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存在环境约束的机器人自适应迭代学习控制

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摘要: 针对存在不确定性和外界干扰的受限机器人系统提出一种自适应迭代学习控制律. 不确定性参数被估计在时间域内, 同时重复性外界干扰在迭代域内得到补偿. 通过引入饱和学习函数, 保证了闭环系统所有信号有界. 借助Lyapunov复合能量函数法, 证明了系统渐进收敛到期望轨迹的同时, 能够保证力跟踪误差有界可调.

关键词: 受限机器人; 自适应控制; 迭代学习控制; 外界干扰

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Adaptive iterative learning control of robot manipulators in the presence of environmental constraint

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Abstract: A novel adaptive iterative learning algorithm is proposed for a class of constraint robotic manipulators with uncertainties and external disturbances. The uncertain parameters are estimated in the time domain whereas the repetitive disturbances is compensated in the iteration domain. With the adoption of saturated learning, all the signals in the closed loop are guaranteed to be bounded. By constructing a Lyapunov-Krasovskii-like composite energy function, the states of the closed system is proved to be asymptotically convergent to the desired trajectory while ensuring the constrained force remains bounded. Simulation results show the effectiveness of the proposed algorithm.

Key words: constrained robots; adaptive control; iterative learning control; external disturbances

1 Introduction

Control of robotic manipulators can be categorized into two modes: free motion control and constrained motion control. Free motion control is used when the robot arm moves in a free space without interacting with the environment. On the contrary, constrained motion control is connected with the robot whose end-effector mechanically interacts with the environment. In this case, the control of the contact forces is at least as important as the position control[1].

The control problem of constrained robots was initially studied in [2]. Then various strategied were proposed to achieve motion and force control for constrained robots[3–5]. Most control methods of motion control of constrained robots are under a general framework in [6], which transformed the constrained robotic systems into reduced-state unconstrained subsystems. Considering parametric uncertain robot manipulators, the adaptive constrained motion control of rigid robots has been developed in [7–8]. If the dynamics are exactly known for the control of a constrained robot, the methods as mentioned above can be used for designing the controllers effectively. However, from a practical viewpoint, exact knowledge about the complex robot dynamics is not available in advance. If there exist external disturbances, the controller so designed may give a degraded performance. This has motivated researches to develop novel controllers to enhance the dynamic performance for constrained robots under parametric uncertainties and external disturbances. To the best of our knowledge, the sliding mode control[9–10], sliding-adaptive control[11–12], and the others[13–16]. Asymptotic convergence to zero tracking error is able to be achieved as time approaches infinity, but control chattering occurs due to the use of discontinuous control laws. Taking advantage of the fact that robot manipulators are generally used in repetitive tasks, several iterative learning control schemes have been proposed in the past two decades. To enhance the robustness of the constrained robotic system in the presence of uncertainties, we present a new learning control strategy in this paper. The main objective of the learning control strategy is to enhance the tracking accuracy from operation to operation for systems executing repetitive tasks. The challenge is how to design the controller to ensure the desired trajectory and constraint force tracking under

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the parametric uncertainties and external disturbances. First, we use a nonlinear transformation that was introduced in [17] to simplify the dynamic model. Then, based on the reduced dynamic model and the defined references signals, taking advantage of the repetition, the adaptive iterative learning scheme is attributed. At each trial, the uncertain parameters are estimated in the time domain whereas the repetitive disturbances is compensated in the iteration domain, which are used for the main part of the controller, and additional part based sliding mode technique is used to compensate for the force tracking error and achieve robustness of the control system.

2 Dynamic equations and problem

The dynamics of a constrained robot with n rigid bodies, is described as

$$M(\boldsymbol{q})\ddot{\boldsymbol{q}} + C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + g(\boldsymbol{q}) + \boldsymbol{\tau}_{\mathrm{d}}(t) = \boldsymbol{\tau} + \boldsymbol{f}, \quad (1)$$

where $\boldsymbol{q} \in \mathbb{R}^n$ is the joint position, $g(\boldsymbol{q})$is the gravity vector, $\boldsymbol{\tau} \in \mathbb{R}^n$ is the generalized torque, $M(\boldsymbol{q}) \in \mathbb{R}^{n\times n}$ is the symmetric and positive definite inertia matrix, $C(\boldsymbol{q},\dot{\boldsymbol{q}}) \in \mathbb{R}^{n\times n}$ is the centripetal-Coriolis matrix, $\boldsymbol{q} \in \mathbb{R}^n$ is the gravity vector, and $\boldsymbol{\tau}_{\mathrm{d}}(t) \in \mathbb{R}^n$ is the unknown bounded disturbance vector that is repetitive for each iteration, $\boldsymbol{f} \in \mathbb{R}^n$ is the interaction force due to contact with the environment.

