

一类有色噪声干扰下的随机时变系统学习辨识

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摘要: 讨论由一类时变ARMAX模型描述的动态系统学习辨识问题, 提出用于估计有限区间上重复运行时变系统时变参数的学习算法. 文中给出最小二乘学习算法的具体形式及实现步骤, 并分析所提出学习算法的收敛性. 分析结果表明, 当重复持续激励条件成立且满足严格正实条件时, 提出的学习算法具有重复一致性, 即参数估值完全收敛于真值. 文中还将结果推广到一类周期时变系统. 通过数值仿真, 进一步对所提学习算法的有效性进行了验证.

关键词: 迭代算法; 学习辨识; 递推辨识; 随机时变系统

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Learning identification of a class of stochastic time-varying systems with colored noise

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Abstract: This paper presents a learning identification method for repetitive systems with time-varying parametric uncertainties. The least squares learning algorithm is derived on the basis of repetitive operations over a pre-specified finite time interval. Sufficient conditions for establishing repetitive consistency of the learning algorithm are given, including the persistent excitation condition and the strictly positive real condition. It is shown that the estimates converge to the time-varying values of the parameters, and the complete estimation can be achieved. The learning identification method is also shown to be applicable to periodically time-varying systems. Numerical simulations are presented to demonstrate the effectiveness of the proposed learning algorithms.

Key words: iterative algorithms; learning identification; recursive identification; time-varying systems

1 引言(Introduction)

离散时变动态系统可以时变ARMAX模型描述, 其时变参数估计方法是令人关注的问题. 递推辨识算法随时间逐步修正参数估值(与系统随时间演化方向一致). 递推最小二乘算法以其收敛速度快, 且可实现在线辨识的特点, 在实际场合应用的较为普遍. 递推算法获得的当前时刻的输入输出数据, 随即用来修正参数估值. 在一定持续激励条件下, 当时间足够大时, 可保证参数估值收敛于真值^[1-4]. 目前已发表的关于递推辨识算法的分析结果多针对定常系统^[5-11]. 对于不可简化的(irreducible)时变系统, 递推辨识对其数学模型中的时变参数是无法得到一致估计的, 因而人们更关注递推算法的跟踪能力. 递推最小二乘算法不具备关于时变参数的跟踪能力^[12]. 为了改进其跟踪能力, 可引入遗忘因子, 构造遗忘因子最小二乘算法. 人们致力于给出跟踪算法的参数估计误差上界, 以便更准确地评价递推算法的参数估计效果^[3-4, 13]. 这些理论结果对于不可简化非平稳过

程的参数估计是十分重要的.

学习辨识借助于系统重复作业的特点构造估计算法, 是时变系统参数估计的一种适用方法. 对于在有限区间上重复运行的系统, 假设时变参数能完全复现上一次的变化规律. 这样固定任一时刻, 沿重复轴来看, 相当于对一定常参数进行估计. 这样本文可沿重复轴构造“递推”算法, 即学习辨识算法^[14-16]. 学习算法沿重复轴进行, 利用新的观测数据, 对上次运行时某时刻的参数估值进行修正, 以获得此次运行该时刻的参数估值. 若给定区间上任一时刻的输入输出数据都满足重复持续激励条件, 则此区间上任一时刻的参数估值会收敛于相应时刻的参数真值, 实现完全估计^[16]. 而且, 无论参数慢变、快变、亦或突变, 学习辨识算法可以给出整个区间上时变参数的一致性收敛结果.

针对以一类ARMAX模型描述的时变系统, 本文给出最小二乘学习算法和具体实现步骤, 并分析这一学习算法的收敛性能. 在重复持续激励和正实条

件下, 证明所提学习算法的重复一致性. 文中还将分析结果推广到周期时变系统.

2 参数不变量原理(Parametric invariant principle)

对于一个实际系统, 当其模型的结构一旦确定, 系统建模任务主要是参数估计问题, 即在参数空间中确定适于该系统的一组参数. 通常, 只考虑参数空间的参数均是定常的. 实际中许多系统的动态特性具有时变性, 为解决时变动态特性的建模问题, 笔者需考虑的参数空间也应具有时变参数, 即参数是时间的函数. 参数随时间可能缓变, 也可能快变, 甚至突变.

参数估计过程可看成是学习过程. 一般地, 一辨识算法针对某种参数不变量能够给出一致性结果. 因此, 寻找参数不变量也应是系统辨识的重要内容.

一个被估参数是可一致估计的条件是它是一种参数不变量. 笔者称这一断言为“参数不变量原理”.

本文研究有限区间时变系统的学习辨识问题, 按照上述不变量原理, 笔者需寻取参数不变量. 设系统在指定的有限区间上重复运行. 时变参数随时间而变化, 但不随重复次数变化. 沿重复轴来看, 固定时刻对应的参数是一固定值. 笔者称这种不变量为重复参数, 满足 $\theta_k(t) = \theta_{k-1}(t), t \in \{0, 1, 2, \dots, N\}, k = 0, 1, 2, \dots$ 表示重复次数. 见图1所示. 当系统连续运行时, 周期参数是一种参数不变量. 周期参数(周期为 N), 它按周期变化, 满足 $\theta(t') = \theta(t' - N)$. 周期参数实际上也是重复参数. 对于周期为 N 的周期参数 $\theta(t')$, 令 $t' = t + kN, t = 0, 1, 2, \dots, N - 1, k = 0, 1, 2, \dots$, 则 $\theta(t') = \theta_k(t)$.

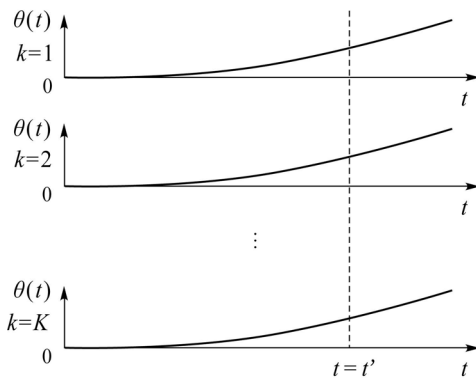


图 1 重复时变参数

Fig. 1 Repetitively time-varying parameters

下面从辨识的3要素(数据、模型类和准则)来说明学习辨识. 非重复系统在无穷区间上持续运行, 记采样点为 $0, 1, 2, \dots$. 采样得到一时间序列, 随着系统运行时间的增加, 该时间序列的长度也不断增加. 相比较地, 为了能够重复运行, 系统的运行区间是有限的, 记采样点为 $0, 1, 2, \dots, N$. 每一次运行采样得

一时间序列, 序列长度为 $N + 1$. 当重复次数不断增加时, 得到的时间序列的个数也不断增加. 沿重复轴来看, 对于每个固定的 t , 通过重复得到一迭代序列; 对于所有的 t , 共得到了 $N + 1$ 个迭代序列. 当重复次数不断增加时, 每个迭代序列的长度也不断增加. 模型类是针对有限区间时变系统给出的, 含 M 个时变参数待估计. 就其中一个参数而言, 假设该参数是时变的但是迭代独立的. 那么, 固定时刻 t , 沿重复轴来看, 该参数是不变的. 这样, 待估参数为 $N + 1$ 个常数. 共计 $(N + 1)M$ 个常数待估计. 每一个待估常数利用相应的迭代序列进行估计, 推导估计算法的准则应该是固定时刻 t , 按重复轴给出. 相比较地, 现有辨识方法中的数据是一时间序列; 模型类通常是定常参数系统; 推导估计算法的准则是按时间轴给出的, 待估常数以采样得到的时间序列来估计.

综上所述, 学习辨识方法可描述如下:

- F1: 系统的作业区间是有限的;
- F2: 系统在指定作业区间上重复运行;
- F3: 系统参数是时变的, 但是重复独立的;
- F4: 每次运行时, 能够获得有效的的信号数据;
- F5: 固定时刻的参数估值沿重复轴修正.

