文章编号: 1000−8152(2012)08−1063−06

非线性迭代学习控制问题的延拓修正牛顿法

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摘要: 对于非线性迭代学习控制问题, 提出基于延拓法和修正Newton法的具有全局收敛性的迭代学习控制新方法. 由于一般的Newton型迭代学习控制律都是局部收敛的, 在实际应用中有很大局限性. 为拓宽收敛范围, 该方法将延拓法引入迭代学习控制问题, 提出基于同伦延拓的新的Newton型迭代学习控制律, 使得初始控制可以较为任意的选择. 新的迭代学习控制算法将求解过程分成N个子问题, 每个子问题由换列修正Newton法利用简单的递推公式解出. 本文给出算法收敛的充分条件, 证明了算法的全局收敛性. 该算法对于非线性系统迭代学习控制具有全局收敛和计算简单的优点.

关键词: 迭代学习控制; 延拓法; 修正Newton法; 全局收敛; 非线性系统
中图分类号: TP273　　**文献标识码**: A

A new iterative learning control algorithm of extension-updated Newton method for nonlinear systems

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Abstract: A new algorithm based on extension method and updated Newton method with global convergence for nonlinear iterative learning control problem is proposed. Since classical Newton-type iterative learning schemes are local convergence, conditions of local convergence can be hardly satisfied in practice. In order to widen the range of convergence, extension method is introduced to iterative learning control problem. A new Newton-type iterative learning control scheme based on homotopy extension is presented, in which the initial control can be chosen arbitrarily. The solving process is subdivided to N subproblem by the new algorithm. The exchange column update Newton method is employed to solve the subproblem by simple recurrent formula. Sufficient conditions for global convergence of this algorithm are given and proved. The implementation of the new algorithm has advantage of guaranteeing global convergence and avoiding complex calculation for nonlinear iterative learning control.

Key words: iterative learning control; extension method; updated Newton method; global convergence; nonlinear systems

1 Introduction

Iterative learning control is a control strategy that needs to improve the control performance of every iterative process by operating in a repetitive mode. In iterative learning control systems, information from previous executions of the task is used in an attempt to generate the updated control iteration and the tracking error between the output trajectory and desired trajectory tends to zero. Such systems include robot arm manipulators, disk drive, chemical batch reactors and other nonlinear industry. Since iterative learning control was originally introduced by Arimoto[1], significant developments in iterative learning research area has stimulated considerable interests in various update algorithms for linear and nonlinear systems[2]. In recent years, the study of iterative learning control has put more and more emphases on nonlinear systems which are the most often seen cases in practice[3–5]. Newton-type iterative learning control schemes are one of the important and effective schemes which have the advantage of improving the convergence speed for nonlinear systems. In [6], a new nonlinear iterative learning control algorithm used a special form of Newton method in continuous time domain. Xu and Tan[7] provided P-type learning and Newton-type learning method for non-affine nonlinear systems. The proposed P-type iterative learning control scheme has the simple form required a prior system knowledge, while Newton-type iterative learning control schemes have faster convergence by incorporating a varying learning gain. For discrete

Received 9 May 2012; revised 8 July 2012.
This work was supported by the National Natural Science Foundation (NNSF) of China (No. 61004056).

nonlinear systems, Lin et al[8], introduced the Newton method into the iterative learning control framework by established the connection between the iterative learning control problem and nonlinear multivariable equations. Then, a nonlinear iterative learning control algorithm with semi-local convergence was presented. By introducing a relaxation index, a Newton method based on iterative learning control for nonlinear systems was shown to converge monotonically in [9]. Optimization was suggested to calculate the index about monotonic convergence and fast convergence speed at the same time. Kang and Tang[10] presented a new iterative learning control algorithm for nonlinear systems based on modified Newton methods. The exchange row updating method was used to construct the approximation of the derivatives of the output function by which the calculation work was reduced largely.

It is well known that iterative learning control based on Newton-type method can greatly speed up the convergence. However, there are some weaknesses. Firstly, all these algorithms mentioned above have local convergence or semi-local convergence. It is implied that the convergence is only guaranteed when the initial control is chosen in a small neighborhood of the target control. In practice, this condition can be hardly satisfied because the control is unknown. Moreover, Newton method need to spent a lot of time on complex calculation of nonlinear inverse systems.

