(t) \in \mathbb{R}$ are scalar control and output, respectively; B and C are matrices with appropriate dimensions with the output matrix C is supposed known and x_0 is a random initial state, called a base initial state; the function $f(x(t), t)$ is an n-dimensional nonlinear vector function satisfying Lipschitz condition, in specific, for all $t \in [0, T_0]$, there exists a constant L_0 such that

$$|C(f(x_2(t), t) - f(x_1(t), t))| \leqslant$$
$$L_0|C(x_2(t) - x_1(t))| = L_0|y_2(t) - y_1(t)|. \quad (2)$$

Here, $|\cdot|$ refers to the absolution operation. Assume that the nonlinear system (1) is repetitive over a finite time interval $[0, T_0]$ with the initial state being iteration-varying.

Note that, for a linear or nonlinear time-invariant repetitive system with its initial state is resettable, it is realizable for a PD-type ILC law with appropriate learning gains to make the system to track a desired trajectory precisely as the iteration index goes to infinity[16]. While the initial state is uncertainly iteration-varying, the precise tracking of the developed ILC law is impossible. In this circumstance, a proper compensation for the existing ILC rule is regarded as an efficacious manner to suppress the tracking error caused by the initial state uncertainty.

In this paper, a sequence of rectangular pulses, practical form of the ideal singlet impulses, is adopted as iteration-wise compensations specified as follows:

Let $\{\delta_k(t), k = 1, 2, \cdots\}$ be a sequence of rectangular pulses expressed as

$$\delta_k(t) = \begin{cases} \dfrac{1}{\varepsilon_k}, & 0 \leqslant t \leqslant \varepsilon_k, \\ 0, & \varepsilon_k < t \leqslant T_0, \end{cases} \quad k = 1, 2, \cdots.$$

For an engineering applicability, it is assumed that the sequence is uniformly bounded, mathematically, $|\delta_k(t)| \leqslant 1/\varepsilon_k \leqslant M$, where M is a tolerance of the system input capability. The subscript k refers to the iteration index.

Suppose that $y_\mathrm{d}(t), \ t \in [0, T_0]$ is a desired trajec-

tory with $y_d(0) \in U(Cx_0)$, where $U(Cx_0)$ refers to a neighborhood of the point Cx_0. With the starting input $u_1(t)$ being arbitrarily given, if the latest historical tracking error and its derivative are available, then a pulse-based first-order PD-type ILC scheme is formulated as

$$
\begin{cases}
u_1(t) : \text{given arbitrarily,} \\
u_{k+1}(t) = u_k(t) + \Gamma_{\mathrm{p1}} e_k(t) + \Gamma_{\mathrm{d1}} \dot{e}_k(t) + \\
\qquad K\delta_k(t)(y_d(0) - Cx_0), \\
\qquad t \in [0, T_0], \ k = 1, 2, 3, \cdots,
\end{cases} \tag{3}
$$

where the subscript k refers to the iteration index, Γ_{p1} and Γ_{d1} are assigned as the first-order proportional and derivative learning gains, respectively. Here K is called as a compensation gain. The term $e_k(t) = y_d(t) - y_k(t)$ represents the tracking error between the desired trajectory $y_d(t)$ and the output $y_k(t)$ at the k-th operation.

If we make use of historical control inputs, tracking errors and their derivatives of the latest two adjacent operations, then a pulse-based second-order PD-type ILC scheme is constructed as

$$
\begin{cases}
u_1(t) : \text{given arbitrarily,} \\
u_2(t) = u_1(t) + \Gamma_{\mathrm{p1}} e_1(t) + \Gamma_{\mathrm{d1}} \dot{e}_1(t) + \\
\qquad K\delta_1(t)(y_d(0) - Cx_0), \\
u_{k+1}(t) = \omega_1(u_k(t) + \Gamma_{\mathrm{p1}} e_k(t) + \Gamma_{\mathrm{d1}} \dot{e}_k(t)) + \\
\qquad \omega_2(u_{k-1}(t) + \Gamma_{\mathrm{p2}} e_{k-1}(t) + \\
\qquad \Gamma_{\mathrm{d2}} \dot{e}_{k-1}(t)) + K\delta_k(t)(y_d(0) - Cx_0), \\
\qquad t \in [0, T_0], \ k = 2, 3, 4, \cdots.
\end{cases} \tag{4}
$$

Here ω_1 and ω_2 are weighting coefficients satisfying $0 \leqslant \omega_1, \omega_2 \leqslant 1$ and $\omega_1 + \omega_2 = 1$. It is seen that $\omega_1 = 1$ induces $\omega_2 = 0$ which implies that the ILC scheme (4) is thus reduced to the scheme (3). In the law (4), the parameters Γ_{p2} and Γ_{d2} are assigned as the second-order proportional and derivative learning gains, respectively.