Suppose the constraint equations is described by

$$\boldsymbol{\Phi}(\boldsymbol{q}) = \boldsymbol{0} \in \mathbb{R}^m, \ m \leqslant n. \quad (2)$$

So we can get that $\dfrac{\partial \boldsymbol{\Phi}}{\partial \boldsymbol{q}}\dot{\boldsymbol{q}} = J(\boldsymbol{q})\dot{\boldsymbol{q}} = 0$, where $J(\boldsymbol{q})$ is an $m \times n$ matrix. The constrained force can be expressed as $\boldsymbol{f} = J^{\mathrm{T}}(q)\boldsymbol{\lambda}$, $\boldsymbol{\lambda} \in \mathbb{R}^m$ is the generalized contact force vector associated with the constraints. When the desired trajectory $\boldsymbol{q}_{\mathrm{d}}$ and a desired constraint force $\boldsymbol{f}_{\mathrm{d}}$, or a desired constraint multiplier $\boldsymbol{\lambda}_{\mathrm{d}}(t)$ is known as a reference input for the system of Eq.(1), the fundamental control problem is to find a control input $\boldsymbol{\tau}(t)$ with which the system output $\boldsymbol{q}(t)$ follows $\boldsymbol{q}_{\mathrm{d}}(t)$ and $\boldsymbol{\lambda}(t)$ follows $\boldsymbol{\lambda}_{\mathrm{d}}(t)$ for all $t \in [0,T]$ as closely as possible. In the framework of learning control this objective can be stated as follows:

Given the desired trajectory $\boldsymbol{q}_{\mathrm{d}}(t) \in \boldsymbol{C}^2(t \in [0,T])$ and a desired constraint multiplier $\boldsymbol{\lambda}_{\mathrm{d}}(t) \in \mathbb{R}^m(t \in [0,T])$ is known as a reference input for the system of Eq.(1), we will propose a sequence of piecewise continuous control input $\boldsymbol{\tau}^j(t) \in \mathbb{R}^n(t \in [0,T])$ which guarantee that:

1) The perfect motion tracking, i.e. $\lim\limits_{j\to\infty} \boldsymbol{q}^j(t) = \boldsymbol{q}^{\mathrm{d}}(t)$.

2) The force tracking error is bounded and adjustable for $t \in [0,T]$.

For learning controller design, we assume the system has the following properties:

A1) Each output trajectory can be measured and hence the error signal $\boldsymbol{e}^j = \boldsymbol{q}^j - \boldsymbol{q}_{\mathrm{d}}$ can be utilized.

A2) The resetting condition is satisfied, i.e. $\boldsymbol{q}^j(0) = \boldsymbol{q}_{\mathrm{d}}(0)$, $\dot{\boldsymbol{q}}^j(0) = \dot{\boldsymbol{q}}_{\mathrm{d}}(0)$, $\forall j \in \mathbb{Z}_+$.

One can partition the joint position vector $\boldsymbol{q}$ as $\boldsymbol{q} = (\boldsymbol{q}_1^{\mathrm{T}}, \boldsymbol{q}_2^{\mathrm{T}})^{\mathrm{T}}$, for $\boldsymbol{q}_1^{\mathrm{T}} \in \mathbb{R}^{n-m}$, $\boldsymbol{q}_2^{\mathrm{T}} \in \mathbb{R}^m$ and $J(\boldsymbol{q}) = [J_1(\boldsymbol{q}) \ J_2(\boldsymbol{q})] = [\dfrac{\partial \boldsymbol{\Phi}}{\partial \boldsymbol{q}_1} \ \dfrac{\partial \boldsymbol{\Phi}}{\partial \boldsymbol{q}_2}]$. Then it follows $J(\boldsymbol{q})\dot{\boldsymbol{q}} = J_1(\boldsymbol{q})\dot{\boldsymbol{q}}_1 + J_2(\boldsymbol{q})\dot{\boldsymbol{q}}_2 = 0$. There exists a proper partition such that $\det[J(\boldsymbol{q})_2] \neq 0$, for all $\boldsymbol{q}$. Then we can get $\dot{\boldsymbol{q}}_2 = -J_2^{-1}(\boldsymbol{q})J_1(\boldsymbol{q})\dot{\boldsymbol{q}}_1$. We can conclude that

$$\dot{\boldsymbol{q}} = L(\boldsymbol{q}_1)\dot{\boldsymbol{q}}_1, \quad (3)$$

$$\ddot{\boldsymbol{q}} = L(\boldsymbol{q}_1)\ddot{\boldsymbol{q}}_1 + \dot{L}(\boldsymbol{q}_1)\dot{\boldsymbol{q}}_1, \quad (4)$$

$$L(\boldsymbol{q}_1) = \begin{pmatrix} I_{n-m} \\ \dfrac{\partial \Omega^{\mathrm{T}}(\boldsymbol{q}_1)}{\partial \boldsymbol{q}_1} \end{pmatrix}. \quad (5)$$