In this paper, a new algorithm of iterative learning control for nonlinear systems is proposed. The iterative learning scheme with a homotopy extension is established. The solving process is divided into N subproblem by the new algorithm. The exchange column update Newton method is used to solve the subproblem by simple recurrent formula. Global convergence of this new algorithm is proved. What distinguishes our work from previous iterative learning control schemes is that the new algorithm has a strong connection to the homotopy extension method and exchange column update Newton method. Iterative learning control scheme has taken advantage of homotopy extension to achieve global convergence. Furthermore, the exchange column update Newton method is considered in this algorithm which makes it possible to reduce complex calculation work. The significance of this paper is that a new iterative learning control algorithm with global convergence is provided instead of local convergence of general Newton-type method.

2 Problem statement

In this paper, we deal with the general setting of nonlinear iterative learning control scheme. Consider nonlinear systems as follows:

$$\begin{cases} \dot{x}(t) = f(x(t), u(t), t), \\ y(t) = \phi(x(t), u(t), t), \end{cases} \tag{1}$$

and initial condition $x(0) = x^{(0)}$, where $x(t) \in D \subset \mathbb{R}^n$, $y(t) \in E \subset \mathbb{R}^m$, $u(t) \in \mathbb{R}^m$, $t \in [0, T]$.

In order to consider the iterative learning control problem, some definitions and assumptions are given.

Definition 1[11] Let the function

$$x(t) : [0, T] \to \mathbb{R}^n,$$

then the λ-norm is defined as

$$\| x \|_\lambda = \sup_{0 \leqslant t \leqslant T} \{ \| x(t) \|_2 \mathrm{e}^{-\lambda t} \},$$

and the supreme norm is

$$\| x \|_s = \sup_{0 \leqslant t \leqslant T} \{ \| x(t) \|_2 \}.$$

Definition 2[11] Let A be a matrix, then the Frobenius norm (F-norm) of A is

$$\| A \|_F = (\sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij}^2)^{\frac{1}{2}} = (\mathrm{tr}(A^{\mathrm{T}} A))^{\frac{1}{2}}.$$

In particular, if an element of the matrix is a function of $t(0 \leqslant t \leqslant T)$, then we can define $\| A \|_{F_s}$ as

$$\| A \|_{F_s} = \max_{0 \leqslant t \leqslant T} \| A \|_{F_s}.$$

Assumption 1 Suppose that $f(x, u, t)$ and $\phi(x, u, t)$ have second Fréchet continuous derivatives in the compact convex subset $\Omega = D \times E \times [0, T]$ with respective to x and u.

From this assumption, the following results can be easily obtained.

Remark 1 $f(x, u, t)$ and $\phi(x, u, t)$ as well as their first derivatives with respective to x and u are uniformly bounded in Ω. The second derivatives of $\phi(x, u, t)$ with respective to u is uniformly bounded in Ω, i.e.

$$\| \phi_{\mathrm{x}}(x, u, t) \| \leqslant K, \ \| \phi_{\mathrm{uu}}(x, u, t) \| \leqslant Q,$$

where K and Q are constants.

Remark 2 There exist constants L_1 and L_2, such that

$$\begin{aligned} &\| f(x_1, u_1, t) - f(x_2, u_2, t) \|_2 \leqslant \\ &L_1(\| x_1 - x_2 \|_2 + \| u_1 - u_2 \|_2), \\ &\| \phi_{\mathrm{u}}(x_1, u_1, t) - \phi_{\mathrm{u}}(x_2, u_2, t) \|_F \leqslant \\ &L_2(\| x_1 - x_2 \|_2 + \| u_1 - u_2 \|_2). \end{aligned}$$

Assumption 2 Suppose that $\phi_{\mathrm{u}}^{-1}(x, u, t)$ exists and is uniformly bounded in Ω, i.e.

$$B_1 \leqslant \| \phi_{\mathrm{u}}^{-1}(x(t), u(t), t) \|_{F_s} \leqslant B,$$

where B_1 and B are constants.