Given that the control input $u(t)$ of the system (1) is undertaken by $u_{k+1}(t)$ generated by the above learning control scheme (3) or (4), the corresponding system dynamics description becomes

$$
\begin{cases}
\dot{x}_{k+1}(t) = f(x_{k+1}(t), t) + Bu_{k+1}(t), \\
y_{k+1}(t) = Cx_{k+1}(t), \\
x_{k+1}(0) \in N(x_0), \ t \in [0, T_0],
\end{cases} \tag{5}
$$

where $x_{k+1}(0)$ is a random initial state which lies in a neighborhood of x_0 denotes as $N(x_0)$. Specifically, we assume that the average of the random initial state values around x_0 from the first iteration to the k-th iteration is subject to the following constraint:

$$
\|\frac{1}{k}\sum_{i=1}^{k}x_i(0) - x_0\|_p \leqslant \beta o(\frac{1}{k}), \tag{6}
$$

where β is a positive constant and $o(\frac{1}{k})$ represents

a high-order infinitesimal with respect to $\frac{1}{k}$ as k approaches to infinity, that is, $\lim_{k\to\infty}[o(\frac{1}{k})/\frac{1}{k}] = 0$. Here, the norm $\|\cdot\|_p$ refers to the Lebesgue-p norm of a vector. Its definition together with the functional Lebesgue-p norm may be referred to the reference [18].

Before the convergence analysis, the property of the initial state shifts satisfying the inequality (6) is discussed in the following:

Lemma 1[16] If the initial state shifts of system (5) satisfies the inequality (6), then

$$
\lim_{k\to\infty}\|x_k(0) - x_0\|_p = 0.
$$

Proof The inequality (6) gives rise to

$$
\|\sum_{i=1}^{k}(x_i(0) - x_0)\|_p \leqslant \beta\frac{o(\frac{1}{k})}{\frac{1}{k}}.
$$

Thus

$$
\lim_{k\to\infty}\|\sum_{i=1}^{k}(x_i(0) - x_0)\|_p = 0. \tag{7}
$$

From the triangular inequality property of Lebesgue-p norm, we have

$$
\|x_k(0) - x_0\|_p =
$$
$$
\|\sum_{i=1}^{k}(x_i(0) - x_0) - \sum_{i=1}^{k-1}(x_i(0) - x_0)\|_p \leqslant
$$
$$
\|\sum_{i=1}^{k}(x_i(0) - x_0)\|_p + \|\sum_{i=1}^{k-1}(x_i(0) - x_0)\|_p.
$$

From the above inequality (7), we get

$$
\lim_{k\to\infty}\|x_k(0) - x_0\|_p = 0.
$$

This completes the proof.

Lemma 2[16] Suppose that initial state values satisfy the inequality

$$
\|\frac{1}{k+1}\sum_{i=0}^{k}x_i(0) - x_0\|_\infty \leqslant \beta\mathrm{e}^{-\gamma k}, \tag{8}
$$

where β and γ are positive constants. Then,

$$
\lim_{k\to\infty}\|x_k(0) - x_0\|_p = 0.
$$

Proof From the L'Hospital's rule of limit for a rational function whose both numerator and denominator are differentiable, we have

$$
\lim_{x\to+\infty}x\mathrm{e}^{-\gamma x} = \lim_{x\to+\infty}\frac{x}{\mathrm{e}^{\gamma x}} = \lim_{x\to+\infty}\frac{1}{\gamma\mathrm{e}^{\gamma x}} = 0, \ \gamma > 0.
$$

According to Cauchy principle for limit theory, the above equality implies that the result $\lim_{k\to\infty}\beta k\mathrm{e}^{-\gamma k} = 0$ guaranteed. Then, the inequality (8) leads to

$$
0 \leqslant \lim_{k\to\infty}\|\sum_{i=0}^{k}(x_i(0) - x_0)\|_\infty \leqslant \lim_{k\to\infty}\frac{\beta\mathrm{e}^{-\gamma k}}{\frac{1}{k+1}} = 0,
$$

that is

$$\lim_{k\to\infty}\|\sum_{i=0}^{k}(x_i(0)-x_0)\|_\infty=0.$$

Similar to the proof of the Lemma 1, we obtain

$$\lim_{k\to\infty}\|x_k(0)-x_0\|_\infty=0.$$

Since

$$\|x_k(0)-x_0\|_p=(\sum_{l=1}^{n}|x_k^{(l)}(0)-x_0^{(l)}|^p)^{\frac{1}{p}},$$

thus

$$\|x_k(0)-x_0\|_p\leqslant(n\max_{1\leqslant l\leqslant n}|x_k^{(l)}(0)-x_0^{(l)}|^p)^{\frac{1}{p}}=$$
$$\sqrt[p]{n}\|x_k(0)-x_0\|_\infty=\sqrt[p]{n}\max_{1\leqslant l\leqslant n}|x_k^{(l)}(0)-x_0^{(l)}|_\infty.$$