So we can conclude that $L^{\mathrm{T}}(\boldsymbol{q})J^{\mathrm{T}}(\boldsymbol{q}) = J_1^{\mathrm{T}}(\boldsymbol{q}) - J_1^{\mathrm{T}}(\boldsymbol{q}) = 0$. Then the constrained system given by Eqs.(1) and (2) can be transformed by substituting Eqs. (3)–(4) into Eq.(1). The result is given by

$$\begin{aligned} &M(\boldsymbol{q})L(\boldsymbol{q}_1)\ddot{\boldsymbol{q}}_1 + M(\boldsymbol{q})\dot{L}(\boldsymbol{q}_1)\dot{\boldsymbol{q}}_1 + \\ &C(\boldsymbol{q},\dot{\boldsymbol{q}})L(\boldsymbol{q}_1)\dot{\boldsymbol{q}}_1 + g(\boldsymbol{q}) + \boldsymbol{\tau}_{\mathrm{d}}(t) = \boldsymbol{\tau} + J^{\mathrm{T}}\boldsymbol{\lambda}. \end{aligned} \quad (6)$$

A multiplication of $L^{\mathrm{T}}(\boldsymbol{q}_1)$ to the above equation leads to

$$\bar{M}(\boldsymbol{q})\ddot{\boldsymbol{q}}_1 + \bar{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}_1 + \bar{g}(\boldsymbol{q}) + \bar{\boldsymbol{\tau}}_{\mathrm{d}}(t) = L^{\mathrm{T}}\boldsymbol{\tau}, \quad (7)$$

where $\bar{M} = L^{\mathrm{T}}ML$, $\bar{C} = L^{\mathrm{T}}(M\dot{L} + CL)$, $\bar{g} = L^{\mathrm{T}}g$, $\bar{\boldsymbol{\tau}}_d = L^{\mathrm{T}}\boldsymbol{\tau}_d$. We now state some fundamental properties of the dynamic motion Eq.(7):

P1) Matrix $\bar{M}$ is uniformly bounded and symmetric positive definite.

P2) Matrix $\dot{\bar{M}} - 2\bar{C}$ is skew-symmetric, i.e.

$$\boldsymbol{y}^{\mathrm{T}}(\dot{\bar{M}} - 2\bar{C})\boldsymbol{y} = 0, \ \forall \boldsymbol{y} \in \mathbb{R}^n.$$

$\bar{M}$ and $\bar{C}$ in Eq.(7) satisfy

$$\dot{\bar{M}} - 2\bar{C} = L^{\mathrm{T}}(\dot{M} - 2C)L,$$

so $\boldsymbol{\xi}^{\mathrm{T}}(\dot{\bar{M}} - 2\bar{C})\boldsymbol{\xi} = 0, \forall \boldsymbol{\xi} \in \mathbb{R}^n$.

P3) Linear parameterization with a suitable selected vector of robot and load parameters

$$M(\boldsymbol{q})\dot{\boldsymbol{\xi}} + C(\boldsymbol{q},\dot{\boldsymbol{q}})\boldsymbol{\xi} + g(\boldsymbol{q}) = Y(\boldsymbol{q},\dot{\boldsymbol{q}},\boldsymbol{\xi},\dot{\boldsymbol{\xi}})\boldsymbol{\theta},$$

where $\boldsymbol{\xi}$ is an intermediate variable, and $Y(\cdot)$ is a known regression matrix, $\boldsymbol{\theta}$ is the unknown parameters of robot.

3 Adaptive iterative learning control design

3.1 Controller design

We assume the desired signals $\boldsymbol{q}_1^{\mathrm{d}}$, $\dot{\boldsymbol{q}}_1^{\mathrm{d}}$, $\ddot{\boldsymbol{q}}_1^{\mathrm{d}}$, $\boldsymbol{\lambda}_{\mathrm{d}}$ and $\dot{\boldsymbol{\lambda}}_{\mathrm{d}}$ and are bounded for all $t \in [0,T]$, while $\boldsymbol{q}_1^{\mathrm{d}}$ satisfies the constraint. For controller design, we define the motion error, force error, and auxiliary signal vector: $\boldsymbol{e}_{\mathrm{m}}^j = \boldsymbol{q}_1^{\mathrm{d}} - \boldsymbol{q}_1^j$, $\boldsymbol{e}_{\mathrm{m}}^j \in \mathbb{R}^{n-m}$, $\boldsymbol{e}_{\lambda}^j = \int(\boldsymbol{\lambda}^j - \boldsymbol{\lambda}_{\mathrm{d}})\mathrm{d}t$, $\boldsymbol{e}_{\lambda}^j \in \mathbb{R}^m$, $\dot{\boldsymbol{q}}_{1\mathrm{r}}^j = \dot{\boldsymbol{q}}_1^{\mathrm{d}} + \Lambda_{\mathrm{m}}\boldsymbol{e}_{\mathrm{m}}^j$, and the filtered tracking error at the jth cycle as

$$\boldsymbol{s}^j = \varLambda_{\rm m}\boldsymbol{e}_{\rm m}^j + \dot{\boldsymbol{e}}_{\rm m}^j,\ \boldsymbol{s}^j \in \mathbb{R}^{n-m}, \tag{8}$$

where $\varLambda_{\rm m}$ are symmetric positive-definite matrices. Consider the dynamics (7) and error dynamics in Eq.(8), we get

$$\begin{aligned}\bar{M}\dot{\boldsymbol{s}}^j = & L^{\rm T}(ML\ddot{\boldsymbol{q}}_{1\rm r}^j + M\dot{L}\dot{\boldsymbol{q}}_{1\rm r}^j + CL\dot{\boldsymbol{q}}_{1\rm r}^j + g) - \\ & \bar{C}\boldsymbol{s}^j + \bar{\boldsymbol{\tau}}_{\rm d}(t) - L^{\rm T}\boldsymbol{\tau}^j.\end{aligned} \tag{9}$$