Assumption 3 For given $y(t)$, there exists a unique $x(t)$ and $u(t)$ such that Eq.(1) is hold. In particular, for given target trajectory $y_{\mathrm{d}}(t)$, there exists a unique control $u_{\mathrm{d}}(t)$ such that

$$\begin{cases} \dot{x}_{\mathrm{d}}(t) = f(x_{\mathrm{d}}(t), u_{\mathrm{d}}(t), t), \\ y_{\mathrm{d}}(t) = \phi(x_{\mathrm{d}}(t), u_{\mathrm{d}}(t), t), \end{cases} \tag{2}$$

for $t \in [0, T]$.

The control target is to find a control sequence $\{u_i(t)\}$ such that the target trajectory $y_d(t)$ can be tracked by the system output $y_k(t)$, i.e.

$$\lim_{k\to\infty} y_k(t) = y_d(t).$$

3 New iterative learning control scheme

3.1 Motivation of the new algorithm

It is well known that Newton-type methods have been applied to nonlinear iterative learning control problems with quick convergence[12–13]. However, only when the initial approximation is sufficiently close to the solution for the problem, the convergence of this algorithm can be guaranteed, i.e, Newton-type methods have only local convergence. Since the solution for the problem is unknown, it is difficult to choose initial approximation such that the convergence of the algorithm is guaranteed.

For example, in [2], the Newton-type iterative learning control scheme was proposed with local convergent in which the convergent range satisfies

$$\| e_i \|_s \leqslant \frac{2r_1}{M_{uu}\alpha_1^{-2}},$$

where $r_1 \in (0,1)$, $\alpha_1 \leqslant \| \phi_u \| \leqslant \alpha_2$ and $\| \phi_{uu} \| \leqslant M_{uu}$. In particular, the initial approximation is subject to

$$\| y_d - \phi(x_0, u_0, t) \|_s = \| e_0 \|_s \leqslant \frac{2\alpha_1^2}{M_{uu}}.$$

Since the target control u_d and the target state x_d are unknown, it is hard to choose u_0.

In order to widen the range of convergence, when $\| e_i \|_s > \dfrac{2r_1}{M_{uu}\alpha_1^{-2}}$, the linear iterative learning control scheme was presented as follows[2]:

$$u_{i+1} = u_i + \Gamma \Delta e_i(t).$$

The constant matrix Γ should be subject to some strict conditions, but it is hardly to be found in fact.

Efficient iterative method should permit rather general initial approximation not only those close to the solution for the problem, i.e., called global convergence algorithms[12–13]. In this paper, a homotopy transformation is constructed into the iterative learning control problem. The homotopy extension and exchanging column update Newton method is utilized to iterative learning control scheme which can achieve global convergence.

3.2 Iterative learning control scheme

In order to illustrate the proposition of the homotopy extension mapping, a lemma is presented, firstly.

Lemma 1 If Assumptions 1–3 are satisfied, then for given $u(t)$, there exists the unique x such that Eq.(1) is hold and x is a continuous function of u.

Proof It can be seen that Eq.(1) is equivalent to

$$x(t) = x^{(0)} + \int_0^t f(x(\tau), u(\tau), \tau)\mathrm{d}\tau.$$

By using the contraction mapping theorem, it is able to prove that for the given $u(t)$, there exists the unique corresponding $x(t)$ satisfies $\dot{x}(t) = f(x(t), u(t), t)$[14]. We can prove the state x is a continuous function with respect to u. In fact,

$$\begin{aligned}
&\| x(u+\Delta u) - x(u) \| = \\
&\| \int_0^t (f(x(u+\Delta u), u+\Delta u, \tau) - \\
&f(x(u), u, \tau))\mathrm{d}\tau \| \leqslant \\
&L_1 \int_0^t (\| x(u+\Delta u) - x(u) \| + \| \Delta u \|)\mathrm{d}\tau.
\end{aligned}$$

From Grownwall Lemma, we get

$$\begin{aligned}
&\| x(u+\Delta u) - x(u) \| \leqslant \\
&\frac{\mathrm{e}^{L_1 T} - 1}{\lambda} L_1 \| \Delta u \| \to 0 \,(\text{if } \| \Delta u \| \to 0).
\end{aligned}$$

