

内模强化学习型模型预测控制及其在人工胰腺上的应用

杨跃男, 王友清

(北京化工大学 信息科学与技术学院, 北京 100029)

摘要: 在学习型模型预测控制的框架里, 迭代学习控制器被用来更新模型预测控制器的设定点. 在已经发表的研究成果里, 学习型模型预测控制用到的是比例型的学习率, 这种学习率的学习能力有限, 而且怎样设计学习增益仍然是一个开放性问题. 在本文中, 基于内模控制理论提出的PID型的迭代学习控制器被用来更新模型预测控制器的设定点. 为了方便起见, 本文提出的结合算法可称为内模强化学习型模型预测控制. 本文提出的算法应用在1型糖尿病病人的人工胰腺闭环控制上. 仿真结果显示, 本算法对比于比例学习型模型预测控制可以达到更好的收敛性能, 而且对非重复干扰有很好的鲁棒性.

关键词: 迭代学习控制; 模型预测控制; 间接型迭代学习控制; 内模控制; 人工胰腺; 1型糖尿病
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Internal model control-enhanced learning-type model predictive control: application to artificial pancreas

YANG Yue-nan, WANG You-qing

(College of Information Science and Technology, Beijing University of Chemical Technology, Beijing 100029, China)

Abstract: In the framework of a learning-type model predictive control (L-MPC), an iterative learning control (ILC) is used to update the setpoint for model predictive control (MPC). In the reported studies, the L-MPC usually has a P-type ILC, which has limited learning capability and also how to design its learning gain remains an open problem. A PID-type ILC was proposed to design the learning-type setpoint for MPC based on internal model control (IMC) theory. For convenience, the proposed combination is named IMC-enhanced L-MPC. The proposed method was applied to the closed-loop control of an artificial pancreatic β -cell for type 1 diabetes mellitus (T1DM). The simulation results show that the proposed algorithm can produce superior convergence performance compared with the P-type L-MPC, and also it has excellent robustness to non-repetitive disturbances.

Key words: iterative learning control (ILC); model predictive control; indirect ILC; internal model control; artificial pancreas; type 1 diabetes mellitus

1 Introduction

Since iterative learning control (ILC) was proposed in 1978^[1], a number of studies have been done in this field. It has primarily been used in the situation where controlled system owns some repetitive natures, such as robot arm for manufacturing^[2], hard disk drive^[3], freeway traffic density control^[4-5] and chemical batch process^[6]. All of these systems execute the periodic tasks and/or suffer the repeating disturbances. In short, iterative learning controller utilizes historical input and output data to design current input signal. Generally speaking, there are P, D, PD, PID^[7-8] and anticipation-type ILC^[9] and so on.

In most cases, iterative learning control is usually applied to design the control signal directly for the controlled system. However, there're some studies

that iterative learning control was implemented indirectly^[10]. In [11-12], for example, ILC was used to update the setpoint for PID controller.

In [13], a model predictive control (MPC) worked as a local controller in the inner loop and an ILC was used to update the setpoint for the MPC in the outer loop, and the proposed method was termed as learning-type model predictive control (L-MPC). ILC in that study is P-type, which might be too simple and was chosen without comparison. In addition, there was no systematic conclusion about how to design the learning gain.

In this paper, the learning-type setpoint was designed based on internal model control (IMC) theory. IMC is a sort of advanced control scheme, which has a good tracking ability and is robust to dis-

turbance. For convenience, the designed IMC controller was often simplified into a proportion-integral-derivation (PID) control, which is named IMC-PID for short^[14–15]. IMC-PID gets all the advantages of IMC, what's more, it keeps PID's simple structure. Generally, there are three tuning parameters in a standard PID controller; however, IMC-PID has only one, which makes the design procedure much more convenient. Through tuning this parameter, we can get excellent tradeoff between tracking performance and robustness with respect to uncertainties and disturbances. In this article, we will see in Section 4, IMC-PID method can be used to design the PID-type learning law of L-MPC. Due to the simplicity and improved performance of IMC-based tuning rule, it's easy to get ideal learning gains.