The definitions of these variables pave the way for the proposed controller, which is chosen in the form:

$$\boldsymbol{\tau}^j = Y\hat{\boldsymbol{\theta}}^j + \hat{\boldsymbol{\tau}}_{\rm d}^j(t) + KL\boldsymbol{s}^j - J^{\rm T}\boldsymbol{u}_{\rm a}^j, \tag{10}$$

$$\boldsymbol{u}_{\rm a}^j = \boldsymbol{\lambda}_{\rm d} - K_{\rm P}\dot{\boldsymbol{e}}_\lambda^j - K_{\rm S}\boldsymbol{e}_\lambda^j, \tag{11}$$

where $K, K_{\rm P}, K_{\rm S}$ are positive definite.

Consider the positive definite function

$$V^j(t) = \frac{1}{2}(\boldsymbol{s}^{j\rm T}\bar{M}\boldsymbol{s}^j + \tilde{\boldsymbol{\theta}}^{j\rm T}\varGamma_1^{-1}\tilde{\boldsymbol{\theta}}^j). \tag{12}$$

After taking the time derivative of Eq.(12) along Eq.(9), we obtain

$$\begin{aligned}\dot{V}^j(t) = & \boldsymbol{s}^{j\rm T}\bar{M}\dot{\boldsymbol{s}}^j + \frac{1}{2}\boldsymbol{s}^{j\rm T}\dot{\bar{M}}\boldsymbol{s}^j + \tilde{\boldsymbol{\theta}}^{j\rm T}\varGamma_1^{-1}\dot{\tilde{\boldsymbol{\theta}}}^j = \\ & \boldsymbol{s}^{j\rm T}\bar{M}\dot{\boldsymbol{s}}^j + \boldsymbol{s}^{j\rm T}\bar{C}\boldsymbol{s}^j + \tilde{\boldsymbol{\theta}}^{j\rm T}\varGamma_1^{-1}\dot{\tilde{\boldsymbol{\theta}}}^j = \\ & -\boldsymbol{s}^{j\rm T}L^{\rm T}KL\boldsymbol{s}^j + \boldsymbol{s}^{j\rm T}L^{\rm T}Y\tilde{\boldsymbol{\theta}}^j - \\ & \boldsymbol{s}^{j\rm T}L^{\rm T}\tilde{\boldsymbol{\tau}}_{\rm d}^j(t) + \tilde{\boldsymbol{\theta}}^{j\rm T}\varGamma_1^{-1}\dot{\tilde{\boldsymbol{\theta}}}^j.\end{aligned} \tag{13}$$

So in order to offset the item $\boldsymbol{s}^{j\rm T}L^{\rm T}Y\tilde{\boldsymbol{\theta}}^j$, the parameter adaptation is chosen as

$$\begin{cases}\dot{\hat{\boldsymbol{\theta}}}^j(t) = \varGamma_1 Y^{\rm T}(t)L\boldsymbol{s}^j(t), \\ \hat{\boldsymbol{\theta}}^j(0) = \hat{\boldsymbol{\theta}}^{j-1}(T),\ \hat{\boldsymbol{\theta}}^{-1}(T) = 0.\end{cases} \tag{14}$$

Substituting Eq.(14) back into Eq.(13) yields

$$\dot{V}^j(t) = -\boldsymbol{s}^{j\rm T}L^{\rm T}K\boldsymbol{s}^jL - \boldsymbol{s}^{j\rm T}L^{\rm T}\tilde{\boldsymbol{\tau}}_{\rm d}^j(t), \tag{15}$$

where $\tilde{\boldsymbol{\tau}}_{\rm d}^j(t) = \boldsymbol{\tau}_{\rm d}(t) - \hat{\boldsymbol{\tau}}_{\rm d}^j(t)$, similar to [17], the learning law is designed as

$$\hat{\boldsymbol{\tau}}_{\rm d}^j(t) = \mathrm{sat}(\hat{\boldsymbol{\tau}}_{\rm d}^{j-1}(t)) + \varGamma_2 L\boldsymbol{s}^j(t),\ \forall t \in [0, T], \tag{16}$$

where $\varGamma_2 > 0$ is learning gain. Initial value $\hat{\boldsymbol{\tau}}_{\rm d}^{-1}(t) = 0$.

3.2 Convergence analysis

The convergence property of the proposed adaptive iterative learning control law is summarized in the following theorem.

Theorem 1 Consider the constrained robot systems (1)–(2) with the external disturbances, using the control law (10) with adaptation law (14) and learning update laws (16), then the following holds:

a) All the signals $\boldsymbol{s}^j$, $\dot{\boldsymbol{s}}^j$, $\boldsymbol{q}_1^j$, $\dot{\boldsymbol{q}}_1^j$ and $\hat{\boldsymbol{\theta}}^j$ in the closed-loop are bounded.

b) $\lim\limits_{j\to\infty}\boldsymbol{q}^j(t) = \boldsymbol{q}^{\rm d}(t)$, $\lim\limits_{j\to\infty}\dot{\boldsymbol{q}}^j(t) = \dot{\boldsymbol{q}}^{\rm d}(t)$, for all $t \in [0, T]$.

c) $\boldsymbol{e}_\lambda^j$ is bounded and adjustable for all $t \in [0, T]$.