The new iterative learning control scheme is presented in the following:

1) Choose initial control u_0, and from the state equation

$$\dot{x}_0(t) = f(x_0(t), u_0(t), t),$$

we can obtain $x_0(t)$. Thus, we have

$$y_0(t) = \phi(x_0(t), u_0(t), t).$$

2) Construct the mapping

$$\begin{aligned}
H(u,\mu) =& (\phi(x(t), u(t), t) - y_d(t)) + \\
& (\mu - 1)(\phi(x_0(t), u_0(t), t) - y_d(t)).
\end{aligned}$$

It is obvious that when $\mu = 0$, we get

$$\begin{aligned}
&H(u,\mu) = H(u,0) = \\
&(\phi(x,u,t) - y_d) - (\phi(x_0,u_0,t) - y_d) = \\
&\phi(x,u,t) - \phi(x_0,u_0,t).
\end{aligned}$$

Thus, (x_0, u_0) satisfies $H(u,0) = 0$. When $\mu = 1$, we have

$$\begin{aligned}
H(u,\mu) =& H(u,1) = \phi(x,u,t) - y_d = \\
& \phi(x,u,t) - \phi(x_d,u_d,t).
\end{aligned}$$

Therefore, (x_d, u_d) satisfies with $H(u,1) = 0$, where u_d and x_d are the target control and target state of systems, respectively. Choose appropriate N, then given points of division $\{\mu_i, i = 1, 2, \cdots, N\}$ which satisfy

$$0 = \mu_0 < \mu_1 < \cdots < \mu_N = 1,$$

and

$$\Delta\mu_i = \mu_i - \mu_{i-1} = \frac{1}{N}.$$

3) The iterative learning control process is derived as follows:

For the ith$(i = 1, 2, \cdots, N)$ iteration, we have

$$\begin{aligned}
&\dot{x}_k(t) = f(x_k^i(t), u_k^i(t), t), \\
&y_k^i(t) = \phi(x_k^i(t), u_k^i(t), t), \\
&u_{k+1}^i = u_k^i + (A_k^i)^{-1} H(u_k^i, \mu_i),
\end{aligned}$$

where

$$H(u_k^i,\mu_i)=(\phi(x_k^i,u_k^i,t)-y_{\rm d})+ \\ (\mu_i-1)(\phi(x_0,u_0,t)-y_{\rm d}), \\ x_0^1=x_0,\ u_0^1=u_0,\ x_0^{i+1}=x_{m_i}^i,\ u_0^{i+1}=u_{m_i}^i, \\ k=0,1,2,\cdots,m_i-1,$$

and m_i is a positive number which is the step of the ith iteration.

4) In order to reduce calculation work, exchange column update Newton method is presented as follows:

From the above iterative learning process, let

$$\begin{cases} A_0^1=A_0=\phi_{\rm u}(x_0,u_0,t), \\ A_0^i=\phi_{\rm u}(x_0^{i+1},u_0^{i+1},t)=\phi_{\rm u}(x_{m_i}^i,u_{m_i}^i,t), \\ A_{k+1}^i=A_k^i+(\phi_{\rm u}(x_{k+1}^i,u_{k+1}^i,t)-A_k^i)e_{l_k}e_{l_k}^{\rm T}, \\ l_k=k\ (\mathrm{mod}\,m),\ k=0,1,\cdots,m_i-1, \end{cases} \tag{3}$$

where e_{l_k} denotes the unit vector of m dimension, i.e. the kth element is one and others are zero. For the method (3), A_{k+1}^{-1} can be calculated by the following recurrent formula:

$$(A_{k+1}^i)^{-1}=(A_k^i)^{-1}-\frac{1}{r}(A_k^i)^{-1}Pe_{l_k}^{\rm T}(A_k^i)^{-1}, \\ P=(\phi_{\rm u}(x_{k+1}^i,u_{k+1}^i,t)-A_k^i)e_{l_k},$$

where $r=1+e_{l_k}^{\rm T}(A_k^i)^{-1}P$. Therefore, the iterative learning control law based on homotopy extension updated Newton method is

$$\begin{cases} u_{k+1}^i=u_k^i(t)+(A_k^i)^{-1}H(u_k^i,\mu_i), \\ (A_{k+1}^i)^{-1}=(A_k^i)^{-1}-\frac{1}{r}(A_k^i)^{-1}Pe_{l_k}^{\rm T}(A_k^i)^{-1}. \end{cases} \tag{4}$$