F1是保证系统能够重复运行的前提; F2说明学习辨识适于重复运行下的系统, 但未限制系统的结构必须随重复轴变化, 也未限定初始条件的具体位置; F3决定怎样的时变参数为不变量; 每次运行时, F4要求获得数据需有效, 如满足重复持续激励条件, 保证参数估值的一致性; F5说明学习算法的形式.

当固定时刻对应的输入输出数据沿重复轴方向满足持续激励条件时, 可获得该固定时刻对应参数的一致性估计. 如果重复激励条件在作业区间上每一点成立, 可获得该时变参数在所有时刻的真值, (即实现完全估计)需要指出的是, 无论参数慢变, 快变甚至突变, 学习辨识算法可以给出其完全估计. 可以看出, 不变量原理给出的只是可学习的必要条件, 但不是充分条件.

周期系统的参数沿时间轴呈周期性变化, 每个周期中某时刻对应的参数是固定不变的, 如图2所示. 学习辨识方法也可用于周期参数的估计.

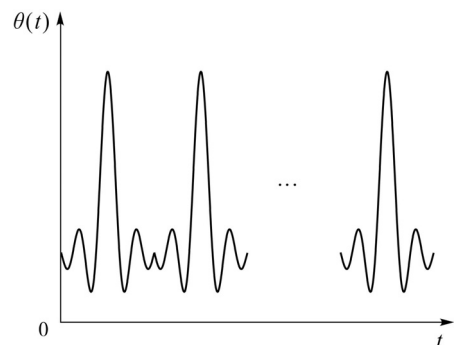


图 2 周期时变参数

Fig. 2 Periodically time-varying parameters

3 时变ARMAX模型与学习辨识算法(Time-varying ARMAX model and learning identification algorithm)

3.1 有限区间上时变ARMAX模型与学习辨识算法(Time-varying ARMAX model and learning identification algorithm over finite interval)

考虑下述ARMAX模型描述的离散时变系统:

$$A(q^{-1}, t)y_k(t) = B(q^{-1}, t)u_k(t) + w_k(t), \quad (1)$$

其中: $u_k(t)$ 和 $y_k(t)$ 分别为系统的输入和输出序列, $w_k(t)$ 为有色噪声序列, $A(q^{-1}, t)$ 和 $B(q^{-1}, t)$ 均为单位后移算子 q^{-1} 的多项式,

$$A(q^{-1}, t) = 1 + a_1(t)q^{-1} + a_2(t)q^{-2} + \dots + a_{n_a}(t)q^{-n_a},$$

$$B(q^{-1}, t) = b_1(t)q^{-1} + b_2(t)q^{-2} + \dots + b_{n_b}(t)q^{-n_b},$$

其中 $a_1(t), \dots, a_{n_a}(t)$ 及 $b_1(t), \dots, b_{n_b}(t)$ 为待估计时变参数.

本文考虑有色噪声干扰 $w_k(t)$ 是白噪声序列经由一定常系统产生的情形, 即

$$w_k(t) = D(q^{-1})v_k(t),$$

式中:

$$D(q^{-1}) = 1 + d_1q^{-1} + d_2q^{-2} + \dots + d_{n_d}q^{-n_d},$$

d_1, d_2, \dots, d_{n_d} 为未知定常参数, $v_k(t)$ 为白噪声.

假设多项式 $A(q^{-1}, t)$, $B(q^{-1}, t)$ 及 $D(q^{-1})$ 中的系数均是重复独立的, 即对于固定的时刻, 它们沿重复轴是恒定不变的.

为了学习算法推导方便, 将式(1)写成如下回归形式:

$$y_k(t) = \phi_k^T(t)\theta(t) + v_k(t), \quad (2)$$

其中:

$$\phi_k(t) = (-y_k(t-1), -y_k(t-2), \dots, -y_k(t-n_a),$$

$$u_k(t-1), u_k(t-2), \dots, u_k(t-n_b),$$

$$v_k(t-1), v_k(t-2), \dots, v_k(t-n_d))^T,$$

$$\theta(t) = (a_1(t), a_2(t), \dots, a_{n_a}(t), b_1(t), b_2(t),$$

$$\dots, b_{n_b}(t), d_1, d_2, \dots, d_{n_d})^T.$$

考虑到 $\phi_k(t)$ 中的 $v_k(t-i)$ ($i = 1, 2, \dots, n_d$)不可测量, 于是用估计值 $\hat{v}_k(t-i)$ ($i = 1, 2, \dots, n_d$)作为替代.

$$y_k(t) = \hat{\phi}_k^T(t)\hat{\theta}(t) + \hat{v}_k(t), \quad (3)$$

其中:

$$\hat{\phi}_k(t) = (-y_k(t-1), -y_k(t-2), \dots, -y_k(t-n_a),$$

$$u_k(t-1), u_k(t-2), \dots, u_k(t-n_b),$$

$$\hat{v}_k(t-1), \hat{v}_k(t-2), \dots, \hat{v}_k(t-n_d))^T,$$

$$\hat{\theta}(t) = (\hat{a}_1(t), \hat{a}_2(t), \dots, \hat{a}_{n_a}(t), \hat{b}_1(t), \hat{b}_2(t),$$

$$\dots, \hat{b}_{n_b}(t), \hat{d}_1, \hat{d}_2, \dots, \hat{d}_{n_d})^T.$$

系统在给定的作业区间上重复运行时, 记录或量测由第1次至第 k 次运行时的输入输出数据 $\{y_i(t), \phi_i(t), 0 \leq t \leq N, i = 0, 1, \dots, k\}$. 对于固定的时刻 $t \in \{0, 1, \dots, N\}$, 考虑极小化下式:

$$J_k(\hat{\theta}_k(t), t) = \sum_{i=1}^k [y_i(t) - \hat{\phi}_i^T(t)\hat{\theta}_i(t)]^2, \quad (4)$$

可给出如下学习算法:

$$\hat{\theta}_k(t) = \hat{\theta}_{k-1}(t) + \mathbf{L}_k(t)e_k(t), \quad (5)$$

$$\mathbf{L}_k(t) = \frac{P_{k-1}(t)\hat{\phi}_k(t)}{1 + \hat{\phi}_k^T(t)P_{k-1}(t)\hat{\phi}_k(t)}, \quad (6)$$

$$P_k(t) = [I - \mathbf{L}_k(t)\hat{\phi}_k^T(t)]P_{k-1}(t), \quad (7)$$

其中: $P_k^{-1}(t) = P_0^{-1}(t) + \sum_{i=1}^k \hat{\phi}_i(t)\hat{\phi}_i^T(t)$, $e_k(t) = y_k(t) - \hat{\phi}_k^T(t)\hat{\theta}_{k-1}(t)$ 为新息, 则式(5)与式(7)构成最小二乘学习算法. 学习算法具体实现步骤如下:

步骤 1 对于 $t = 0, 1, \dots, N$, 给定参数初始估值 $\hat{\theta}_0(t) = 0$ 和迭代公式中所需的 $P_0(t)$ 值, 并置 $k = 1$.

步骤 2 第 k 次重复运行时, 记录或量测数据 $u_k(t), y_k(t)$.

步骤 3 计算 $\hat{\phi}_k(t)$ 及 $e_k(t)$.

步骤 4 依照式(5)计算 $\hat{\theta}_k(t)$.

步骤 5 依照式(7)计算 $P_k(t)$.

步骤 6 利用式(3)更新 $\hat{v}_k(t)$.

步骤 7 检验迭代停止条件. 若条件满足则停止运行; 否则置 $k = k + 1$, 转至步骤1.

3.2 周期时变ARMAX模型与学习辨识算法(Periodically time-varying ARMAX model and learning identification algorithm)

周期时变系统沿时间轴连续运行, 其参数的周期为 N , 则时间变量可表示为 $t + kN, t = 0, 1, \dots, N - 1, k = 1, 2, \dots$. 参数满足 $\theta(t + kN) = \theta(t + (k - 1)N)$. 当系统在有限区间上重复运行时, 作业区间为 N , 其时间变量为 $t = 0, 1, \dots, N$, 重复次数 $k = 1, 2, \dots$. 显然, 这类重复系统不能保证 $\theta(kN) = \theta((k - 1)N)$.

