

## 迭代学习控制技术回顾与长期学习控制展望

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**摘要:** 本文首先回顾了迭代学习控制中初始状态漂移问题和单调收敛性分析的研究技术. 其次, 综述了高阶迭代学习控制机制及其收敛速度比较和有效性. 再次, 评述了重复运行大系统和变幅值大工业过程的迭代学习控制机理. 最后, 展望了长期学习控制的研究趋势等.

**关键词:** 迭代学习控制; 收敛性分析; 初始状态漂移; 高阶学习律; 大系统; 长期学习控制

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## Retrospective review of some iterative learning control techniques with a comment on prospective long-term learning

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**Abstract:** This paper firstly makes a retrospective review of some iterative learning control techniques and results regarding to the initial state shift issue and the monotone convergence analysis. Secondly, the paper presents a review of the higher-order iterative learning control scheme including its convergence speed comparison and effectiveness. Then, the paper exhibits a review of iterative learning control mechanism for large-scale systems including repetitive systems and magnitude-varying industrial processes. Lastly, the paper gives a comment on prospective long-term learning control for the future.

**Key words:** iterative learning control; convergence analysis; initial state shift; higher-order learning law; large-scale systems; long-term learning control

### 1 Introduction

'Learning' is a common notion that pervades through wide ranges from biology and psychology to sociology and philosophy, and so on, both in theoretical investigations and in practical applications. For example, human beings must learn not only survival capabilities including living resource acquirement and language communication but also socially communicative and cooperative ability with the environment. In addition, for advanced and quality-living convenience, human beings have invented various kinds of intelligent machines such as robot. In order to drive such an intelligent machine to work in accordance with human directions and intention, some forms of human being-like learning mechanism must be artificially embedded into the machine to guide its operation.

The notion 'learning' has been dealt with in various disciplines, including mathematics, computer science, linguistics, psychology and philosophy, etc. Its inves-

tigative interests include theories of scientific discovery, formal learning, machine inductive inference, computational learning and empirical inquiry and so on. Typical learning plan involves specification of target function, choice of learning algorithm, selection of data/data subsets, preprocessing, measure of performance as well as halt criteria and so on. Generally speaking, learning is a process for an intelligent system to acquire knowledge or experience on the basis of its perception and cognition of the environment and then to act on the environment referring the knowledge/experience so as to improve its behavior performance the next time.

For the task of a robotic system to track a desired trajectory, the technique of iterative learning control (ILC) is well-known as one of the key artificial intelligent strategies. The ILC is a recursive algorithm that utilizes the tracking discrepancy/error of the system output from the desired trajectory and iteratively updates its current control command so as to generate

an upgraded control command for the next operation, when the system operates repetitively over a fixed finite time interval<sup>[1]</sup>. The aim is to improve the tracking performance more efficiently and effectively as the system operation repeats. As the basic ILC mechanism is simply the system input-output data driven scheme and does not need a priori knowledge of the system dynamics, it is adoptable for the systems with uncertainty, nonlinearity and complexity and so on<sup>[2]</sup>. As such, it has drawn much attention for its development and investigation in three decades since the ILC technique has been invented, and has been applied not only to robotic systems but also to batch types of industrial processes and video driver, etc<sup>[3-9]</sup>.

The topics of ILC mainly concern algorithmic construction<sup>[10-19]</sup>, convergence analytical techniques<sup>[20-23]</sup>, initial state shift issue<sup>[24-35]</sup>, adaptive ILC updating law design<sup>[36-37]</sup>, and optimal ILC mechanism<sup>[38-39]</sup> as well as practical applications<sup>[40]</sup>. Regarding to those constructed strategies, one of the key problems to be solved is convergence clarification. The analytical techniques hinge mainly on contraction mapping and fixed point theory. Due to the fact that the tracking performance is measured by some types of norm, the corresponding convergent results may not coincide with each other to some extent. Besides, the initial state shift issue is of course an ordinary phenomenon while the proposed scheme is implemented for practical systems. Its investigations involve tracking performance evaluation and compensation or rectifying strategies. For adaptive and optimal ILC rules, some cost/energy functions are constructed and the system parameters and state information are utilized to design the algorithms. Though the surveying status of the ILC investigations is updated successively<sup>[41-42]</sup>, some specific ILC techniques and its focuses are prominent in clarifying some controversial assertions as well as enriching the contents of the ILC development. This paper presents a retrospective review of some specific iterative learning control techniques mainly explored by Bien's group and then gives a comment of prospective long-term learning control. The paper is organized as follows. Section 2 reviews the investigation of the initial state shift issue and the monotone convergence. The higher-order ILC schemes and some newly-developed results are addressed in Section 3 and ILC architecture for large-scale system is exhibited in Section 4. Section 5 presents a prospective view on long-term learning control, and lastly, Section 6 concludes the paper.