Proof Define a Lyapunov-Krasovskii-like composite energy function (CEF) at the jth iteration as

$$E^j(t) = V^j(t) + \frac{1}{2}\int_0^t \tilde{\boldsymbol{\tau}}_{\rm d}^{j\rm T}\varGamma_2^{-1}\tilde{\boldsymbol{\tau}}_{\rm d}^j{\rm d}s. \tag{17}$$

a) Considering $t = T$, we compute the difference of $E^j(T)$ at the jth iteration, which is

$$\begin{aligned}& \Delta E^j(T) = E^j(T) - E^{j-1}(T) = \\ & \frac{1}{2}\int_0^T[\tilde{\boldsymbol{\tau}}_{\rm d}^{j\rm T}\varGamma_2^{-1}\tilde{\boldsymbol{\tau}}_{\rm d}^j - (\tilde{\boldsymbol{\tau}}_{\rm d}^{j-1})^{\rm T}\varGamma_2^{-1}\tilde{\boldsymbol{\tau}}_{\rm d}^{j-1}]{\rm d}s + \\ & V^j(0) - V^{j-1}(T) + \int_0^T \dot{V}^j(s){\rm d}s.\end{aligned} \tag{18}$$

By simple algebraic manipulation, we have the following inequality:

$$\begin{aligned}& \frac{1}{2}\int_0^T[\tilde{\boldsymbol{\tau}}_{\rm d}^{j\rm T}\varGamma_2^{-1}\tilde{\boldsymbol{\tau}}_{\rm d}^j - (\tilde{\boldsymbol{\tau}}_{\rm d}^{j-1})^{\rm T}\varGamma_2^{-1}\tilde{\boldsymbol{\tau}}_{\rm d}^{j-1}]{\rm d}s \leqslant \\ & -\int_0^T[\hat{\boldsymbol{\tau}}_{\rm d}^j(s) - \hat{\boldsymbol{\tau}}_{\rm d}^{j-1}(s)]^{\rm T}\varGamma_2^{-1}\tilde{\boldsymbol{\tau}}_{\rm d}^j{\rm d}s \leqslant \\ & -\int_0^T \boldsymbol{s}^{j\rm T}L^{\rm T}\tilde{\boldsymbol{\tau}}_{\rm d}^j{\rm d}s.\end{aligned} \tag{19}$$

So we get

$$\Delta E^j(T) \leqslant V^j(0) - V^{j-1}(T) - \int_0^T \boldsymbol{s}^{j\rm T}L^{\rm T}KL\boldsymbol{s}^j{\rm d}s, \tag{20}$$

which implies

$$\begin{aligned}& E^j(T) - E^{j-1}(T) \leqslant \\ & V^j(0) - V^{j-1}(T) - \int_0^T \boldsymbol{s}^{j\rm T}L^{\rm T}KL\boldsymbol{s}^j{\rm d}s.\end{aligned} \tag{21}$$

Since A2) ensures that $\boldsymbol{s}^j(0) = 0$, and from Eq.(14) we know $\tilde{\boldsymbol{\theta}}^j(0) = \tilde{\boldsymbol{\theta}}^{j-1}(T)$, which results in

$$V^j(0) - V^{j-1}(T) = -\frac{1}{2}\boldsymbol{s}^{j\rm T}(T)\bar{M}\boldsymbol{s}^j(T) \leqslant 0. \tag{22}$$

Thus, we obtain

$$E^j(T) - E^{j-1}(T) \leqslant -\int_0^T \boldsymbol{s}^{j\rm T}L^{\rm T}KL\boldsymbol{s}^j{\rm d}s. \tag{23}$$

It can be seen that the boundedness of $E^j(T)$ is ensured provided $E^0(T)$ is uniformly finite for $t \in [0, T]$. So next, we prove the uniform finiteness of $E^0(T)$,