将周期时变ARMAX模型写成如下的回归形式:

$$y(t + kN) = \phi^T(t + kN)\theta(t + kN) + v(t + kN), \quad (8)$$

其中

$$\phi(t + kN) = (-y(t + kN - 1), -y(t + kN - 2), \dots,$$

$$\begin{aligned}
 & -y(t+kN-n_a), u(t+kN-1), u(t+kN-2), \\
 & \dots, u(t+kN-n_b), v(t+kN-1), \\
 & v(t+kN-2), \dots, v(t+kN-n_d))^T.
 \end{aligned}$$

考虑到其中的 $v(t+kN-i)(i=1, 2, \dots, n_d)$ 不可测量, 于是用估计值 $\hat{v}(t+kN-i)$ 作为替代.

$$y(t+kN) = \hat{\phi}^T(t+kN)\hat{\theta}(t+kN) + \hat{v}(t+kN). \quad (9)$$

系统可在无限区间上周期运行, 则针对一个周期上的固定时刻 $t \in \{0, 1, \dots, N-1\}$, 考虑极小化下式:

$$\begin{aligned}
 & J(\hat{\theta}(t+kN), t+kN) = \\
 & \sum_{i=1}^k [y(t+iN) - \hat{\phi}^T(t+iN)\hat{\theta}(t+iN)]^2. \quad (10)
 \end{aligned}$$

学习算法如下:

$$\begin{aligned}
 & \hat{\theta}(t+kN) = \\
 & \hat{\theta}(t+(k-1)N) + \mathbf{L}(t+kN)e(t+kN), \quad (11)
 \end{aligned}$$

$$\begin{aligned}
 & \mathbf{L}(t+kN) = \\
 & \frac{P(t+(k-1)N)\hat{\phi}(t+kN)}{1 + \hat{\phi}^T(t+kN)P(t+(k-1)N)\hat{\phi}(t+kN)}, \quad (12)
 \end{aligned}$$

$$\begin{aligned}
 & P(t+kN) = \\
 & [I - \mathbf{L}(t+kN)\hat{\phi}^T(t+kN)]P(t+(k-1)N). \quad (13)
 \end{aligned}$$

周期学习辨识的具体实现步骤如下:

步骤 1 给定初始参数估值 $\hat{\theta}(t) = 0, P(t)$. 其中, $t = 0, 1, \dots, N-1$, 并置 $k = 1$.

步骤 2 在第 k 个周期内, 记录或量测数据 $u(t+kN), y(t+kN)$.

步骤 3 计算 $\phi(t+kN)$ 及 $e(t+kN)$.

步骤 4 依照式(11)计算 $\hat{\theta}(t+kN)$.

步骤 5 依照式(13)计算 $P(t+kN)$.

步骤 6 利用式(9)更新 $\hat{v}(t+kN)$.

步骤 7 检验迭代停止条件, 若条件满足则停止运行; 否则置 $k = k + 1$, 转至步骤 1.

4 收敛性分析(Convergence analysis)

迭代学习算法依据系统在作业区间上重复运行中的输入输出数据, 获得参数估值. 由于作业区间是有限的, 无法得到常规意义下的估计一致性, 只能获得重复一致性, 即, 对于 $t \in \{0, 1, 2, \dots, N\}$,

$$\lim_{k \rightarrow \infty} \hat{\theta}_k(t) = \theta(t), \text{ a.s.} \quad (14)$$

周期学习算法的重复一致性是指, 对于 $t \in \{0, 1, 2, \dots, N-1\}$,

$$\lim_{k \rightarrow \infty} \hat{\theta}(t+kN) = \theta(t), \text{ a.s.}, \quad (15)$$

这里, $\theta(t+kN) = \theta(t)$, 对于所有 k 成立.

4.1 迭代学习最小二乘算法的收敛性分析 (Convergence analysis of iterative learning least squares algorithm)

对于固定时刻 $t \in \{0, 1, \dots, N\}$, 记 $F_k(t)$ 为由经历 k 次重复作业获得的输入输出数据所构成的 σ 代数. 为了对提出的学习算法进行收敛性分析, 做如下假设.

假设 1 $v_k(t)$ 满足

$$E[v_k(t)|F_{k-1}(t)] = 0, \text{ a.s.} \quad (16)$$

假设 2 存在关于 t 一致有界的 $\sigma_v(t)$ 使得

$$E[v_k^2(t)|F_{k-1}(t)] \leq \sigma_v^2(t), \text{ a.s.} \quad (17)$$

假设 3 存在 k_0 , 当 $k \geq k_0$ 时, 下述重复持续激励条件成立:

$$\alpha(t)I \leq \frac{1}{k} \sum_{i=1}^k \phi_i(t)\phi_i^T(t) \leq \beta(t)I, \text{ a.s.}, \quad (18)$$

式中 $\alpha(t), \beta(t) > 0$.

假设 4 $\frac{1}{D(q^{-1})} - \frac{1}{2}$ 是严格正实的.

引理 1 如果存在 k_0 , 使得 $|P_{k_0-1}^{-1}(t)| > 1$ 但有界, 那么

$$\sum_{i=k_0}^k \frac{|P_i^{-1}(t)| - |P_{i-1}^{-1}(t)|}{|P_i^{-1}(t)|(\ln |P_i^{-1}(t)|)^{\delta(t)}} < \infty, \quad (19)$$

其中: $\delta(t) > 1, |\cdot|$ 表示矩阵 (\cdot) 的行列式.

证 对于 $k \geq k_0$,

$$\begin{aligned}
 & \sum_{i=k_0}^k \frac{|P_i^{-1}(t)| - |P_{i-1}^{-1}(t)|}{|P_i^{-1}(t)|(\ln |P_i^{-1}(t)|)^{\delta(t)}} \leq \\
 & \int_{|P_{k_0-1}^{-1}(t)|}^{|P_k^{-1}(t)|} \frac{dx}{x(\ln x)^{\delta(t)}} = \\
 & \frac{-1}{\delta(t) - 1} \cdot \frac{1}{(\ln x)^{\delta(t)-1}} \Big|_{|P_{k_0-1}^{-1}(t)|}^{|P_k^{-1}(t)|} = \\
 & \frac{1}{\delta(t) - 1} \left[\frac{1}{(\ln |P_{k_0-1}^{-1}(t)|)^{\delta(t)-1}} - \frac{1}{(\ln |P_k^{-1}(t)|)^{\delta(t)-1}} \right] \leq \\
 & \frac{1}{\delta(t) - 1} \frac{1}{(\ln |P_{k_0-1}^{-1}(t)|)^{\delta(t)-1}}.
 \end{aligned}$$

由 $\ln |P_{k_0-1}^{-1}(t)| > 0$ 有界, 证得式(19). 证毕.

定理 1 在假设1-4成立时, 由式(5)-(7)给出的参数估值使得

$$\|\tilde{\theta}_k(t)\| = O\left(\sqrt{\frac{\sum_{i=0}^t A_k(i)}{\lambda_{\min}[P_k^{-1}(t)]}}\right), \text{ a.s.}, \quad (20)$$

式中: 记 $A_k(i) = (\ln |P_k^{-1}(i)|)^{\delta(i)}$, $\tilde{\theta}_k(t) = \hat{\theta}_k(t) - \theta(t)$, $\lambda_{\min}[P_k^{-1}(t)]$ 为 $P_k^{-1}(t)$ 的最小特征值.