## 2 Initial state uncertainty and monotone convergence issue

In most ILC algorithms, it is assumed that the initial state value of the plant is equal to that of the desired trajectory for perfect tracking, even though it is hard to set

the initial state value of the plant at that of the desired trajectory exactly. Lee and Bien reported the possibility of divergence of control input due to the initial state error<sup>[24]</sup>. Later, Lee and Bien showed that a proportional term of error can be positively solicited to get a better performance against initial state error<sup>[26]</sup>.

To be more specific, consider the linear system described by Eq.(1).

$$\begin{cases} \dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{u}(t), \\ \mathbf{y}(t) = C\mathbf{x}(t). \end{cases} \quad (1)$$

Here,  $\mathbf{x} \in \mathbb{R}^n$ ,  $\mathbf{u} \in \mathbb{R}^r$  and  $\mathbf{y} \in \mathbb{R}^q$  denote the state, the input and the output, respectively.  $A$ ,  $B$  and  $C$  are matrices with appropriate dimensions and it is assumed that  $C$ ,  $B$  is a full rank matrix. Let  $\mathbf{x}_d(\cdot)$  be the desired state trajectory and  $\mathbf{y}_d(\cdot)$  be the corresponding output trajectory. Assume that  $\mathbf{y}_d(\cdot)$  and  $\mathbf{x}_d(\cdot)$  are continuously differentiable on  $[0, T]$ . It is shown that when the ILC algorithm (2) is applied to the system (1), the output trajectory converges to the form in Eq.(3).

$$\mathbf{u}_{k+1}(t) = \mathbf{u}_k(t) + \Gamma(\dot{\mathbf{e}}_k(t) - R\mathbf{e}_k(t)), \quad (2)$$

$$\lim_{k \rightarrow \infty} \mathbf{y}_k(t) = \mathbf{y}_d(t) + e^{Rt}C(\mathbf{x}_0 - \mathbf{x}_d(0)), \quad (3)$$

where the subscript  $k$  is employed to mark the iteration index and hereafter.  $\mathbf{y}_k(\cdot)$ ,  $\mathbf{u}_k(\cdot)$  and  $\mathbf{e}_k(\cdot)$  are output trajectory, control input trajectory, and output error trajectory at the  $k$ -th iteration, respectively. Eq.(3) shows that the effect of initial state error can be controlled by tuning the gain  $R$  of the ILC algorithm, and the error asymptotically converges to zero if  $R$  is chosen so that all the eigenvalues have negative real parts.

Sometimes an intentional overshoot may be positively utilized to get a better performance. Park et al, generalized the previous result to the PID-type ILC algorithm and showed that the performance can be improved by adding an integral term<sup>[27]</sup>. It is shown that when the ILC algorithm (4) is applied to the system (1), the output trajectory converges to the form in Eq.(5).

$$\mathbf{u}_{k+1}(t) = \mathbf{u}_k(t) + \Gamma(\dot{\mathbf{e}}_k(t) + Q_0\mathbf{e}_k(t) + Q_1 \int_0^t \mathbf{e}_k(\tau)d\tau), \quad (4)$$

$$\lim_{k \rightarrow \infty} \mathbf{y}_k(t) = \mathbf{y}_d(t) + C_R e^{A_R t} \xi_0, \quad (5)$$

where

$$A_R = \begin{bmatrix} 0 & I \\ -Q_1 & -Q_0 \end{bmatrix}, \quad C_R = [I \quad 0],$$

$$\xi_0 = \begin{bmatrix} I \\ -Q_0 \end{bmatrix} C(\mathbf{x}_0 - \mathbf{x}_d(0)).$$

Eq.(5) implies that the output trajectory can be controlled in a variety of ways by introducing the integral term.