Therefore,

$$\lim_{k\to\infty}\|x_k(0)-x_0\|_p=0.$$

Lemma 3[17]   Let $\{a_k,\ k=1,2,\cdots\}$ be a real sequence defined as

$$a_k\leqslant\rho_1 a_{k-1}+\rho_2 a_{k-2}+\cdots+$$
$$\rho_M a_{k-M}+d_k,\ k\geqslant M+1,$$

with initial conditions

$$a_1=\bar{a}_1,\ a_2=\bar{a}_2,\cdots,a_M=\bar{a}_M,$$

where d_k is a specified real sequence. If $\rho_1,\ \rho_2,\cdots,$ ρ_M are nonnegative numbers satisfying

$$\rho=\sum_{j=1}^{M}\rho_j<1.$$

Then

i) $d_k\leqslant\bar{d},k\geqslant M+1$ implies that

$$a_k\leqslant\max\{\bar{a}_1,\bar{a}_2,\cdots,\bar{a}_M\}+\frac{\bar{d}}{1-\rho},\ k\geqslant M+1.$$

ii) $\limsup\limits_{k\to\infty}d_k\leqslant d_\infty$ implies that

$$\limsup_{k\to\infty}a_k\leqslant\frac{d_\infty}{1-\rho}.$$

Lemma 4[19] (Bellman-Gronwall inequality)   Let Ω denote an interval of the real line of the form $[a,\infty)$ or $[a,b]$ or $[a,b)$ with $a<b$. Let $\alpha(t),\ \beta(t)$ and $f_1(t)$ be real-valued functions defined on Ω. Assume that $\beta(t)$ and $f_1(t)$ are continuous and that the negative part of $\alpha(t)$ is integrable on every closed bounded subinterval of Ω. If $\beta(t)$ is non-negative and $f_1(t)$ satisfies the integral inequality

$$f_1(t)\leqslant\alpha(t)+\int_a^t\beta(\tau)f_1(\tau)\mathrm{d}\tau,\ \forall t\in\Omega,$$

then

$$f_1(t)\leqslant\alpha(t)+\int_a^t\alpha(\tau)\beta(\tau)\exp(\int_\tau^t\beta(s)\mathrm{d}s)\mathrm{d}\tau,$$
$$\forall t\in\Omega.$$

Let $\Omega=[0,T_0],\ \alpha(t)=h_1(t)+\int_0^t g_1(\tau)\mathrm{d}\tau$ and $\beta(t)=\eta$, where η is a non-negative constant. Then it is immediate to get a corollary as follows:

Corollary 1   Let $f_1(t),\ g_1(t)$ and $h_1(t)$ are posi-

tive continuous functions over the time interval $[0,T_0]$. If there exists a constant $\eta\geqslant0$ such that the inequality

$$f_1(t)\leqslant h_1(t)+\int_0^t\eta f_1(\tau)\mathrm{d}\tau+\int_0^t g_1(\tau)\mathrm{d}\tau$$

holds, then

$$f_1(t)\leqslant$$
$$h_1(t)+\int_0^t\exp(\eta\cdot(t-\tau))[\eta h_1(\tau)+g_1(\tau)]\mathrm{d}\tau.$$

As mentioned above, the scheme (3) is a specific case of the scheme (4) when the coefficients are set as $\omega_1=1$ and $\omega_2=1$. We thus only prove the scheme (4), and remark the performance of the scheme (3).

Before giving the proof, for simplicity of the analysis of convergence in the next section, we list a group of denotations as follows:

$$\rho_1=|1-CB\varGamma_{\mathrm{d1}}|+(L_0+|CB\varGamma_{\mathrm{p1}}|+$$
$$L_0|1-CB\varGamma_{\mathrm{d1}}|)\times\|\exp(L_0(\cdot))\|_1,$$
$$\rho_2=|1-CB\varGamma_{\mathrm{d2}}|+(L_0+|CB\varGamma_{\mathrm{p2}}|+$$
$$L_0|1-CB\varGamma_{\mathrm{d2}}|)\times\|\exp(L_0(\cdot))\|_1,$$
$$\tilde{\rho}=\omega_1\rho_1+\omega_2\rho_2,$$
$$W_k(t)=\begin{cases}\dfrac{t}{\varepsilon_k},&0\leqslant t\leqslant\varepsilon_k,\\[2mm]1,&\varepsilon_k\leqslant t\leqslant T_0,\end{cases}$$
$$\varPsi_k=C(x_{k+1}(0)-\omega_1 x_k(0)-\omega_2 x_{k-1}(0)),$$
$$\varOmega_k^1=CB\omega_1\varGamma_{\mathrm{d1}}C(x_k(0)-x_0),$$
$$\varOmega_{k-1}^2=CB\omega_2\varGamma_{\mathrm{d2}}C(x_{k-1}(0)-x_0),$$
$$\varPhi_k(t)=(CB\omega_1\varGamma_{\mathrm{d1}}+CB\omega_2\varGamma_{\mathrm{d2}}-$$
$$CBKW_k(t))\times(y_{\mathrm{d}}(0)-Cx_0),$$
$$H_k(t)=CB(\varGamma_{\mathrm{d1}}-KW_k(t))(y_{\mathrm{d}}(0)-Cx_0),$$
$$H=\limsup_{k\to\infty}\|H_k(\cdot)\|_p(1+L_0\|\exp(L_0(\cdot))\|_1),$$
$$\varPhi=\limsup_{k\to\infty}(1+L_0\|\exp(L_0(\cdot))\|_1)\|\varPhi_k(t)\|_p.$$