4 Convergence of new iterative learning control scheme

Lemma 2[10] If the Assumptions 1–3 hold, the iterative process is produced by Eqs.(3)–(4), then we can get the following results.

1) If $k\leqslant m$, then

$$\|A_k-\phi_{\rm u}(x,u,t)\|_{F_s}\leqslant \\ L(\sum_{i=0}^{k}(\|x_{k-i}-x\|_s+\|u_{k-i}-u\|_s)).$$

2) If $k>m$, then

$$\|A_k-\phi_{\rm u}(x,u,t)\|_{F_s}\leqslant \\ L(\sum_{i=k-m-1}^{k}(\|x_{k-i}-x\|_s+\|u_{k-i}-u\|_s)),$$

where L is a constant.

According to Lemma 2, the following result is given.

Theorem 1 Let the initial error $\varepsilon_0=y_{\rm d}(t)-\phi(x_0,u_0,t)$, if Assumptions 1–3 are satisfied, and in the above iterative learning process, $\Delta\mu$ satisfies the following inequality:

$$\Delta\mu\leqslant\frac{1}{2(4LmB^2+QB^2)\|\varepsilon_0\|_s},$$

then the new iterative learning control algorithm is convergent

$$\lim_{k\to\infty}\|H(u_k^N,1)\|_\lambda=\lim_{k\to\infty}\|H(u_k^N,1)\|_s=0,$$

i.e.

$$\lim_{k\to\infty}y_k^N(t)=y_{\rm d}(t).$$

Proof From iterative learning control scheme in Section 3, we obtain

$$u_1^1=u_0^1+A_0^{-1}H(u_0^1,\mu_1)= \\ u_0-\phi_{\rm u}^{-1}(x_0,u_0,t)\Delta\mu\varepsilon_0,$$

then

$$\|u_1^1-u_0\|=\|\phi_{\rm u}^{-1}(x_0,u_0,t)\||\Delta\mu|\|\varepsilon_0\|\leqslant \\ B|\Delta\mu|\|\varepsilon_0\|=B\|H(u_0^1,\mu_1)\|.$$

Moreover,

$$\|x_1^1-x_0^1\|= \\ \|\int_0^t(f(x_1^1,u_1^1,\tau)-f(x_0^1,u_0^1,\tau))\mathrm{d}\tau\|\leqslant \\ L_1\int_0^t(\|x_1^1-x_0^1\|+\|u_1^1-u_0^1\|)\mathrm{d}\tau\leqslant \\ L_1\int_0^t\|x_1^1-x_0^1\|\,\mathrm{d}\tau+L_1B\,|\Delta\mu|\int_0^t\|\varepsilon_0\|\,\mathrm{d}\tau.$$

In terms of Bellman-Gronwall Lemma, we get

$$\|x_1^1-x_0^1\|\leqslant L_1\mathrm{e}^{L_1t}B\,|\Delta\mu|\,\frac{\mathrm{e}^{\lambda t-1}}{\lambda}\,\|\varepsilon_0\|\leqslant \\ O(\frac{1}{\lambda})\Delta\mu\,\|\varepsilon_0\|,$$

and

$$\|A_1^1-\phi_{\rm u}(x_1^1,u_1^1,t)\|_{F_s}\leqslant \\ L_2(\|x_1^1-x_1^0\|_s+\|u_1^1-u_1^0\|_s)\leqslant \\ O(\frac{1}{\lambda})\,\|\varepsilon_0\|_s+B\Delta\mu\,\|\varepsilon_0\|_s\,.$$