$$\begin{aligned}& \dot{E}^0(t) = \dot{V}^0(t) + \frac{1}{2}\tilde{\boldsymbol{\tau}}_{\rm d}^{0\rm T}\varGamma_2^{-1}\tilde{\boldsymbol{\tau}}_{\rm d}^0 = \\ & -\boldsymbol{s}^{0\rm T}L^{\rm T}KL\boldsymbol{s}^0 - \boldsymbol{s}^{0\rm T}L^{\rm T}\tilde{\boldsymbol{\tau}}_{\rm d}^0(t) + \frac{1}{2}\tilde{\boldsymbol{\tau}}_{\rm d}^{0\rm T}\varGamma_2^{-1}\tilde{\boldsymbol{\tau}}_{\rm d}^0 \leqslant \\ & -\boldsymbol{s}^{0\rm T}L^{\rm T}\tilde{\boldsymbol{\tau}}_{\rm d}^0(t) + \frac{1}{2}\tilde{\boldsymbol{\tau}}_{\rm d}^{0\rm T}\varGamma_2^{-1}\tilde{\boldsymbol{\tau}}_{\rm d}^0 \leqslant \\ & \hat{\boldsymbol{\tau}}_{\rm d}^{0\rm T}(t)\varGamma_2^{-1}\tilde{\boldsymbol{\tau}}_{\rm d}^0(t) + \frac{1}{2}\tilde{\boldsymbol{\tau}}_{\rm d}^{0\rm T}\varGamma_2^{-1}\tilde{\boldsymbol{\tau}}_{\rm d}^0 \leqslant \\ & \boldsymbol{\tau}_{\rm d}^{\rm T}(t)\varGamma_2^{-1}\tilde{\boldsymbol{\tau}}_{\rm d}^0(t) - \frac{1}{2}\tilde{\boldsymbol{\tau}}_{\rm d}^{0\rm T}\varGamma_2^{-1}\tilde{\boldsymbol{\tau}}_{\rm d}^0 \leqslant \frac{1}{2}\boldsymbol{\tau}_{\rm d}^{\rm T}\varGamma_2^{-1}\boldsymbol{\tau}_{\rm d}.\end{aligned} \tag{24}$$

Since $\boldsymbol{\tau}_{\rm d}$ is bounded, we get the boundedness of $\dot{E}^0(t)$. Noting $E^0(0) = 0$, which implies that $E^0(t)$ is uniformly continuous and bounded over $[0, T]$, thus $E^0(T)$ is bounded, so we can conclude that $E^j(T)$ is bounded. We obtain from Eq.(20) the following:

$$\begin{aligned}& E^j(t) \leqslant \\ & E^{j-1}(t) + V^j(0) - V^{j-1}(t) - \int_0^t \boldsymbol{s}^{j\rm T}L^{\rm T}KL\boldsymbol{s}^j{\rm d}s = \\ & \frac{1}{2}\int_0^t \tilde{\boldsymbol{\tau}}_{\rm d}^{j\rm T}\varGamma_2^{-1}\tilde{\boldsymbol{\tau}}_{\rm d}^j{\rm d}s - \int_0^t \boldsymbol{s}^{j\rm T}L^{\rm T}KL\boldsymbol{s}^j{\rm d}s + V^j(0).\end{aligned} \tag{25}$$

b) Summing $E^j(t) - E^{j-1}(t)$ from $j = 1$ to k

gives

$$E^k(t) = E^0(t) + \sum_{j=1}^{k} \Delta E_t^j \leqslant$$
$$E^0(t) - \sum_{j=1}^{k} \int_0^t \boldsymbol{s}^{j\mathrm{T}} L^{\mathrm{T}} K L \boldsymbol{s}^j \mathrm{d}s, \quad (26)$$

which implies that $\sum_{j=1}^{k} \int_0^t \boldsymbol{s}^{j\mathrm{T}} L^{\mathrm{T}} K L \boldsymbol{s}^j \mathrm{d}s \leqslant E^0(t)$. Since $E^0(t)$ is finite and $E^k(t)$ is positive, $\sum_{j=1}^{\infty} \int_0^t \boldsymbol{s}^{j\mathrm{T}} L^{\mathrm{T}} K L \boldsymbol{s}^j \mathrm{d}s$ converges. According to the convergence theorem of the sum of series, $\lim_{j\to\infty} \int_0^t \boldsymbol{s}^{j\mathrm{T}} \cdot \boldsymbol{s}^j \mathrm{d}s = 0$. In addition, from Eqs.(7)–(8), $\dot{\boldsymbol{s}}^j$ is bounded on $[0, T]$. Thus, $\boldsymbol{s}^j$ converges to zero on $[0, T]$, as $j \to \infty$. From Eq.(8) we can conclude that

$$\lim_{j\to\infty} \boldsymbol{e}_{\mathrm{m}}^j = \lim_{j\to\infty} \dot{\boldsymbol{e}}_{\mathrm{m}}^j = 0.$$

c) Substituting the control law (10) into the reduced order dynamic system model (6) yields

$$\begin{aligned} &J^{\mathrm{T}}[(I + K_{\mathrm{P}})\dot{\boldsymbol{e}}_\lambda^j + K_{\mathrm{S}}\boldsymbol{e}_\lambda^j] = \\ &M(\boldsymbol{q}^j)L(\boldsymbol{q}_1^j)\ddot{\boldsymbol{q}}_1^j + M(\boldsymbol{q}^j)\dot{L}(\boldsymbol{q}_1^j)\dot{\boldsymbol{q}}_1^j + g(\boldsymbol{q}^j) + \boldsymbol{\tau}_{\mathrm{d}}(t) + \\ &C(\boldsymbol{q}^j, \dot{\boldsymbol{q}}^j)L(\boldsymbol{q}_1^j)\dot{\boldsymbol{q}}_1^j - Y\hat{\boldsymbol{\theta}}^j - \hat{\boldsymbol{\tau}}_{\mathrm{d}}^j(t) - KL\boldsymbol{s}^j = \\ &\boldsymbol{\xi}(\boldsymbol{q}_1, \dot{\boldsymbol{q}}_1, \boldsymbol{q}_{1\mathrm{r}}, \dot{\boldsymbol{q}}_{1\mathrm{r}}). \end{aligned} \quad (27)$$