From Lemma 1 and Lemma 2, it is immediate to attain that

Corollary 2   If the initial state of the system (5) satisfies either the constraint (6) or (8), then

$$\lim_{k\to\infty}\|\varPsi_k\|_p=0,\ \lim_{k\to\infty}\|\varOmega_k^1\|_p=0,\ \lim_{k\to\infty}\|\varOmega_{k-1}^2\|_p=0.$$

3   Convergence analysis

Theorem 1   Assume that the pulse-based second-order PD-type ILC scheme (4) is applied to the system (1) and the average of initial states of the corresponding system (5) satisfies the constraint (6). If the system matrix B, C and the Lipschitz constant L_0 together with the learning gains $\varGamma_{\mathrm{p1}}$, $\varGamma_{\mathrm{d1}}$, $\varGamma_{\mathrm{p2}}$ and $\varGamma_{\mathrm{d2}}$ satisfy the inequalities $\rho_1<1$ and $\rho_2<1$, then we have

$$\limsup_{k\to\infty}\|y_{\mathrm{d}}(\cdot)-y_{k+1}(\cdot)\|_p\leqslant\frac{\varPhi}{1-\tilde{\rho}},$$

where $\tilde{\rho}=\omega_1\rho_1+\omega_2\rho_2$.

Proof

$e_{k+1}(t) =$

$y_{\mathrm{d}}(t) - y_{k+1}(t) =$

$\omega_1(y_{\mathrm{d}}(t) - y_k(t)) + \omega_2(y_{\mathrm{d}}(t) - y_{k-1}(t)) -$

$(y_{k+1}(t) - \omega_1 y_k(t) - \omega_2 y_{k-1}(t)) =$

$\omega_1 e_k(t) + \omega_2 e_{k-1}(t) -$

$C(x_{k+1}(t) - \omega_1 x_k(t) - \omega_2 x_{k-1}(t)) =$

$\omega_1 e_k(t) + \omega_2 e_{k-1}(t) - C[x_{k+1}(0) +$

$\displaystyle\int_0^t (f(x_{k+1}(\tau), \tau) + Bu_{k+1}(\tau))\mathrm{d}\tau] +$

$C\omega_1[x_k(0) + \displaystyle\int_0^t (f(x_k(\tau), \tau) + Bu_k(\tau))\mathrm{d}\tau] +$

$C\omega_2[x_{k-1}(0) +$

$\displaystyle\int_0^t (f(x_{k-1}(\tau), \tau) + Bu_{k-1}(\tau))\mathrm{d}\tau] =$

$\omega_1 e_k(t) + \omega_2 e_{k-1}(t) -$

$C(x_{k+1}(0) - \omega_1 x_k(0) - \omega_2 x_{k-1}(0)) -$

$C\omega_1 \displaystyle\int_0^t (f(x_{k+1}(\tau), \tau) - f(x_k(\tau), \tau))\mathrm{d}\tau -$

$C\omega_2 \displaystyle\int_0^t (f(x_{k+1}(\tau), \tau) - f(x_{k-1}(\tau), \tau))\mathrm{d}\tau -$

$CB \displaystyle\int_0^t (u_{k+1}(\tau) - \omega_1 u_k(\tau) - \omega_2 u_{k-1}(\tau))\mathrm{d}\tau =$

$\omega_1 e_k(t) + \omega_2 e_{k-1}(t) -$

$C(x_{k+1}(0) - \omega_1 x_k(0) - \omega_2 x_{k-1}(0)) -$

$C\omega_1 \displaystyle\int_0^t (f(x_{k+1}(\tau), \tau) - f(x_k(\tau), \tau))\mathrm{d}\tau -$

$C\omega_2 \displaystyle\int_0^t (f(x_{k+1}(\tau), \tau) - f(x_{k-1}(\tau), \tau))\mathrm{d}\tau -$

$CB \displaystyle\int_0^t (\omega_1 \Gamma_{\mathrm{p}1} e_k(\tau) + \omega_1 \Gamma_{\mathrm{d}1} \dot{e}_k(\tau) +$

$\qquad \omega_2 \Gamma_{\mathrm{p}2} e_{k-1}(\tau) + \omega_2 \Gamma_{\mathrm{d}2} \dot{e}_{k-1}(\tau))\mathrm{d}\tau -$

$CB \displaystyle\int_0^t K\delta_k(\tau)(y_{\mathrm{d}}(0) - Cx_0)\mathrm{d}\tau =$

$\omega_1 e_k(t) + \omega_2 e_{k-1}(t) -$

$C(x_{k+1}(0) - \omega_1 x_k(0) - \omega_2 x_{k-1}(0)) -$

$C\omega_1 \displaystyle\int_0^t (f(x_{k+1}(\tau), \tau) - f(x_k(\tau), \tau))\mathrm{d}\tau -$