The algorithm proposed in this paper is applied to the closed-loop control of an artificial pancreatic β -cell for type 1 diabetes mellitus (T1DM). T1DM is a kind of incretion disease of human body, which is induced by the deficiency of insulin secretion. Long-term hyperglycemia (high blood glucose concentration) induced by the deficiency of insulin injection will result in many chronic disease, such as hypertension, heart disease, retinopathy and so on. To reduce the glucose level, diabetic patients have to depend on exogenous insulin infusion. It's not easy for patient to decide how much insulin to be injected. If excessive insulin was delivered, hypoglycemia (low blood glucose concentration) event will occur, which is very dangerous to human health. Therefore, it's essential to control the insulin infusion for normal glucose.

In Section 5, the feasibility and effectiveness of the proposed control scheme were validated using some simulation tests on artificial pancreas. The proposed scheme showed better tracking performance than the P-type L-MPC. What's more, tuning one parameter will produce different control performances. Through tuning the parameter, the ideal tracking performance can be achieved conveniently.

2 Virtual subject and auto-regressive exogenous (ARX) model

This section is a brief introduction of an in silico model for T1DM. The in silico patient comprises three subsystems: the glucose subsystem, the insulin subsystem, and the meal subsystem. Many parameters need to be set in this virtual patient, which is the same as in [16–17]. However, the in silico patient is too complicated and actually it is impossible to attain the accurate model for the subject. Therefore, it's indispensable to identify a model. In clinical practice, there're three variables available for model identifi-

cation, namely, insulin delivery rate, glucose concentration and carbohydrate (CHO) count, however, the CHO count is estimated by a human, so we cannot get an accurate value. Based on the above mentioned, an auto-regressive exogenous (ARX) model is used to approximate the relationship between the insulin and the glucose,

$$A(z^{-1})\bar{y}(\bar{t}) = B(z^{-1})\bar{u}(\bar{t} - nd) + \bar{w}(\bar{t}), \quad (1)$$

where $\bar{u}(\bar{t})$, the input, represents insulin delivery rate, and $\bar{y}(\bar{t})$, the output, represents glucose concentration, and $\bar{w}(\bar{t})$ represents uncertainties or disturbances, z^{-1} is the backward shift operator, nd is the time delay, \bar{t} is the time step index, and the sample time was set at 5 min in this study. Through indentifying the model, we can obtain the relationship between $\bar{u}(\bar{t})$ and $\bar{y}(\bar{t})$.

3 Problem statement

3.1 Model predictive control

In this paper, MPC was used as a local controller in the inner loop because of its ability to deal with input and state constraints and robustness to running uncertainties over conventional control strategies^[18–19]. MPC consists of three key parts: prediction model, cost function, and receding horizon optimization. Assuming the setpoint for MPC is $\bar{y}_r(\bar{t})$, we have the cost function

$$\Omega \triangleq \sum_{j=1}^N \alpha_1 (\bar{y}_r(\bar{t} + j) - \hat{y}(\bar{t} + j | \bar{t}))^2 + \sum_{i=0}^M [\alpha_2 (\bar{u}(\bar{t} + i | \bar{t}))^2 + \alpha_3 (\Delta \bar{u}(\bar{t} + i | \bar{t}))^2], \quad (2)$$

where integers N is the predictive horizon and M is the control horizon ($N > M$), $\hat{y}(\bar{t} + j | \bar{t})$ denotes the prediction of $\bar{y}(\bar{t} + j)$ based on the known information at time \bar{t} and meantime $\bar{u}(\bar{t} + i | \bar{t})$ denotes the possible control sequence in the control horizon; $\Delta \bar{u}(\bar{t} + i | \bar{t})$ denotes variations of the control signal; weights α_1 , α_2 , and α_3 are respectively used to adjust the relative importance of tracking error suppression, input penalty and input variation penalty. And how to choose $\{N, M, \alpha_1, \alpha_2, \alpha_3\}$ can be found in [20]. The following optimization problem is solved to get the control sequence:

$$\bar{u}(\bar{t} + i | \bar{t})|_{i=0}^M = \arg \min_{\bar{u}(\bar{t} + i | \bar{t})} \Omega. \quad (3)$$