证

$$y_k(t) = \phi_k^T(t)\theta(t) + v_k(t) = \hat{\phi}_k^T(t)\hat{\theta}_k(t) + \hat{v}_k(t). \quad (21)$$

记

$$\begin{aligned} \hat{y}_k(t) &= \hat{\phi}_k^T(t)\hat{\theta}_{k-1}(t), \\ e_k(t) &= y_k(t) - \hat{\phi}_k^T(t)\hat{\theta}_{k-1}(t), \\ \hat{v}_k(t) &= y_k(t) - \hat{\phi}_k^T(t)\hat{\theta}_k(t). \end{aligned}$$

于是

$$\begin{aligned} \hat{v}_k(t) &= y_k(t) - \hat{\phi}_k^T(t)\hat{\theta}_k(t) = \\ &= y_k(t) - \hat{y}_k(t) + \hat{y}_k(t) - \hat{\phi}_k^T(t)\hat{\theta}_k(t) = \\ &= e_k(t) + \hat{\phi}_k^T(t)(\hat{\theta}_{k-1}(t) - \hat{\theta}_k(t)) = \\ &= e_k(t) - \hat{\phi}_k^T(t)P_k(t)\hat{\phi}_k(t)e_k(t) = \\ &= (1 - \hat{\phi}_k^T(t)P_k(t)\hat{\phi}_k(t))e_k(t). \end{aligned} \quad (22)$$

由 $P_k(t)$ 可得

$$1 - \hat{\phi}_k^T(t)P_k(t)\hat{\phi}_k(t) = \frac{1}{1 + \hat{\phi}_k^T(t)P_{k-1}(t)\hat{\phi}_k(t)}. \quad (23)$$

故

$$\hat{v}_k(t) = \frac{1}{1 + \hat{\phi}_k^T(t)P_{k-1}(t)\hat{\phi}_k(t)}e_k(t). \quad (24)$$

于是

$$\begin{aligned} \tilde{\theta}_k(t) &= \tilde{\theta}_{k-1}(t) + P_k(t)\hat{\phi}_k(t)e_k(t) = \\ &= \tilde{\theta}_{k-1}(t) + P_{k-1}(t)\hat{\phi}_k(t)\hat{v}_k(t), \end{aligned} \quad (25)$$

$$\begin{aligned} P_{k-1}^{-1}(t)\tilde{\theta}_k(t) &= \\ &= P_{k-1}^{-1}(t)\tilde{\theta}_{k-1}(t) + \hat{\phi}_k(t)\hat{v}_k(t), \end{aligned} \quad (26)$$

$$\begin{aligned} \tilde{\theta}_{k-1}^T(t)P_{k-1}^{-1}(t)\tilde{\theta}_k(t) &= \\ &= \tilde{\theta}_{k-1}^T(t)P_{k-1}^{-1}(t)\tilde{\theta}_{k-1}(t) + \tilde{\theta}_{k-1}^T(t)\hat{\phi}_k(t)\hat{v}_k(t), \end{aligned} \quad (27)$$

其中, 式(27)右边第2项

$$\begin{aligned} \tilde{\theta}_{k-1}^T(t)\hat{\phi}_k(t)\hat{v}_k(t) &= \\ [\tilde{\theta}_k(t) - P_{k-1}(t)\hat{\phi}_k(t)\hat{v}_k(t)]^T\hat{\phi}_k(t)\hat{v}_k(t) &= \\ \tilde{\theta}_k^T(t)\hat{\phi}_k(t)\hat{v}_k(t) - \hat{\phi}_k^T(t)P_{k-1}(t)\hat{\phi}_k(t)\hat{v}_k^2(t), \end{aligned} \quad (28)$$

式(27)左边可写成

$$\begin{aligned} \tilde{\theta}_{k-1}^T(t)P_{k-1}^{-1}(t)\tilde{\theta}_k(t) &= \\ [\tilde{\theta}_k(t) - P_{k-1}(t)\hat{\phi}_k(t)\hat{v}_k(t)]^T P_{k-1}^{-1}(t)\tilde{\theta}_k(t) &= \\ \tilde{\theta}_k^T(t)P_{k-1}^{-1}(t)\tilde{\theta}_k(t) - \hat{v}_k(t)\hat{\phi}_k^T(t)\tilde{\theta}_k(t) &= \\ \tilde{\theta}_k^T(t)[P_{k-1}^{-1}(t) - \hat{\phi}_k(t)\hat{\phi}_k^T(t)]\tilde{\theta}_k(t) - \\ \hat{v}_k(t)\hat{\phi}_k^T(t)\tilde{\theta}_k(t) &= \\ \tilde{\theta}_k^T(t)P_{k-1}^{-1}(t)\tilde{\theta}_k(t) - \tilde{\theta}_k^T(t)\hat{\phi}_k(t)\hat{\phi}_k^T(t)\tilde{\theta}_k(t) - \\ \hat{v}_k(t)\hat{\phi}_k^T(t)\tilde{\theta}_k(t). \end{aligned} \quad (29)$$

记 $\mu_k(t) = -\hat{\phi}_k^T(t)\tilde{\theta}_k(t)$. 则上式右边第1项

$$\begin{aligned} \tilde{\theta}_k^T(t)P_{k-1}^{-1}(t)\tilde{\theta}_k(t) &= \\ \tilde{\theta}_{k-1}^T(t)P_{k-1}^{-1}(t)\tilde{\theta}_{k-1}(t) &+ \\ \tilde{\theta}_k^T(t)\hat{\phi}_k(t)\hat{\phi}_k^T(t)\tilde{\theta}_k(t) + \hat{v}_k(t)\hat{\phi}_k^T(t)\tilde{\theta}_k(t) &+ \\ \tilde{\theta}_k^T(t)\hat{\phi}_k(t)\hat{v}_k(t) - \hat{\phi}_k^T(t)P_{k-1}(t)\hat{\phi}_k(t)\hat{v}_k^2(t) &= \\ \tilde{\theta}_{k-1}^T(t)P_{k-1}(t)\tilde{\theta}_{k-1}(t) + \mu_k^2(t) - \hat{v}_k(t)\mu_k(t) - \\ \mu_k(t)\hat{v}_k(t) - \hat{\phi}_k^T(t)P_{k-1}(t)\hat{\phi}_k(t)\hat{v}_k^2(t) &\leq \\ \tilde{\theta}_{k-1}^T(t)P_{k-1}^{-1}(t)\tilde{\theta}_{k-1}(t) + \mu_k^2(t) - 2\hat{v}_k(t)\mu_k(t). \end{aligned} \quad (30)$$

构造指标函数

$$V_k = \tilde{\theta}_k^T(t)P_k^{-1}(t)\tilde{\theta}_k(t). \quad (31)$$

进而可得

$$V_k(t) \leq V_{k-1}(t) + \mu_k^2(t) - 2[\hat{v}_k(t) - v_k(t)]\mu_k(t), \quad (32)$$

其中, 上式右边第4项中

$$\begin{aligned} -\mu_k(t) &= \tilde{\theta}_k^T(t)\hat{\phi}_k(t) = \\ [\tilde{\theta}_{k-1}(t) + P_{k-1}(t)\hat{\phi}_k(t)\hat{v}_k(t)]^T\hat{\phi}_k(t) &= \\ \tilde{\theta}_{k-1}^T(t)\hat{\phi}_k(t) + \hat{\phi}_k^T(t)P_{k-1}(t)\hat{\phi}_k(t)\hat{v}_k(t) &= \\ \tilde{\theta}_{k-1}^T(t)\hat{\phi}_k(t) + \hat{\phi}_k^T(t)P_{k-1}(t)\hat{\phi}_k(t)[\hat{v}_k(t) - \\ v_k(t)] + \hat{\phi}_k^T(t)P_{k-1}(t)\hat{\phi}_k(t)v_k(t). \end{aligned} \quad (33)$$

故指标函数

$$\begin{aligned} V_k(t) &\leq \\ V_{k-1}(t) + \mu_k^2(t) - 2[\hat{v}_k(t) - v_k(t)]\mu_k(t) &+ \\ 2[\tilde{\theta}_{k-1}^T(t)\hat{\phi}_k(t) + \hat{\phi}_k^T(t)P_{k-1}(t)\hat{\phi}_k(t)(\hat{v}_k(t) - \\ v_k(t))]v_k(t) + 2\hat{\phi}_k^T(t)P_{k-1}(t)\hat{\phi}_k(t)v_k^2(t). \end{aligned} \quad (34)$$