$C\omega_2 \displaystyle\int_0^t (f(x_{k+1}(\tau), \tau) - f(x_{k-1}(\tau), \tau))\mathrm{d}\tau -$

$CB\omega_1 \Gamma_{\mathrm{p}1} \displaystyle\int_0^t e_k(\tau)\mathrm{d}\tau -$

$CB\omega_1 \Gamma_{\mathrm{d}1} e_k(t) + CB\omega_1 \Gamma_{\mathrm{d}1} e_k(0) -$

$CB\omega_2 \Gamma_{\mathrm{p}2} \displaystyle\int_0^t e_{k-1}(\tau)\mathrm{d}\tau -$

$CB\omega_2 \Gamma_{\mathrm{d}2} e_{k-1}(t) + CB\omega_2 \Gamma_{\mathrm{d}2} e_{k-1}(0) -$

$CB \displaystyle\int_0^t K\delta_k(t)(y_{\mathrm{d}}(0) - Cx_0)\mathrm{d}\tau. \qquad (9)$

It is known that $\delta_k(t) = \dfrac{1}{\varepsilon_k}$, for $0 \leqslant t \leqslant \varepsilon_k$. Thus, we have

$$-CB \int_0^t K\frac{1}{\varepsilon_k}(y_{\mathrm{d}}(0) - Cx_0)\mathrm{d}\tau =$$

$$-CB \int_0^t K\frac{1}{\varepsilon_k}(y_{\mathrm{d}}(0) - Cx_0)\mathrm{d}\tau =$$

$$-CBK\frac{t}{\varepsilon_k}(y_{\mathrm{d}}(0) - Cx_0). \qquad (10)$$

And then, we consider that $\delta_k(t) = 0$, for $\varepsilon_k \leqslant t \leqslant T_0$, we get that

$$-CB \int_0^t K\delta_k(\tau)(y_{\mathrm{d}}(0) - Cx_0)\mathrm{d}\tau =$$

$$-CB \int_0^{\varepsilon_k} K\frac{1}{\varepsilon_k}(y_{\mathrm{d}}(0) - Cx_0)\mathrm{d}\tau =$$

$$-CBK(y_{\mathrm{d}}(0) - Cx_0). \qquad (11)$$

So from the above definitions, we get that

$$-CB \int_0^t K\delta_k(\tau)(y_{\mathrm{d}}(0) - Cx_0)\mathrm{d}\tau =$$

$$-CBKW_k(t)(y_{\mathrm{d}}(0) - Cx_0). \qquad (12)$$

Notice that

$$e_k(0) = y_{\mathrm{d}}(0) - Cx_k(0) =$$

$$(y_{\mathrm{d}}(0) - Cx_0) - C(x_k(0) - x_0),$$

$$e_{k-1}(0) = (y_{\mathrm{d}}(0) - Cx_0) - C(x_{k-1}(0) - x_0).$$

So the Eq.(9) can be changed to

$e_{k+1}(t) =$

$\omega_1 e_k(t) + \omega_2 e_{k-1}(t) -$

$C(x_{k+1}(0) - \omega_1 x_k(0) - \omega_2 x_{k-1}(0)) -$

$C\omega_1 \displaystyle\int_0^t (f(x_{k+1}(\tau), \tau) - f(x_k(\tau), \tau))\mathrm{d}\tau -$

$C\omega_2 \displaystyle\int_0^t (f(x_{k+1}(\tau), \tau) - f(x_{k-1}(\tau), \tau))\mathrm{d}\tau -$

$CB\omega_1 \Gamma_{\mathrm{p}1} \displaystyle\int_0^t e_k(\tau)\mathrm{d}\tau - CB\omega_1 \Gamma_{\mathrm{d}1} e_k(t) -$

$CB\omega_1 \Gamma_{\mathrm{d}1} C(x_k(0) - x_0) -$

$CB\omega_2 \Gamma_{\mathrm{p}2} \displaystyle\int_0^t e_{k-1}(\tau)\mathrm{d}\tau - CB\omega_2 \Gamma_{\mathrm{d}2} e_{k-1}(t) -$

$CB\omega_2 \Gamma_{\mathrm{d}2} C(x_{k-1}(0) - x_0)$

$CB\omega_1 \Gamma_{\mathrm{d}1}(y_{\mathrm{d}}(0) - Cx_0) +$

$CB\omega_2 \Gamma_{\mathrm{d}2}(y_{\mathrm{d}}(0) - Cx_0) - CBK(y_{\mathrm{d}}(0) - Cx_0).$

$$(13)$$

Takeing absolute on both sides of the Eq.(13), and applying the Lipschitz condition, we get

$|e_{k+1}(t)| \leqslant \omega_1|1 - CB\Gamma_{\mathrm{d}1}||e_k(t)| +$

$\omega_2|1 - CB\Gamma_{\mathrm{d}2}||e_{k-1}(t)| +$

$\omega_1 \displaystyle\int_0^t |C(f(x_{k+1}(\tau), \tau) - f(x_k(\tau), \tau))|\mathrm{d}\tau +$