3.2 L-MPC

When designing controller, we take measures to make the output to be close to the target Y_r . This target Y_r is generally the same as the setpoint, which is the command for the controller. However, this produces a problem that we can't make sure whether the setpoint is the optimal choice. Therefore, L-MPC

was proposed, which was applied to glucose control in [21]. The authors utilized ILC to update the setpoint in respect that glucose-insulin dynamic process is disturbed by meals periodically every day. And it was proved that L-MPC produced better performance than MPC and showed good convergence property under repetitive disturbances. In previous work, the P-type learning rate was proposed^[21]

$$\bar{y}_r(\bar{t}) = \bar{y}_r(\bar{t} - T) + K\bar{e}(\bar{t} - T), \quad 0 < K < 1, \quad (4)$$

where T is the period of the considered system or external disturbances, T was chosen as 24 h in paper [21]; K is the designed learning gain. For convenience, the following operator is introduced:

$$\xi(t, k) \hat{=} \bar{\xi}(\bar{t}), \quad \xi = u, y, e, y_r, \quad (5)$$

where $k = \lfloor t/T \rfloor$, $t = \bar{t} - kT$. $\lfloor * \rfloor$ returns the nearest integer for $*$ towards minus infinity. Operation (5) divides the continuous sequence into several batches. By using this operator, Eq.(4) can be rewritten as

$$y_r(t, k) = y_r(t, k - 1) + Ke(t, k - 1), \quad (6)$$

where k denotes the k -th operation of the system; t is the time index during k -th iteration, and $t \in [0, T]$, $e(t, k)$ is the tracking error, and if the objective output is represented by $Y_r(t)$ and the real output is denoted by $y(t, k)$, then the error is $e(t, k) \hat{=} Y_r(t) - y(t, k)$, K is the learning gain, and was set at 0.5 for the compromise between convergence rate and robustness. For convenience, all formulas were expressed in 2-D form in the subsequent part of the paper.

4 IMC-enhanced learning-type setpoint

The learning law of (6) was chosen without comparisons, and it seemed that the P-type learning law was too simple. In this section, the relationship between the setpoint $y_r(t, k)$ and the output $y(t, k)$ was indentified. Using this model, an IMC-based learning-type MPC scheme was proposed.

4.1 Setpoint-to-output model

As seen from Fig.1, the controlled system consists of a T1DM and his/her local MPC controller. The setpoint $y_r(t, k)$ is considered the input, and the glucose output $y(t, k)$ remains as the output. When the setpoint has a step change, therefore it's easy to identify the relationship between $y(t, k)$ and the setpoint $y_r(t, k)$, as below:

$$y(s, k) = G_m(s)y_r(s, k). \quad (7)$$

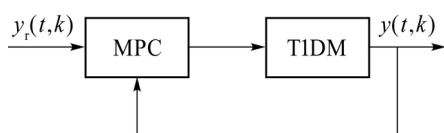


Fig. 1 Relationship between $y_r(t, k)$ and $y(t, k)$

Discretize formula (7), it can be obtained the following discrete time, linear time invariant system

$$y(t, k) = G_m(z^{-1})y_r(t, k). \quad (8)$$

4.2 IMC-based learning-type setpoint

Consider a learning-type setpoint

$$y_r(t, k) = y_r(t, k - 1) + L(t, k - 1), \quad (9)$$

where $L(t, k - 1)$ is the updating law.

Assuming

$$\Delta y_r(t, k) = y_r(t, k) - y_r(t, k - 1), \quad (10)$$

similarly

$$\Delta y(t, k) = y(t, k) - y(t, k - 1), \quad (11)$$

so, we can get

$$\Delta y_r(t, k) = L(t, k - 1). \quad (12)$$

Let the updating law be a PID type of the tracking error

$$\Delta y_r(t, k) = k_p[e(t, k - 1) + k_i \sum_{i=1}^t e(i, k - 1)\Delta T + k_d \frac{e(t, k - 1) - e(t - 1, k - 1)}{\Delta T}], \quad (13)$$

where ΔT is sample time, k_p , k_i , k_d are proportional gain, integral gain, and derivative gain, respectively.

Motivated by this structure, we used IMC scheme to get the proper value for k_p , k_i , k_d . The next part showed the relationship between $\Delta y_r(t, k)$ and error term $e(t, k - 1)$ in the controlled system.