From Assumption 2, we have

$$\|A_1^{-1}\|_{F_s}\leqslant\frac{B}{1-(O(\frac{1}{\lambda})+B\Delta\mu)\|\varepsilon_0\|_s\,B}\leqslant 2B.$$

Thus,

$$\|u_{k+1}^1-u_k^1\|=\|(A_k^i)^{-1}H(u_k^1,\mu_1)\|\leqslant \\ \|(A_k^i)^{-1}\|\|H(u_k^1,\mu_1)\|,$$

and

$$\|x_{k+1}^1-x_k^1\|= \\ \|\int_0^t(f(x_{k+1}^1,u_{k+1}^1,\tau)-f(x_k^1,u_k^1,\tau))\mathrm{d}\tau\|\leqslant \\ L_1\int_0^t(\|x_{k+1}^1-x_k^1\|+\|u_{k+1}^1-u_k^1\|)\mathrm{d}\tau\leqslant \\ L_1O(\frac{1}{\lambda})\,\|(A_k^i)^{-1}\|\|H(u_k^1,\mu_1)\|_\lambda\,.$$

We also have

$$H(u_{k+1}^1,\mu_1)-H(u_k^1,\mu_1)= \\ \phi(x_{k+1}^1,u_{k+1}^1,t)-\phi(x_k^1,u_k^1,t)= \\ -\int_0^1\phi_x(x_k^1+\tau(x_{k+1}^1-x_k^1),u_{k+1}^1,\tau)\mathrm{d}\tau\,\cdot$$

$$(x_{k+1}^1 - x_k^1) - \phi_{\rm u}(x_k^1, u_k^1, t)(u_{k+1}^1 - u_k^1) - \int_0^1 (1-\tau)\phi_{\rm uu}(x_k^1, u_k^1 + \tau(u_{k+1}^1 - u_k^1), \tau){\rm d}\tau \cdot (u_{k+1}^1 - u_k^1)(u_{k+1}^1 - u_k^1).$$

Therefore,

$$H(u_1^1, \mu_1) = H(u_0^1, \mu_1) - \int_0^1 \phi_x(x_0^1 + \tau(x_1^1 - x_0^1), u_1^1, \tau){\rm d}\tau \cdot (x_1^1 - x_0^1) - \phi_{\rm u}(x_0^1, u_0^1, t)(u_1^1 - u_0^1) - \int_0^1 (1-\tau)\phi_{\rm uu}(x_0^1, u_0^1 + \tau(u_1^1 - u_0^1), \tau){\rm d}\tau \cdot (u_1^1 - u_0^1)(u_1^1 - u_0^1).$$

Then, we get

$$\|H(u_1^1, \mu_1)\|_\lambda \leqslant L_1 O(\frac{1}{\lambda}) \|H(u_0^1, \mu_1)\|_\lambda + \frac{1}{2} QB^2 \|H(u_0^1, \mu_1)\|_s \|H(u_0^1, \mu_1)\|_\lambda < \frac{1}{2} \|H(u_0^1, \mu_1)\|_\lambda .$$

If $j \leqslant k$, the following inequalities are hold:

$$\|(A_j^1)^{-1}\|_{F_s} \leqslant 2B,$$

$$\|H(u_j^1, \mu_1)\|_\lambda \leqslant \frac{1}{2} \|H(u_{j-1}^1, \mu_1)\|_s,$$

$$\|x_j - x_{j+1}\|_\lambda \leqslant L_1 O(\frac{1}{\lambda}) \|\varepsilon_0\|_\lambda .$$

If $j = k+1$, from Lemma 2 and Eq.(3), we get

$$\|A_{k+1}^1 - \phi_{\rm u}(x_{k+1}^1, u_{k+1}^1, t)\|_{F_s} \leqslant L(\sum_{i=1}^{m-1}(\|x_k^1 - x_{k-i}^1\|_s + \|u_k^1 - u_{k-i}^1\|_s)),$$

and

$$\|x_{k+1}^1 - x_k^1\|_2 = \|\int_0^t (f(x_{k+1}^1, u_{k+1}^1, \tau) - f(x_{k+1}^1, u_{k+1}^1, \tau)){\rm d}\tau\|_2 \leqslant L_1 \int_0^t (\|x_{k+1}^1 - x_k^1\|_2 + \|(A_k^1)^{-1}\|_{F_s} \|H(u_k^1, \mu_1)\|_2){\rm d}\tau \leqslant 2BL_1 {\rm e}^{L_1 t} \frac{{\rm e}^{\lambda t} - 1}{\lambda} \|H(u_k^1, \mu_1)\|_\lambda .$$