$\omega_2 \displaystyle\int_0^t |C(f(x_{k+1}(\tau), \tau) - f(x_{k-1}(\tau), \tau))|\mathrm{d}\tau +$

$|CB\omega_1 \Gamma_{\mathrm{p}1}| \displaystyle\int_0^t |e_k(\tau)|\mathrm{d}\tau +$

$|CB\omega_2 \Gamma_{\mathrm{p}2}| \displaystyle\int_0^t |e_{k-1}(\tau)|\mathrm{d}\tau +$

$|\Phi_k| + |\Omega_k^1| + |\Omega_{k-1}^2| + |\Psi_k(t)| \leqslant$

$\omega_1|1 - CB\Gamma_{\mathrm{d}1}||e_k(t)| + \omega_2|1 - CB\Gamma_{\mathrm{d}2}||e_{k-1}(t)| +$

$$\omega_1 L_0 \int_0^t |y_{k+1}(\tau) - y_d(\tau) + y_d(\tau) - y_k(\tau)| d\tau +$$

$$\omega_2 L_0 \int_0^t |y_{k+1}(\tau) - y_d(\tau) + y_d(\tau) - y_{k-1}(\tau)| d\tau +$$

$$|CB\omega_1 \Gamma_{p1}| \int_0^t |e_k(\tau)| d\tau +$$

$$|CB\omega_2 \Gamma_{p2}| \int_0^t |e_{k-1}(\tau)| d\tau +$$

$$|\Phi_k| + |\Omega_k^1| + |\Omega_{k-1}^2| + |\Psi_k(t)| \leqslant$$

$$\omega_1 |1 - CB\Gamma_{d1}||e_k(t)| + \omega_2 |1 - CB\Gamma_{d2}||e_{k-1}(t)| +$$

$$\omega_1 L_0 \int_0^t |e_{k+1}(\tau)| d\tau + \omega_1 L_0 \int_0^t |e_k(\tau)| d\tau +$$

$$\omega_2 L_0 \int_0^t |e_{k+1}(\tau)| d\tau + \omega_2 L_0 \int_0^t |e_{k-1}(\tau)| d\tau +$$

$$|CB\omega_1 \Gamma_{p1}| \int_0^t |e_k(\tau)| d\tau +$$

$$|CB\omega_2 \Gamma_{p2}| \int_0^t |e_{k-1}(\tau)| d\tau +$$

$$|\Phi_k| + |\Omega_k^1| + |\Omega_{k-1}^2| + |\Psi_k(t)| =$$

$$\omega_1 |1 - CB\Gamma_{d1}||e_k(t)| + \omega_2 |1 - CB\Gamma_{d2}||e_{k-1}(t)| +$$

$$\int_0^t (\omega_1 (L_0 + |CB\Gamma_{p1}|)|e_k(\tau)| +$$

$$\omega_2 (L_0 + |CB\Gamma_{p2}|)) d\tau + L_0 \int_0^t |e_{k+1}(\tau)| d\tau +$$

$$|\Psi_k| + |\Omega_k^1| + |\Omega_{k-1}^2| + |\Phi_k(t)|. \tag{14}$$

Applying Corollary 1 to the above inequality (14) yields

$$|e_{k+1}(t)| \leqslant$$
$$\omega_1 |1 - CB\Gamma_{d1}||e_k(t)| +$$
$$\omega_2 |1 - CB\Gamma_{d2}||e_{k-1}(t)| +$$
$$\omega_1 (L_0 + |CB\Gamma_{p1}| + L_0 |1 - CB\Gamma_{d1}|) \times$$
$$\int_0^t \exp(L_0(t-\tau))|e_k(\tau)| d\tau +$$
$$\omega_2 (L_0 + |CB\Gamma_{p2}| + L_0 |1 - CB\Gamma_{d2}|) \times$$
$$\int_0^t \exp(L(t-\tau))|e_{k-1}(\tau)| d\tau +$$
$$L_0 \int_0^t \exp(L(t-\tau))(|\Psi_k| + |\Omega_k^1| +$$
$$|\Omega_{k-1}^2| + |\Phi_k(t)|) d\tau +$$
$$|\Psi_k| + |\Omega_k^1| + |\Omega_{k-1}^2| + |\Phi_k(t)|. \tag{15}$$