The control objective is to make the output $y(t, k)$ to be close to the given target $Y_r(t)$ as closely as possible. The issue is equivalent to taking measure to make

$$Y_r(t) - y(t, k) = 0, \quad (14)$$

because of Eq.(11), we have

$$\begin{aligned} Y_r(t) - y(t, k) &= \\ Y_r(t) - y(t, k - 1) - \Delta y(t, k) &= 0. \end{aligned} \quad (15)$$

Assuming $Y_r^*(t)$ is the target of $\Delta y(t, k)$, so we have

$$Y_r^*(t) = Y_r(t) - y(t, k - 1). \quad (16)$$

Applying Eqs.(10) and (11) to Eq.(8), and according to the assumption that $G_m(z^{-1})$ is discrete time, linear time invariant system, we can get a result which is very similar to Eq.(8),

$$\Delta y(t, k) = G_m(z^{-1})\Delta y_r(t, k), \quad (17)$$

So it's easily to get

$$Y_r^*(t) - \Delta y(t, k) = Y_r^*(t) - G_m(z^{-1})\Delta y_r(t, k). \quad (18)$$

Define $e^*(t, k)$ is the error between $Y_r^*(t)$ and $\Delta y(t, k)$, which is

$$\begin{aligned} e^*(t, k) &= Y_r^*(t) - \Delta y(t, k) = \\ Y_r(t) - y(t, k - 1) - [y(t, k) - y(t, k - 1)] &= \\ Y_r(t) - y(t, k) &= e(t, k). \end{aligned} \quad (19)$$

It's clearly seen from Eq.(19) that the error between $Y_r^*(t)$ and $\Delta y(t, k)$ is the same as the error between $Y_r(t)$ and $y(t, k)$.

From Eqs.(17) and (19),

$$\begin{aligned} Y_r^*(t) - G_m(z^{-1})\Delta y_r(t, k) = \\ Y_r(t) - y(t, k - 1) - G_m(z^{-1})\Delta y_r(t, k) = \\ e(t, k - 1) - G_m(z^{-1})(y_r(t, k) - y_r(t, k - 1)). \end{aligned} \quad (20)$$

To make $e^*(t, k) = 0$, we need

$$e(t, k - 1) - G_m(z^{-1})(y_r(t, k) - y_r(t, k - 1)) = 0. \quad (21)$$

So it's straightforward to find

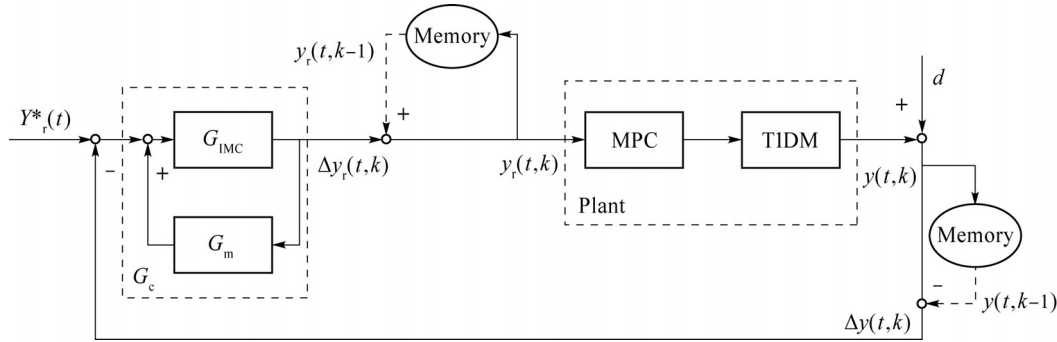


Fig. 2 Structure of IMC-enhanced L-MPC equivalent to classical control structure

1) The plant model G_m is decomposed into an all pass portion G_+ and a minimum phase portion G_- .

2) For a better robustness and making the internal model controller to be rational, a filter is needed. Letting the filter as follows:

$$f(s) = \frac{1}{\lambda s + 1}.$$

According to the strategy of designing internal model controller, we obtain the internal model controller

$$G_{IMC} = G_-^{-1} \cdot f(s).$$

Using IMC-PID design method^[22], we can have feed-back controller $G_c(s)$ which is the controller of the equivalent feedback system of standard IMC in Fig.2.

$$G_c(s) = \frac{G_{IMC}(s)}{1 - G_{IMC}(s)G_m(s)} = \frac{G_-^{-1}(s)}{f^{-1}(s) - G_+(s)}.$$

Through the decomposition of the formula above, we obtain the proportional gain, derivative time constant and integral time constant which contains only one unknown variable λ . By tuning this parameter, we will design the learning law of L-MPC conveniently.

5 Simulations

For comparing the tracking performance of the control scheme proposed and P-type L-MPC, we utilize the following calculation formula:

$$ATE(k) \hat{=} \sum_{t=(k-1)T+1}^{kT} |y(t, k) - Y_r|/T,$$

$$y_r(t, k) = y_r(t, k - 1) + G_m^{-1}(z^{-1})e(t, k - 1), \quad (22)$$

i.e.

$$\Delta y_r(t, k) = G_m^{-1}(z^{-1})e(t, k - 1). \quad (23)$$

As expected, Eq.(23) suggests that $\Delta y_r(t, k)$ do have linear relationship with error term. It's a feasible plan that using IMC scheme designs the updating law of ILC.