Since $\boldsymbol{s}^j$, $\dot{\boldsymbol{s}}^j$, $\boldsymbol{q}_1^j$, $\dot{\boldsymbol{q}}_1^j$ are all bounded, $\boldsymbol{\xi}$ is bounded and we get $J^{\mathrm{T}}[(I + K_{\mathrm{P}})\dot{\boldsymbol{e}}_\lambda^j + K_{\mathrm{S}}\boldsymbol{e}_\lambda^j]$ is bounded. If we appropriately choose $K_{\mathrm{P}} = \mathrm{diag}\{k_{\mathrm{p},i}\}$, $k_{\mathrm{p},i} > -1$ and $K_{\mathrm{S}} = \mathrm{diag}\{k_{\mathrm{s},i}\}$, $k_{\mathrm{s},i} > 0$ to make $G_i(p) = \dfrac{1}{(k_{\mathrm{p},i} + 1)p + k_{\mathrm{s},i}}$, $p = \dfrac{\mathrm{d}}{\mathrm{d}t}$, a strictly proper exponential stable transfer function, then it can be concluded that $\boldsymbol{e}_\lambda^j \in L_\infty$, $\dot{\boldsymbol{e}}_\lambda^j \in L_\infty$. The size of $\boldsymbol{e}_\lambda^j \in L_\infty$ can be adjusted by choosing the proper gain matrices K_{P} and K_{S}.

Since all three terms on the right hand side of Eq.(23) are bounded, $E^j(t)$ is bounded on $[0, T]$. From the definition of $E^j(t)$, we know that $\boldsymbol{s}^j$ and $\hat{\boldsymbol{\theta}}^j$ are all bounded on $[0, T]$ for all j. From the fact that $\boldsymbol{q}_1^{\mathrm{d}}$ and $\dot{\boldsymbol{q}}_1^{\mathrm{d}}$ are all bounded, we can conclude that both $\boldsymbol{q}_1^j$ and $\dot{\boldsymbol{q}}_1^j$ are bounded on $[0, T]$ for all j.

4 Simulation example

A two-link robot with a circular path constraint is used to verify the validity of the learning controller presented in this paper. The matrices of the original model which is in the form of Eq.(1) can be written as

$$M(\boldsymbol{q}) = \begin{pmatrix} \theta_1 + \theta_2 + 2\theta_3 \cos q_2 & \theta_2 + \theta_3 \cos q_2 \\ \theta_2 + \theta_3 \cos q_2 & \theta_1 \end{pmatrix},$$

$$C(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \begin{pmatrix} -\theta_3 \dot{q}_2 \sin q_2 & -\theta_3(\dot{q}_1 + \dot{q}_2)\sin q_2 \\ \theta_3 \dot{q}_1 \sin q_2 & 0 \end{pmatrix},$$

$$g(\boldsymbol{q}) = \frac{9.81}{l_1} \begin{pmatrix} \theta_1 \cos q_2 + \theta_3 \cos(q_1 + q_2) \\ \theta_3 \cos(q_1 + q_2) \end{pmatrix}.$$

The constraint surface is expressed as

$$\boldsymbol{\Phi}(\boldsymbol{q}) = \sqrt{l_1^2 + 2l_1 l_2 \cos q_2} - a,$$

where a is a constraint and l_1 and l_2 are the lengths of the two robot links. It then follows that $J_{\mathrm{c}}(\boldsymbol{q}) = [0 \;\; -2l_1 l_2 \sin q_2]$, $L = [1 \;\; 0]^{\mathrm{T}}$ i.e., $\dot{q}_2 = 0$. The desired trajectory and the desired constraint force are respectively assumed to $q_{1\mathrm{d}} = \cos(3t)$, $q_{2\mathrm{d}} = \dfrac{\pi}{4}$ $\lambda_{\mathrm{d}} = 20$ for $t \in [0, 20]$. The disturbances are chosen as $\tau_{\mathrm{d}} = [0.5 + 0.5\cos(10t) \;\; 1 + 0.5\sin(10t)]^{\mathrm{T}}$. The three unknown parameters are: $\theta_1 = (m_1 + m_2)l_1^2$, $\theta_2 = m_2 l_2^2$, $\theta_3 = m_2 l_1 l_2$. The robot parameters are: $T = 20\,\mathrm{s}$, $l_1 = l_2 = 0.5$, $m_1 = 10$, $m_2 = 5$, $\varGamma_1 = \mathrm{diag}\{0.015, 0.015, 0.015\}$, $\varGamma_2 = \mathrm{diag}\{0.05, 0.05\}$, $K = \mathrm{diag}\{60, 60\}$, $\boldsymbol{\theta} = [\theta_1 \;\; \theta_2 \;\; \theta_3]^{\mathrm{T}}$, $K_{\mathrm{P}} = 30$, $K_{\mathrm{S}} = 50$.