Taking Lebesgue-p norm on both sides of the above inequality (15), and applying the generalized Young inequality of convolution integral, we get

$$\|e_{k+1}(\cdot)\|_p \leqslant$$
$$\omega_1 |1 - CB\Gamma_{d1}|\|e_k(\cdot)\|_p +$$
$$\omega_2 |1 - CB\Gamma_{d2}|\|e_{k-1}(\cdot)\|_p +$$
$$\omega_1 (L_0 + |CB\Gamma_{p1}| + L_0 |1 - CB\Gamma_{d1}|) \times$$
$$\|\exp(L_0(\cdot))\|_1 \|e_k(\cdot)\|_p +$$
$$\omega_2 (L_0 + |CB\Gamma_{p2}| + L_0 |1 - CB\Gamma_{d2}|) \times$$
$$\|\exp(L_0(\cdot))\|_1 \|e_{k-1}(\cdot)\|_p +$$
$$(1 + L_0 \|\exp(L_0(\cdot))\|_1) \times$$
$$(\|\Psi_k\|_p + \|\Omega_k^1\|_p + \|\Omega_{k-1}^2\|_p) +$$

$$(1 + L_0 \|\exp(L_0(\cdot))\|_1)\|\Phi_k(t)\|_p. \tag{16}$$

From above denotations, the inequality (16) can be simplified to

$$\|e_{k+1}(\cdot)\|_p \leqslant$$
$$\omega_1 \rho_1 \|e_k(\cdot)\|_p + \omega_2 \rho_2 \|e_{k-1}(\cdot)\|_p +$$
$$(1 + L_0 \|\exp(L_0(\cdot))\|_1) \times$$
$$(\|\Psi_k\|_p + \|\Omega_k^1\|_p + \|\Omega_{k-1}^2\|_p) +$$
$$(1 + L_0 \|\exp(L_0(\cdot))\|_1)\|\Phi_k(t)\|_p. \tag{17}$$

It is obvious that $\tilde{\rho} = \omega_1 \rho_1 + \omega_2 \rho_2 < 1$ under the assumption that $\rho_1 < 1$ and $\rho_2 < 1$. Hence, from Corollary 2 and Lemma 3, the inequality (17) leads to

$$\limsup_{k \to \infty} \|e_{k+1}(\cdot)\|_p \leqslant \frac{\Phi}{1 - \tilde{\rho}}. \tag{18}$$

This completes the proof.

Remark 1 If the learning gains Γ_{p1}, Γ_{d1}, Γ_{p2} and Γ_{d2} are chosen in such a way that the convergence factor $\tilde{\rho}$ is sufficiently small, the expression (18) indicates that the limit superior of the tracking error sequence is bounded by the upper bound $\dfrac{\Phi}{1 - \tilde{\rho}}$ which in turn can be confined as small as possible.

Remark 2 In the scheme (4), if the weighting coefficients are $\omega_1 = 1$ and $\omega_2 = 0$, the scheme (4) turns to be the first-order scheme (3). Hence, the convergence condition of the first-order scheme (3) is $\rho_1 < 1$ and the upper bound of the limit superior of the tracking errors sequence becomes $\dfrac{H}{1 - \rho_1}$. Comparing the bound $\dfrac{\Phi}{1 - \tilde{\rho}}$ with the $\dfrac{H}{1 - \rho_1}$, it is observed that the upper bounded $\dfrac{\Phi}{1 - \tilde{\rho}}$ is smaller than $\dfrac{H}{1 - \rho_1}$ if the learning gains and the weighting coefficients are appropriately chosen such that the conditions $\tilde{\rho} < \rho_1$ and $\Phi < H$ hold simultaneously. Under this condition, the second-order scheme is convergent faster than the first-order one.

Remark 3 From the expression of $W_k(t)$, it is further observed that $W_k(t) \leqslant 1$. Hence, a feasible way to choose the compensation gain K is to let it be approximately equal to $\omega_1 \Gamma_{d1} + \omega_2 \Gamma_{d2}$, which results that $\Phi_k(t)$ is sufficiently small and thus Φ is also sufficiently small concurrently.

4 Numerical simulations

Consider the following nonlinear system:

$$\begin{cases} \begin{bmatrix} \dot{x}^{(1)}(t) \\ \dot{x}^{(2)}(t) \end{bmatrix} = \begin{bmatrix} 0.5x^{(1)}(t) \\ 0.1x^{(2)}(t) + 0.3\cos(x^{(1)}(t)) \end{bmatrix} + \\ \qquad\qquad \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t), \\ y(t) = [0 \ \ 1] \begin{bmatrix} x^{(1)}(t) \\ x^{(2)}(t) \end{bmatrix}. \end{cases} \tag{19}$$

The desired trajectory is set as $y_d(t) = 12t^2(1-t)$, $t \in [0, 1]$. The initial state shifts are produced as $x_0 =$

$[0 \ 0.2]^{\mathrm{T}}$ and $x_k(0) = x_0 + \dfrac{0.2}{k^2}(\mathrm{rand} - 0.5)$, $k = 1$, $2, \cdots$, where rand refers to a uniformly distributed random number over the interval $(0, 1)$. A sequence of pulse signals is set as

$$\delta_k(t) = \begin{cases} \dfrac{1}{0.1 - (-1)^k \dfrac{0.05}{k^2}}, \\ \quad 0 \leqslant t \leqslant 0.1 - (-1)^k \dfrac{0.05}{k^2}; \\ 0, \ 0.1 - (-1)^k \dfrac{0.05}{k^2} < t \leqslant 1. \end{cases} \quad (20)$$