4.3 Updating law design

As seen from Fig.2, G_m is the model of the plant that consists of model predictive controller and TIDM, and G_{IMC} denotes the internal model controller.

which is the abbreviation of the average tracking error. Since the repetitive nature of meal intake, glucose measurement, and insulin delivery over a 24 h period and the sample time is 5 min, the period for time step t is $T = 288$.

5.1 Repetitive diets

In this section, it's assumed that the subject has three meals a day as main disturbances at time 7:00, 12:00, 18:00 with fixed carbohydrate of 40 g, 60 g, 85 g. The target of glucose output is set $Y_r(t) = 110$ mg/dl. The value of hyperglycemia and hypoglycemia is defined as 180 mg/dl and 70 mg/dl, so the range of the outputs between 70 mg/dl and 180 mg/dl is safe.

Firstly, compare the tracking performance of IMC-enhanced L-MPC and P-type L-MPC. In this case, K , the learning gain of P-type L-MPC, is chosen as 0.5 for the tradeoff between convergence rate and robustness. Because it begins learning the setpoint in third day, the average tracking errors are the same in first two days. As seen from Fig. 3, IMC-enhanced L-MPC demonstrates good convergence property and has better tracking performance than P-type L-MPC. It is showed that the learning law designed by IMC theory is better than P-type learning law.

The control performance under IMC-enhanced L-MPC for 30 days are given in Fig.4: Fig.4(a) shows the glucose levels for 30 days; Fig.4(b) shows the

insulin delivery rates; for having a good look at the control results, Fig.4(c) and Fig.4(d) give the glucose level and insulin delivery amount in 30-th day. It can be noticed that the glucose outputs are among the safe range [70 mg/dl, 180 mg/dl] and track the target more and more closely.

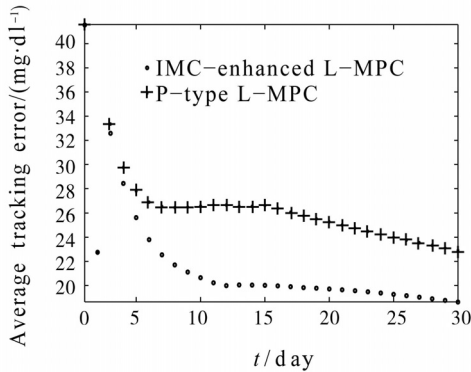


Fig. 3 Comparison of ATE under IMC-enhanced L-MPC and P-type L-MPC

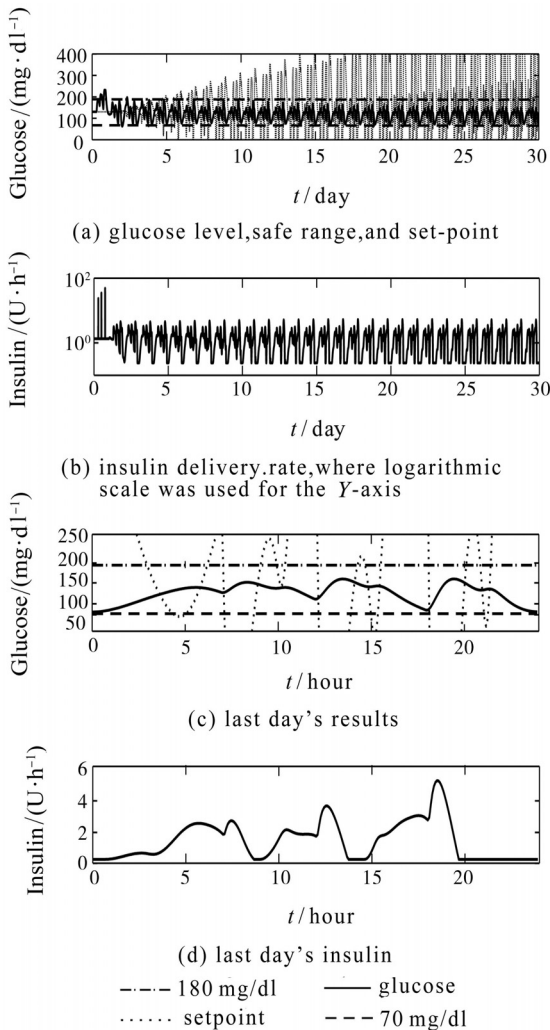


Fig. 4 Control performance under IMC-enhanced L-MPC ($\lambda = 0.4$) in 30 days

Then we keep all the other conditions identical, and only change the parameter λ , observing the differences based upon $\lambda = 0.25$, $\lambda = 0.4$ and $\lambda = 0.6$.