Thus,

$$\|x_{k+1}^1 - x_k^1\|_\lambda \leqslant O(\frac{1}{\lambda}) \|H(u_k^1, \mu_1)\|_\lambda .$$

When $k \leqslant m$, from recurrent method, we can obtain

$$\|H(u_{k+1}^1, \mu_1)\|_\lambda \leqslant \frac{1}{2} \|H(u_k^1, \mu_1)\|_\lambda .$$

When $k > m$ and $j = k+1$, we have

$$\phi_{\rm u}(x_k^1, u_k^1, t)(u_{k+1}^1 - u_k^1) = \phi_{\rm u}(x_k^1, u_k^1, t)(A_k^1)^{-1} H(u_k^1, \mu) = (\phi_{\rm u}(x_k^1, u_k^1, t) - (A_k^1))(A_k^1)^{-1} H(u_k^1, \mu_1) + H(u_k^1, \mu_1).$$

Thus,

$$H(u_{k+1}^1, \mu_1) = -(\phi_{\rm u}(x_k^1, u_k^1, t) - A_k^1)(A_k^1)^{-1} H(u_k^1, \mu_1) - \int_0^1 \phi_x(x_k^1 + \tau(x_{k+1}^1 - x_k^1), u_{k+1}^1, t){\rm d}\tau (x_{k+1}^1 - x_k^1) - \int_0^1 (1-\tau)\phi_{\rm uu}(x_{k+1}^1, u_{k+1}^1 + \tau(u_{k+1}^1 - u_k^1), \tau){\rm d}\tau \cdot (A_k^1)^{-1} H(u_k^1, \mu_1)(A_k^1)^{-1} H(u_k^1, \mu_1).$$

From Bellman-Gronwall lemma, we have

$$\begin{aligned}
&\|H(u_{k+1}^1, \mu_1)\|_\lambda \leqslant \\
&\|\phi_{\rm u}(x_k^1, u_k^1, t) - A_k\|_{F_s} \|(A_k^1)^{-1}\|_{F_s} \|H(u_k^1, \mu_1)\|_\lambda + \\
&\frac{1}{2} \|\phi_{\rm uu}(x_k^1, u_k^1, t)\|_{F_s} \|(A_k^1)^{-1}\|_{F_s} \cdot \\
&\|H(u_k^1, \mu_1)\|_{F_s} \|(A_k^1)^{-1}\|_{F_s} \|H(u_k^1, \mu_1)\|_\lambda + \\
&\|\phi_x(x_k^1, u_k^1, t)\|_{F_s} \|x_{k+1}^1 - x_k^1\|_\lambda \leqslant \\
&L(\sum_{i=1}^{m-1}(\|x_k^1 - x_{k-i}^1\|_s + \|u_k^1 - u_{k-i}^1\|_s)) \cdot \\
&2B\|H(u_k^1, \mu_1)\|_\lambda + \frac{Q}{2} 4B^2 \|H(u_k^1, \mu_1)\|_s \cdot \\
&\|H(u_k^1, \mu_1)\|_\lambda + KO(\frac{1}{\lambda}) \|H(u_k^1, \mu_1)\|_\lambda \leqslant \\
&8LmB^2 \|H(u_{k-m+1}^1, \mu_1)\|_s \|H(u_k^1, \mu_1)\|_\lambda + \\
&2QB^2 \|H(u_k^1, \mu_1)\|_s \|H(u_k^1, \mu_1)\|_\lambda + \\
&O_1(\frac{1}{\lambda}) \|H(u_k^1, \mu_1)\|_\lambda \leqslant \\
&O_1(\frac{1}{\lambda}) \|H(u_k^1, \mu_1)\|_\lambda + 2(4LmB^2 + QB^2) \cdot \\
&\|H(u_k^1, \mu_1)\|_s \|H(u_k^1, \mu_1)\|_\lambda \leqslant \\
&\frac{1}{2} \|H(u_k^1, \mu_1)\|_\lambda .
\end{aligned} \tag{5}$$