Here, we illustrate tracking performances operated by the proposed scheme (4). The weighting coefficients are assigned as $\omega_1 = 0.4$ and $\omega_2 = 0.6$. The compensation gain $K = 0.8$ and $K = 0$, respectively. The first order learning gains in both scheme (3) and (4) are identically chosen as $\Gamma_{\mathrm{p1}} = 0.1$ and $\Gamma_{\mathrm{d1}} = 1.3$, whilst the second order learning gains are selected as $\Gamma_{\mathrm{p2}} = 0.4$ and $\Gamma_{\mathrm{d2}} = 0.6$, respectively. It is computed that $\rho_1 = 0.8898 < 1$, $\rho_2 = 0.8822 < 1$ and thus $\tilde{\rho} = \omega_1 \rho_1 + \omega_2 \rho_2 = 0.8852$. Their tracking performances at the 5th and the 8th iterations are shown in Figs.1–2, respectively, where the dash-dotted curves present the desired trajectories, the solid curves depict the outputs stimulated by the pulse-based second-order PD-type ILC scheme with $K = 0.8$ and the dash curves denote the rectifying action-based second-order PD-type ILC scheme with $K = 0$.

In term of the convergence speed of the pulse compensated first and second-order ILC scheme, the tracking errors is shown in Fig.3, which indicates that the asymptotic tracking error of the pulse-based second-order PD-type ILC scheme is smaller than that of pulse-based first-order PD-type ILC scheme after the third iteration.
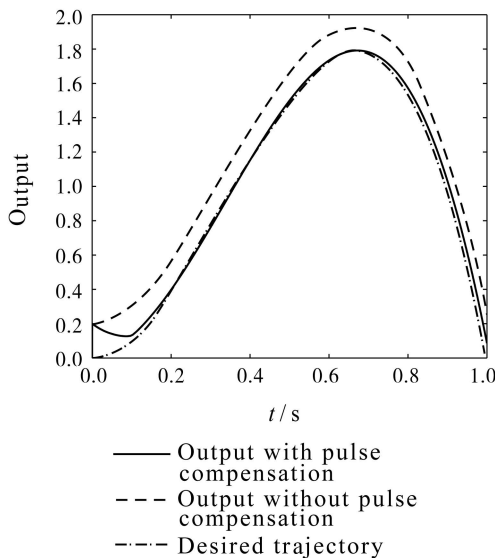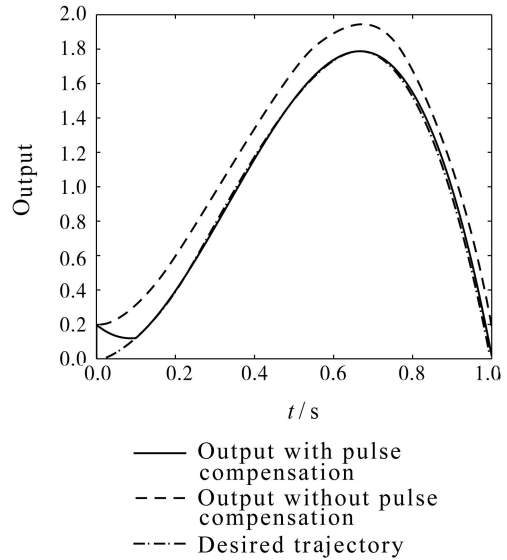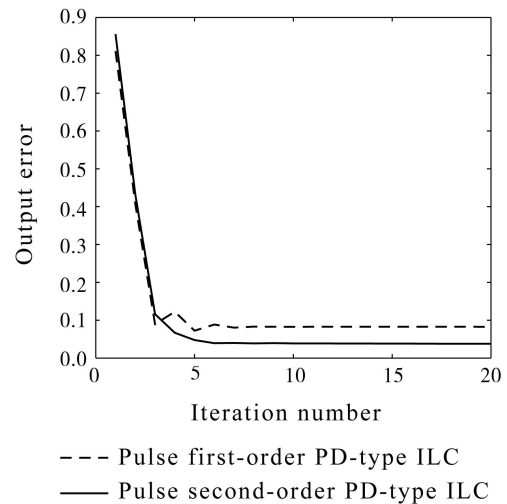
Fig. 2 Tracking performance at the 8th iteration

Fig. 3 Tracking error

5 Conclusions

In this paper, for nonlinear systems a type of rectangular pulse signal is adopted to compensate for a class of PD-type ILC updating laws so as to suppress the tracking discrepancy caused by the nonzero initial state shift. By means of the generalized Young inequality of convolution integral, the tracking performance is quantified with the tracking error being measured in the sense of the Lebesgue-p norm. It is observed that the pulse signal can suppress the tracking error incurred by the initial state shift effectively. In comparison of the pulse-based first-order ILC scheme with the pulse-based second-order PD-type ILC scheme, it is noted that the second-order scheme can improve the transient tracking performance better.

Fig. 1 Tracking performance at the 5th iteration

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