上式右边第2, 3项

$$\begin{aligned} \mu_k^2(t) - 2[\hat{v}_k(t) - v_k(t)]\mu_k(t) &= \\ -2\mu_k(t)[\hat{v}_k(t) - v_k(t) - \frac{1+\rho}{2}\mu_k(t)] - \rho\mu_k^2(t). \end{aligned} \quad (35)$$

此外, 容易得到

$$\begin{aligned} D(q^{-1})[\hat{v}_k(t) - v_k(t)] &= \\ D(q^{-1})\hat{v}_k(t) - D(q^{-1})v_k(t) &= \\ \hat{v}_k(t) - y_k(t) + \hat{\phi}_k^T(t)\theta(t) &= \\ -\hat{\phi}_k^T(t)(\hat{\theta}(t) - \theta(t)) &= \mu_k(t). \end{aligned} \quad (36)$$

记 $\eta_k(t) = \hat{v}_k(t) - v_k(t) - \frac{1+\rho}{2}\mu_k(t)$. 于是

$$\eta_k(t) = [\frac{1}{D(q^{-1})} - \frac{1+\rho}{2}]\mu_k(t). \quad (37)$$

由假设4, $\frac{1}{D(q^{-1})} - \frac{1}{2}$ 严格正实, 总可以找到一个小的

的常数 ρ , 使得 $\frac{1}{D(q^{-1})} - \frac{1+\rho}{2}$ 也满足严格正实条件, 于是

$$V_k(t) \leq V_{k-1}(t) - 2\mu_k(t)\eta_k(t) - \rho\mu_k^2(t) + 2\{\tilde{\theta}_{k-1}^T(t)\hat{\phi}_k(t) + \hat{\phi}_k^T(t)P_{k-1}(t)\hat{\phi}_k(t) [\hat{v}_k(t) - v_k(t)]\}v_k(t) + 2\hat{\phi}_k^T(t)P_{k-1}(t)\hat{\phi}_k(t)v_k^2(t). \quad (38)$$

上式两边在时间区间 $[0, t]$ 上求和.

$$\sum_{i=0}^t V_k(i) \leq \sum_{i=0}^t V_{k-1}(i) - 2 \sum_{i=0}^t \mu_k(i)\eta_k(i) - \rho \sum_{i=0}^t \mu_k^2(i) + 2 \sum_{i=0}^t \{\tilde{\theta}_{k-1}^T(i)\hat{\phi}_k(i) + \hat{\phi}_k^T(i)P_{k-1}(i)\hat{\phi}_k(i)[\hat{v}_k(i) - v_k(i)]\}v_k(i) + 2 \sum_{i=0}^t \hat{\phi}_k^T(i)P_{k-1}(i)\hat{\phi}_k(i)v_k^2(i). \quad (39)$$

记 $L_k(t) = \sum_{j=k_0}^k \{2 \sum_{i=0}^t \mu_j(i)\eta_j(i)\}$, 根据正实性引理

$$L_k(t) = \sum_{j=k_0}^k \{2 \sum_{i=0}^t \mu_j(i)\eta_j(i)\} \geq 0,$$

容易得到如下递推形式:

$$L_k(t) = \sum_{j=k_0}^{k-1} \{2 \sum_{i=0}^t \mu_j(i)\eta_j(i)\} + 2[\sum_{i=0}^t \mu_k(i)\eta_k(i)] = L_{k-1}(t) + 2[\sum_{i=0}^t \mu_k(i)\eta_k(i)]. \quad (40)$$

记 $W_k(t) = L_k(t) + \sum_{i=0}^t V_k(i)$. 于是

$$W_k(t) \leq L_k(t) + \sum_{i=0}^t V_{k-1}(i) - 2 \sum_{i=0}^t \mu_k(i)\eta_k(i) - \rho \sum_{i=0}^t \mu_k^2(i) + 2 \sum_{i=0}^t [\tilde{\theta}_{k-1}^T(i)\hat{\phi}_k(i) + \hat{\phi}_k^T(i)P_{k-1}(i)\hat{\phi}_k(i)[\hat{v}_k(i) - v_k(i)]]v_k(i) + 2 \sum_{i=0}^t \hat{\phi}_k^T(i)P_{k-1}(i)\hat{\phi}_k(i)v_k^2(i) = L_{k-1}(t) + 2 \sum_{i=0}^t \mu_k(i)\eta_k(i) + \sum_{i=0}^t V_{k-1}(i) - 2 \sum_{i=0}^t \mu_k(i)\eta_k(i) - \rho \sum_{i=0}^t \mu_k^2(i) + 2 \sum_{i=0}^t [\tilde{\theta}_{k-1}^T(i)\hat{\phi}_k(i) + \hat{\phi}_k^T(i)P_{k-1}(i)\hat{\phi}_k(i)[\hat{v}_k(i) - v_k(i)]]v_k(i) + 2 \sum_{i=0}^t \hat{\phi}_k^T(i)P_{k-1}(i)\hat{\phi}_k(i)v_k^2(i) =$$

$$W_{k-1}(t) - \rho \sum_{i=0}^t \mu_k^2(i) + 2 \sum_{i=0}^t [\tilde{\theta}_{k-1}^T(i)\hat{\phi}_k(i) + \hat{\phi}_k^T(i)P_{k-1}(i)\hat{\phi}_k(i)[\hat{v}_k(i) - v_k(i)]]v_k(i) + 2 \sum_{i=0}^t \hat{\phi}_k^T(i)P_{k-1}(i)\hat{\phi}_k(i)v_k^2(i). \quad (41)$$

上式两边取数学期望

$$E\{W_k(t)|F_{k-1}(t)\} \leq W_{k-1}(t) - \rho E\{\sum_{i=0}^t \mu_k^2(i)|F_{k-1}(t)\} + 2 \sum_{i=0}^t \hat{\phi}_k^T(i)P_{k-1}(i)\hat{\phi}_k(i)\sigma_v^2. \quad (42)$$

记

$$B_k(t) = \rho \sum_{j=k_0}^k \sum_{i=0}^t \mu_j^2(i) = \rho \sum_{j=k_0}^{k-1} \sum_{i=0}^t \mu_j^2(i) + \rho \sum_{i=0}^t \mu_k^2(i) = B_{k-1}(t) + \rho \sum_{i=0}^t \mu_k^2(i). \quad (43)$$

令

$$X_k(t) = \frac{W_k(t) + B_k(t)}{\sum_{i=0}^t A_k(i)}. \quad (44)$$

对上式两边取数学期望

$$E\{X_k(t)|F_{k-1}(t)\} = E\{\frac{W_k(t) + B_k(t)}{\sum_{i=0}^t A_k(i)}|F_{k-1}(t)\} = \frac{1}{\sum_{i=0}^t A_k(i)} E\{W_k(t) + B_k(t)|F_{k-1}(t)\} \leq \frac{1}{\sum_{i=0}^t A_k(i)} \{W_{k-1}(t) - \rho E\{\sum_{i=0}^t \mu_k^2(i)|F_{k-1}(t)\} + \sum_{i=0}^t A_k(i)\} + 2 \sum_{i=0}^t \hat{\phi}_k^T(i)P_{k-1}(i)\hat{\phi}_k(i)\sigma_v^2 + B_{k-1}(t) + \rho E\{\sum_{i=0}^t \mu_k^2(i)|F_{k-1}(t)\} = \frac{W_{k-1}(t) + B_{k-1}(t)}{\sum_{i=0}^t A_{k-1}(i)} - \frac{W_{k-1}(t) + B_{k-1}(t)}{\sum_{i=0}^t A_{k-1}(i)} + \frac{W_{k-1}(t) + B_{k-1}(t)}{\sum_{i=0}^t A_k(i)} + \frac{2 \sum_{i=0}^t \hat{\phi}_k^T(i)P_{k-1}(i)\hat{\phi}_k(i)\sigma_v^2}{\sum_{i=0}^t A_k(i)}, \quad (45)$$

即

$$E\{X_k(t)|F_{k-1}(t)\} \leq X_{k-1}(t) - [W_{k-1}(t) + B_{k-1}(t)] \left(\frac{1}{\sum_{i=0}^t A_{k-1}(i)} - \frac{1}{\sum_{i=0}^t A_k(i)} + 2 \frac{\sum_{i=0}^t \hat{\phi}_k^T(i) P_{k-1}(i) \hat{\phi}_k(i)}{\sum_{i=0}^t A_k(i)} \right) \sigma_v^2. \quad (46)$$

由鞅收敛定理, $X_k(t)$ 几乎处处收敛于一有穷随机变量. 即有

$$\lim_{k \rightarrow \infty} \frac{W_k(t)}{\sum_{i=0}^t A_k(i)} < \infty.$$

根据 $W_k(t)$ 定义式, 可得

$$\frac{\sum_{i=0}^t V_k(i)}{\sum_{i=0}^t A_k(i)} < \infty,$$

即

$$V_k(t) = O\left(\sum_{i=0}^t A_k(i)\right).$$

至此容易证得定理. 证毕.