The simulation results in Fig.5 denote that the convergence property is the best when $\lambda = 0.4$.

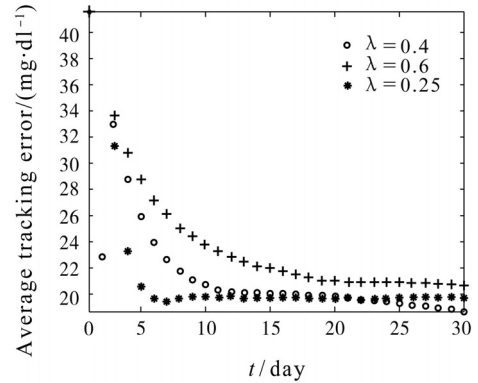


Fig. 5 Comparison of ATE under IMC-enhanced L-MPC when $\lambda = 0.25, 0.4, 0.6$, respectively

5.2 Robustness to meal variations

In all of the simulations above, the meal amount is fixed. In fact, we have to consider the situation that meal amount is variable from day to day. We give $\pm 50\%$ variation in meal amounts and find the control scheme demonstrating excellent robustness. As shown in Fig. 6, most blood glucose concentrations are controlled within the safe range, even though there are big meal size variations.

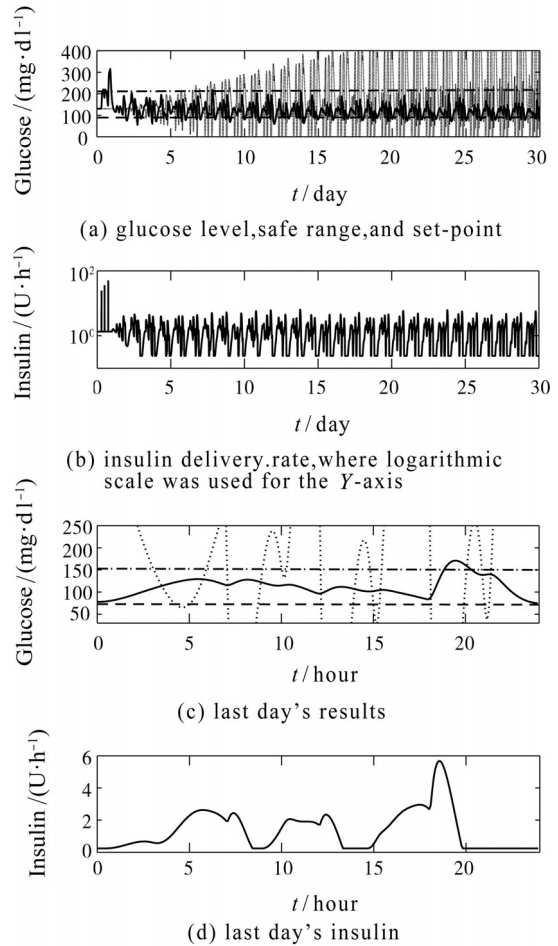


Fig. 6 Control results under IMC-enhanced L-MPC with $\pm 50\%$ variations in meal amounts

6 Conclusions

On the basis of reported P-type L-MPC, a novel IMC-enhanced L-MPC was proposed in this paper and was applied to the artificial pancreatic β -cell for T1DM. Only by tuning a parameter, we can obtain a proper PID-type updating law for ILC. The simulation results demonstrate good convergence property and robustness to non-repetitive disturbances. Moreover, it has superior tracking performance compared with the P-type L-MPC.

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作者简介:

杨跃男 (1987–), 男, 目前研究方向为迭代学习控制、内模控制和人工胰脏, E-mail: 2010000836@grad.buct.edu.cn;

王友清 (1981–), 男, 教授, 博士生导师, 目前研究方向包括生物医学过程(例如人工胰脏、麻醉控制)的建模和控制、间歇过程、迭代学习控制和容错控制, E-mail: wang_youqing@mail.buct.edu.cn, 通讯作者。