Then, we get

$$\lim_{k\to\infty} \|H(u_k^1, \mu_1)\|_\lambda = \lim_{k\to\infty} \|H(u_k^1, \mu_1)\|_s = 0.$$

There exists the positive number m_1 such that

$$\|H(u_{m_1}^1, \mu_1)\|_\lambda \leqslant \frac{1}{2} \Delta\mu \|\varepsilon_0\|_s .$$

We can take $u_0^2 = u_{m_1}^1$ and $x_0^2 = x_{m_1}^1$ for solving the $i = 2$ subproblem iteration.

Similarly, we can also obtain

$$\|H(u_{k+1}^i, \mu_i)\|_\lambda \leqslant 8LmB^2 \|H(u_{k-m+1}^i, \mu_i)\|_s \|H(u_k^i, \mu_i)\|_\lambda + 2QB^2 \|H(u_k^i, \mu_i)\|_s \|H(u_k^i, \mu_i)\|_\lambda + O_1(\frac{1}{\lambda}) \|H(u_k^i, \mu_i)\|_\lambda \leqslant \frac{1}{2} \|H(u_k^i, \mu_i)\|_\lambda .$$

Thus,

$$\lim_{k\to\infty} \|H(u_k^i, \mu_i)\|_\lambda = \lim_{k\to\infty} \|H(u_k^i, \mu_i)\|_s = 0.$$

There exists the positive number m_i such that

$$\|H(u_{m_i}^i, \mu_i)\|_\lambda \leqslant \Delta\mu \|\varepsilon_0\|_s .$$

We can take $u_0^{i+1} = u_{m_i}^i$ and $x_0^{i+1} = x_{m_i}^i$ for solving the ith problem, then

$$\|H(u_{k+1}^{i+1}, \mu_{i+1})\|_\lambda < \frac{1}{2}\|H(u_k^{i+1}, \mu_{i+1})\|_\lambda .$$

Therefore, for $i = 1, 2, \cdots, N-1$, we have

$$\lim_{k\to\infty}\| H(u_k^{i+1}, \mu_{i+1})\|_\lambda = \lim_{k\to\infty}\| H(u_k^{i+1}, \mu_{k+1})\|_s = 0.$$

In particular,

$$\lim_{k\to\infty}\| H(u_k^N, 1)\|_\lambda = \lim_{k\to\infty}\| H(u_k^N, 1)\|_s = 0.$$

i.e.

$$\lim_{k\to\infty}\phi(x_k(t), u_k(t), t) = y_{\rm d}(t).$$

Since $H(u_k^i, \mu_i)$ converges to 0 in supreme norm, $H(u_k^i, \mu_i)$ uniformly converges to 0 on $[0, T]$ from Ascoliarzela theorem. From Eq.(5), we can see that $\{u_k^i\}$ is a Cauchy sequence which implies $\{u_k^i\}$ is a convergent sequence. Therefore,

$$\lim_{k\to\infty}\| H(u_k^N, 1)\|_\lambda = \lim_{k\to\infty}\| H(u_k^N, 1)\|_s = 0,$$

i.e.

$$\lim_{k\to\infty}\phi(x_k^N, u_k^N, t) = \lim_{k\to\infty} y_k^N(t) = y_{\rm d}(t).$$

5 Conclusions

In this paper, a new iterative learning control algorithm based on extension-updated Newton method for nonlinear systems is presented. In terms of homotopy extension methods, a homotopy is constructed in iterative learning control problem. The solving process is divided into N subproblem by the new algorithm. The exchange column update Newton method is used to solve the subproblem by simple recurrent formula. The new iterative learning control algorithm is proposed to wide the range of convergence and the iterative learning process of new algorithm is derived. Sufficient conditions for the convergence of the new algorithm are given and proved. This new algorithm has global convergence instead of local convergence of classical Newton-type methods.

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