更进一步分析, 当重复持续激励条件满足, 根据 $\ln |P_k^{-1}(t)| = O(\ln \lambda_{\max}[P_k^{-1}(t)])$, 可得 $\|\tilde{\theta}_k(t)\|$ 收敛速率的估计

$$\|\tilde{\theta}_k(t)\| = O\left(\sqrt{\frac{\sum_{i=0}^t (\ln k)^{\delta(i)}}{k}}\right), \text{ a.s.} \quad (47)$$

4.2 周期学习最小二乘算法的收敛性分析 (Convergence analysis of periodically learning least squares algorithm)

下面针对周期学习最小二乘算法进行收敛性分析. 对于固定时刻 $t \in \{0, 1, \dots, N-1\}$, 记 $F(t+kN)$ 为系统经由 k 个周期的输入输出数据构成的 σ 代数.

假设 5

$$E[v(t+kN)|F(t+(k-1)N)] = 0, \text{ a.s.}$$

假设 6 存在关于 t 一致有界的 $\sigma_v(t)$ 使得

$$E[v^2(t+kN)|F(t+(k-1)N)] \leq \sigma_v^2(t), \text{ a.s.} \quad (48)$$

假设 7 存在 k_0 , 当 $k \geq k_0$ 时, 下述持续激励条件成立:

$$\alpha(t)I \leq \frac{1}{k} \sum_{i=1}^k \phi(t+iN)\phi^T(t+iN) \leq \beta(t)I, \text{ a.s.}, \quad (49)$$

式中 $\alpha(t), \beta(t) > 0$.

假设 8 $\frac{1}{D(q^{-1})} - \frac{1}{2}$ 是严格正实的.

定理 2 当假设 5-8 成立时, 由式(11)-(13)给出的参数估值使得

$$\|\tilde{\theta}(t+kN)\| = O\left(\sqrt{\frac{\sum_{i=0}^t (\ln |P^{-1}(i+kN)|)^{\delta(i)}}{\lambda_{\min}[P^{-1}(i+kN)]}}\right), \text{ a.s.}, \quad (50)$$

式中: $\delta(i) > 1, \tilde{\theta}(t+kN) = \hat{\theta}(t+kN) - \theta(t+kN), \lambda_{\min}[P^{-1}(t+kN)]$ 为 $P^{-1}(t+kN)$ 的最小特征值.

应该指出的是, 上述采用的分析手段和所获得的理论结果与递推算法类似. 文中笔者借鉴现有的递推算法分析方法, 给出学习算法能够实现完全估计的理论证明. 就有限区间上重复时变系统而言, 学习辨识方法的提出是必要的.

5 数值结果(Numerical results)

这一节完成数值算例, 以说明学习辨识算法可用于动态系统中时变参数的估计. 首先, 考虑下述有限区间时变系统:

$$\begin{aligned} y_k(t+1) + a_1(t)y_k(t) + a_2(t)y_k(t-1) = \\ b_1(t)u_k(t) + b_2(t)u_k(t-1) + v_k(t+1) + \\ d_1v_k(t) + d_2v_k(t-1), \end{aligned}$$

其中:

$$\begin{aligned} a_1(t) &= -1.5 + 0.2 \sin(\pi t/40), \\ a_2(t) &= 0.7 + 4.5 \sin(t/10)(1/(t^{\frac{3}{4}} + 3)), \\ b_1(t) &= 0.6 + 0.2\sqrt{t+1}(\sin(\pi t/20) + 30), \\ b_2(t) &= 0.5 + 0.1 \sin(2\pi t/N), \\ d_1 &= 0.45, d_2 = -0.35. \end{aligned}$$

仿真中, $N = 500$. 对于 $t = 0, 1, 2, \dots, N, k = 0, 1, 2, \dots, u_k(t)$ 取值为 $[-0.5, 0.5]$ 上的均匀分布的随机变量, $w_k(t) = 0.01 \text{randn}$. 这里, randn 为服从 $(0, 1)$ 正态分布的随机变量的产生函数. 在最小二乘学习算法(5)及(7)中, 设置初值 $P_{-1}(t) = 10^6 I_{6 \times 6}, \theta_{-1}(t) = (0, 0, 0, 0, 0, 0)^T$. 为了考查收敛性能, 定义

$$\begin{aligned} J_k &= \max_{0 \leq t \leq N} |e_k(t)|, \\ e_k(t) &= y_k(t) - \phi_k^T(t)\hat{\theta}_{k-1}(t), \\ J_{a_i,k} &= \max_{0 \leq t \leq N} |a_i(t) - \hat{a}_{i,k}(t)|, \\ J_{b_i,k} &= \max_{0 \leq t \leq N} |b_i(t) - \hat{b}_{i,k}(t)|, \\ J_{d_i,k} &= \max_{0 \leq t \leq N} |d_i(t) - \hat{d}_{i,k}(t)|, \quad i = 1, 2. \end{aligned}$$

重复进行 1000 次, 仿真结果见图 3-7. 其中: 预报误差见图 3, 系统参数估计结果见图 4, 系统参数估计误

差见图5, 有色噪声项参数估计结果见图6, 有色噪声项参数估计误差见图7. 由图3可以看出, 预报误差随着迭代次数的增加而迅速减小, 并最终收敛于一个较小的界内. 在图4中, 最后一次迭代所获得的系统参数估值与参数真值基本吻合, 在图5中, 随着迭代次数的增加, 系统参数估计误差很快收敛于零. 在图6中, 最后一次迭代所取得的有色噪声项的参数估值与有色噪声项真值基本吻合. 在图7中, 随着迭代次数的增加, 有色噪声项参数的估计误差迅速收敛于零. 仿真结果表明了参数估值的一致收敛性, 可以实现对待估计参数的完全估计.