The simulation results are shown in Figs.1–5. As shown in Fig.1 and Fig.2, the controller we designed for the constrained robot cannot achieve good performance mainly due to the disturbances and parameter uncertainties. However, we increase the number of iterations into 20 times, the system trajectory can achieve perfect tracking and the force error remains bounded as small as possible as the results shown in Fig.3 and Fig.4. From Fig.5 we can easily get that with the iteration number increasing, the maximum tracking error of link 1 gradually tends to zero. The example validates the effectiveness of the control law in Theorem 1.

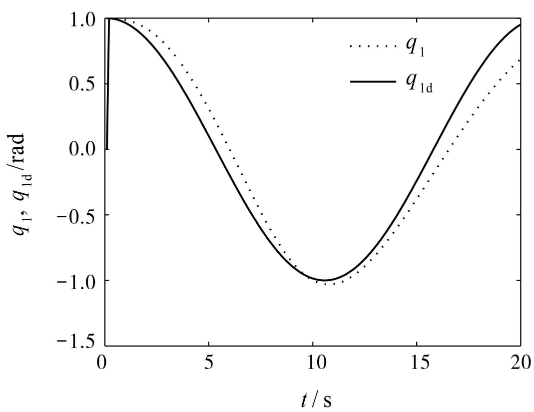

Fig. 1 Simulated responses of q_1, $q_{1\mathrm{d}}$ at $j = 1$

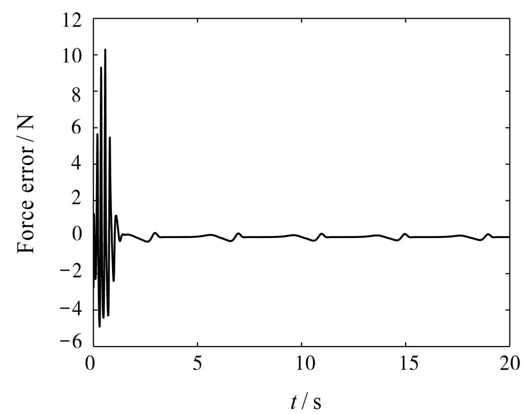

Fig. 2 Simulated responses of force tracking error at $j = 1$

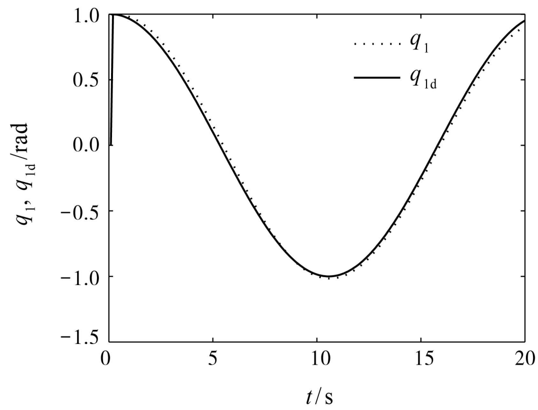

Fig. 3 Simulated responses of q_1, $q_{1\mathrm{d}}$ at $j = 20$

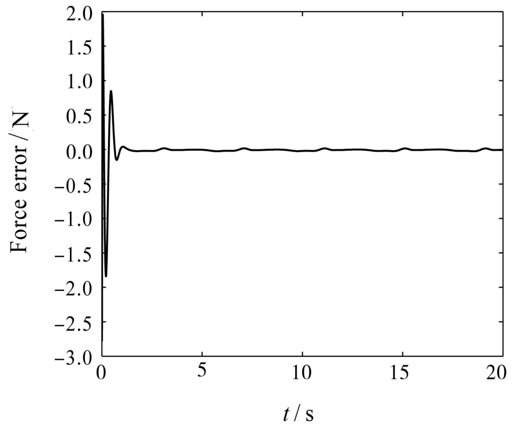

Fig. 4 Simulated responses of force tracking error at $j = 20$

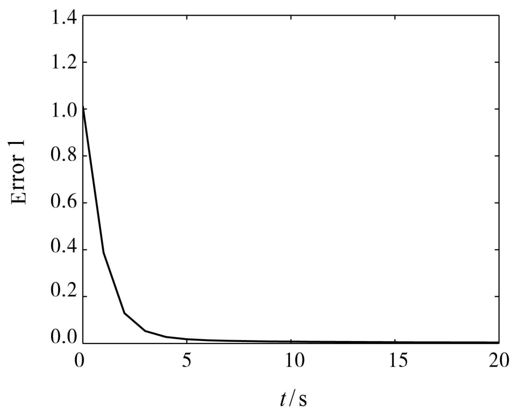

Fig. 5 The maximum tracking error 1 with iteration numbers

5 Conclusions

We have presented a new adaptive iterative learning controller for constrained robots under uncertainties and external disturbances. Based on the reduced dynamic model, the uncertain parameters are estimated in the time domain whereas repetitive disturbances are compensated in the iteration domain. Both theoretical analysis and simulation demonstrate that the proposed approach can achieve the perfect tracking at the same time the force error is bounded and adjustable and rejects repetitive disturbances.

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