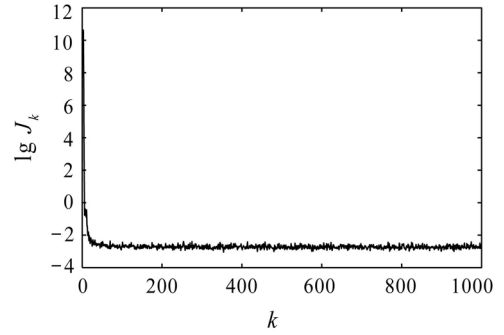


图 3 预报误差

Fig. 3 Predictive error with respect to repetition

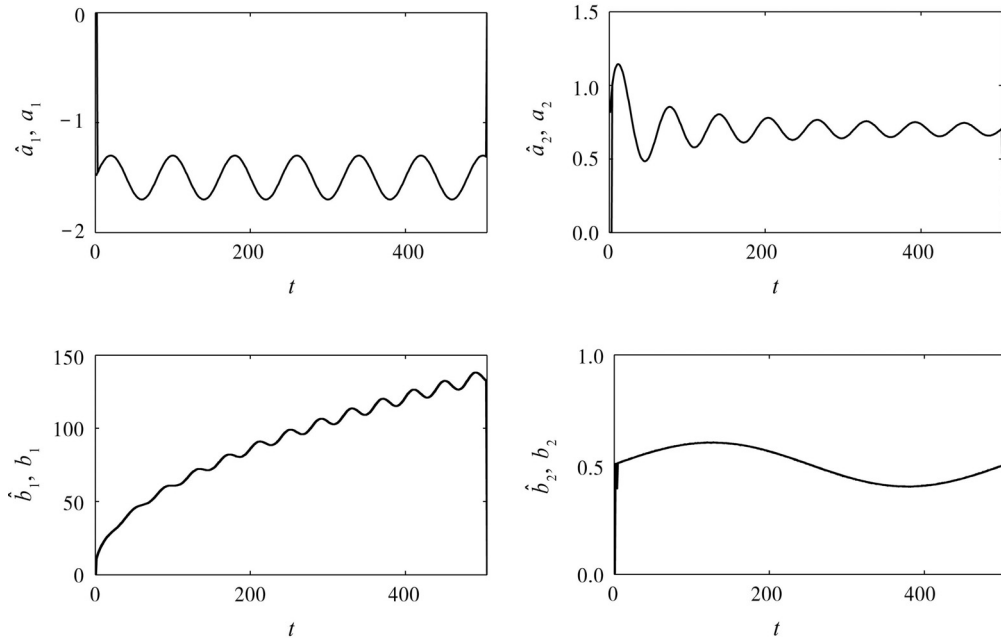


图 4 系统参数估计结果

Fig. 4 Parameters of system estimation after learning

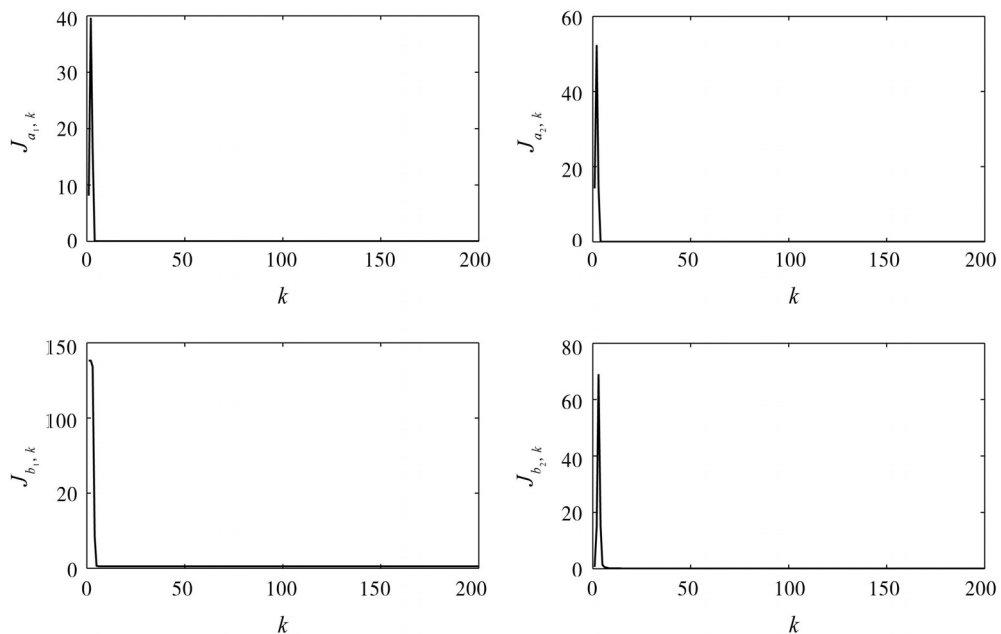


图 5 系统参数估计误差

Fig. 5 Parameters of system estimation error after learning

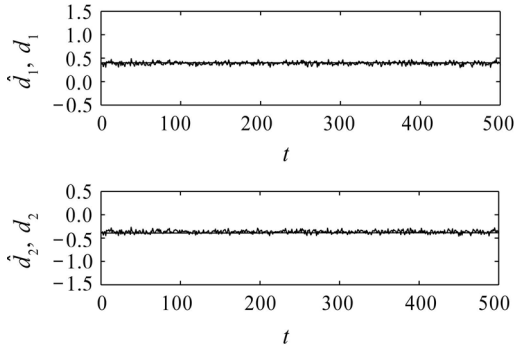


图6 有色噪声项参数估计结果

Fig. 6 Parameters estimation after learning

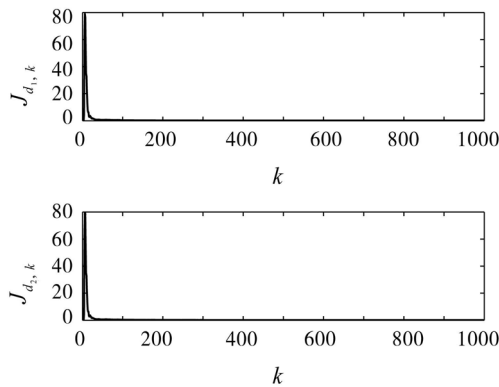


图7 有色噪声项参数估计误差

Fig. 7 Parameters estimation error after learning

下面考虑周期时变系统:

$$y(t+kN+1) + a_1(t+kN)y(t+kN) + a_2(t+kN)y(t+kN-1) = b_1(t+kN)u(t+kN) + b_2(t+kN)u(t+kN-1) + v(t+kN+1) + d_1v(t+kN) + d_2v(t+kN-1),$$

其中:

$$a_1(t+kN) = 1 + 0.3 \sin(2\pi(t+kN)/N_1),$$

$$a_2(t+kN) = \begin{cases} 0, & t \in A_1, \\ 1, & t \in A_2, \end{cases}$$

其中: $A_1 = [kN + n_2N_2, kN + (n_2 + \frac{1}{2})N_2]$, $A_2 = (kN + (n_2 + \frac{1}{2})N_2, kN + (n_2 + 1)N_2)$,

$$b_1(t+kN) = \begin{cases} 0.5t, & t \in B_1, \\ 7.5 - 0.25t, & t \in B_2, \end{cases}$$

其中: $B_1 = [kN + n_3N_3, kN + (n_3 + \frac{1}{3})N_3]$, $B_2 = (kN + (n_3 + \frac{1}{3})N_3, kN + (n_3 + 1)N_3)$,

$$b_2(t+kN) = 0.5 + 0.3 \sin(2\pi(t+kN)/N_4),$$

$$d_1 = 0.5, d_2 = -0.3.$$

在仿真中, 设定4个周期时变参数的周期分别为: $N_1 = 60, N_2 = 10, N_3 = 30, N_4 = 12$, 选取4个时变周期参数的最小公倍数作为系统周期 N , 本例 $N = 60, t = 0, 1, \dots, N - 1$. 重复次数 $k = 0, 1, 2, \dots, n_j = (0, 1, \dots, \frac{N}{N_j} - 1)(j = 1, 2, 3, 4)$, $u(t+kN)$ 取值为 $[-0.5, 0.5]$ 上的均匀分布的随机变量, $v(t+kN) = 0.01\text{randn}$. 这里, randn 为服从 $(0, 1)$ 正态分布的随机变量的产生函数. 在最小二乘学习算法(11)及(13)中, 设置初值 $P(t-N) = 10^6 I_{6 \times 6}, \theta(t-N) = (0, 0, 0, 0, 0, 0)^T$. 为了考查收敛性能, 定义 $J_k = \max_{0 \leq t \leq N-1} |e(t+kN)|$, 其中 $e(t+kN) = y(t+kN) - \phi^T(t+kN)\hat{\theta}(t+(k-1)N)$. 定义

$$J_{a_i,k} = \max_{0 \leq t \leq N-1} |a_i(t+kN) - \hat{a}_i(t+kN)|,$$

$$J_{b_i,k} = \max_{0 \leq t \leq N-1} |b_i(t+kN) - \hat{b}_i(t+kN)|,$$

$$J_{d_i,k} = \max_{0 \leq t \leq N-1} |d_i(t+kN) - \hat{d}_i(t+kN)|, i = 1, 2.$$

重复进行1000次, 仿真结果见图8-12. 其中: 预报误差见图8, 系统参数估计结果见图9, 系统参数估计误差见图10, 有色噪声项参数估计结果见图11, 有色噪声项参数估计误差见图12. 由图8可以看出, 预报误差随着周期数的增加而减小, 并最终稳定在一个较小的界内. 在图9中, 可以看出在最后的几个周期中, 系统参数估值与系统参数真值基本吻合, 在图10中, 可以看出随着周期数的增加, 系统的参数估值误差迅速收敛于零. 在图11中, 在最后一个周期中, 有色噪声项参数估值与有色噪声项真值基本吻合. 在图12中, 随着周期数的增加, 有色噪声项的参数估值迅速收敛于零. 仿真结果表明了参数估值的一致收敛性, 周期学习算法可以实现对周期时变参数的完全估计.

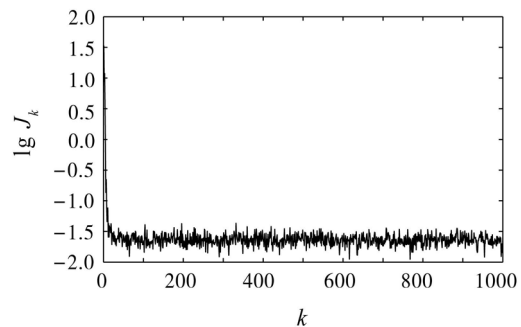


图8 预报误差

Fig. 8 The error with respect to repetition

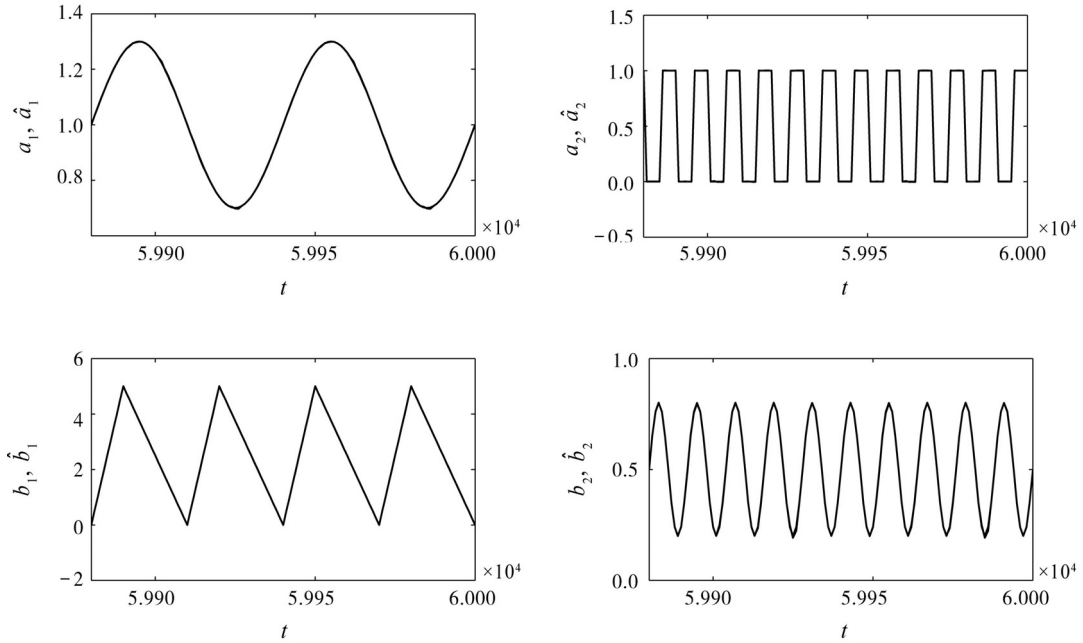


图 9 系统参数估计结果

Fig. 9 Parameters of system estimation after learning

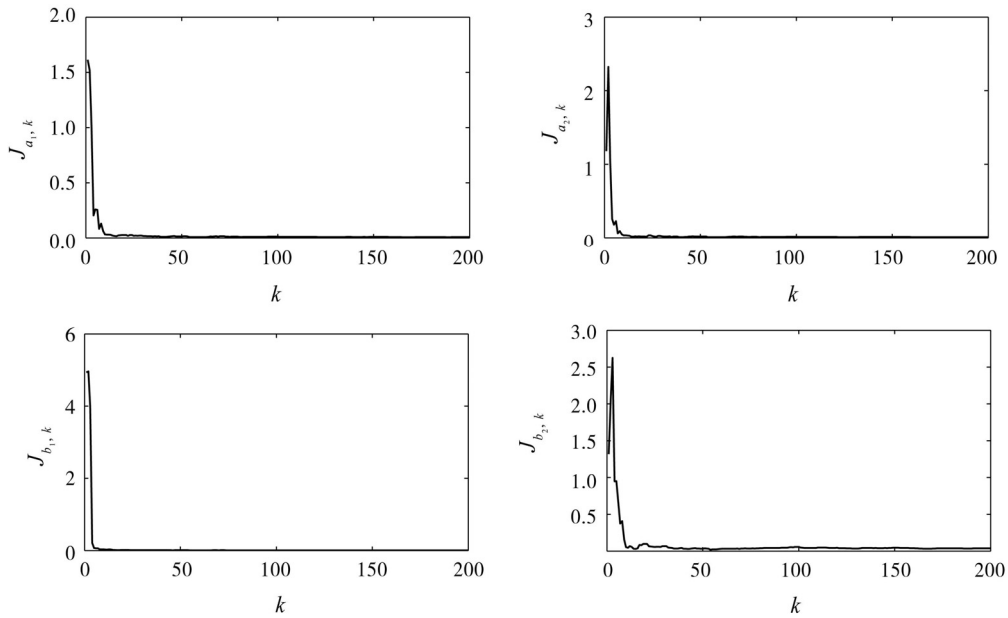


图 10 系统参数估计误差

Fig. 10 Parameters of system estimation error after learning

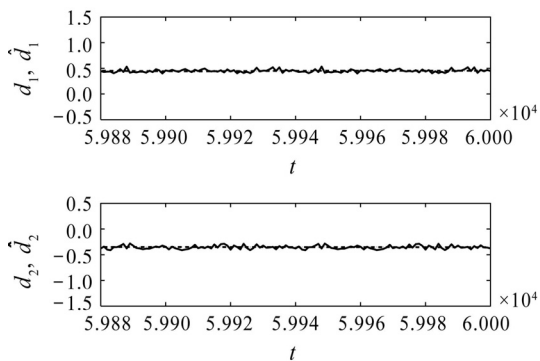


图 11 有色噪声项参数估计结果
Fig. 11 Parameters estimation after learning

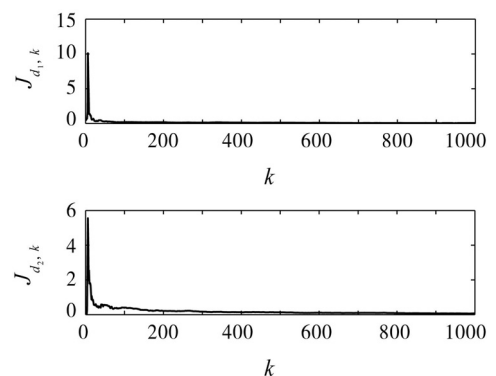


图 12 有色噪声项参数估计误差
Fig. 12 Parameters estimation error after learning

6 结论(Conclusions)

本文提出的学习辨识算法可用于解决一类 ARMAX系统的时变参数估计问题. 文中推导了重复作业下的迭代学习最小二乘学习算法和周期学习最小二乘算法. 在重复持续激励和正实条件下, 证明了迭代学习最小二乘学习算法和周期学习最小二乘算法的重复一致性. 对于有限区间时变系统而言, 这种重复一致性保证参数估值在整个区间上收敛于真值.

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