Park and Bien also extended the algorithms in the form of linear controller to the generalized ILC algorithm by adopting continuous operators<sup>[28]</sup>. It is shown

that when the ILC algorithm (6) is applied to the system (1), the output trajectory converges to the form in Eq.(7).

$$\begin{aligned} \mathbf{u}_{k+1}(t) &= \mathbf{u}_k(t) + \Gamma(\dot{\mathbf{e}}_k(t) + (P\mathbf{e}_k(\cdot))(t)), \quad (6) \\ \lim_{k \rightarrow \infty} \mathbf{y}_k(t) &= \mathbf{y}_d(t) - \tilde{\mathbf{e}}(t), \quad (7) \end{aligned}$$

where  $P$  is an operator of error function  $\mathbf{e}_k(t)$ , which is continuous in  $\mathbf{e}_k(\cdot)$ , and  $\tilde{\mathbf{e}}(t)$  is the solution of

$$\begin{cases} \dot{\tilde{\mathbf{e}}}(t) + (P\tilde{\mathbf{e}}(\cdot))(t) = \mathbf{0}, \\ \tilde{\mathbf{e}}(0) = C(\mathbf{x}_d(0) - \mathbf{x}_0). \end{cases}$$

Eq.(7) implies that the trend of error reduction can be effectively controlled by adopting relevant operators.

In spite of these conspicuous progresses in ILC, it is still required that the initial state value of the system should be same at each iteration even though it can be different from that of the desired trajectory. It is however inevitable to have deviation in initialization from the initial state value at last iteration although it may be very small. To resolve this problem, Park introduced an average operator-based ILC algorithm<sup>[29]</sup>. It is shown that when the ILC algorithm (8) is applied to the system (1), the output trajectory converges to the form in Eq.(9).

$$\begin{aligned} \mathbf{u}_{k+1}(t) &= \text{avg} \{ \mathbf{u}_i(t) \}_{i=0}^k + \frac{k+2}{k+1} \Gamma \times \\ &\quad \left( \sum_{i=0}^k \dot{\mathbf{e}}_i(t) - R \sum_{i=0}^k \mathbf{e}_i(t) \right), \quad (8) \end{aligned}$$

$$\begin{aligned} \lim_{k \rightarrow \infty} (\mathbf{y}_k(t) + C e^{At} (\mathbf{x}_0 - \mathbf{x}_k(0))) &= \\ \mathbf{y}_d(t) - e^{Rt} C (\mathbf{x}_d(0) - \mathbf{x}_0), \quad (9) \end{aligned}$$

where  $\text{avg} \{ \cdot \}_{i=0}^k$  denotes an average operator which is defined as

$$\text{avg} \{ h_i(\cdot) \}_{i=0}^k = \frac{1}{k+1} \sum_{i=0}^k h_i(\cdot).$$

Later, Ruan and Bien introduced a rectangular pulse compensation for PD-type ILC algorithms to suppress the tracking discrepancy incurred by initial state error, and showed that the upper bound of the asymptotical tracking error can be improved by tuning the compensation gain properly<sup>[43]</sup>.

Recall that, for a vector function  $f : [0, T] \rightarrow \mathbb{R}^m$ ,  $f(t) = [f^1(t) \cdots f^m(t)]^T$  and a real number  $\lambda > 0$ , the  $\lambda$ -norm is defined in [1] as

$$\|f(\cdot)\|_\lambda = \sup_{0 \leq t \leq T} e^{-\lambda t} \left( \sup_{1 \leq i \leq m} |f^i(t)| \right).$$

By comparison, the sup-norm<sup>[44]</sup> and Lebesgue- $p$  norm<sup>[45]</sup> are also utilized for ILC design, which are respectively defined as

$$\begin{aligned} \|f(\cdot)\|_{\text{sup}} &= \sup_{0 \leq t \leq T} \left( \sup_{1 \leq i \leq m} |f^i(t)| \right), \\ \|f(\cdot)\|_p &= \left[ \int_0^T \left( \sup_{1 \leq i \leq m} |f^i(t)| \right)^p dt \right]^{\frac{1}{p}}, \quad 1 \leq p \leq \infty. \end{aligned}$$

The convergence property in the sensor of sup-norm may seem to be equivalent to that obtained in the sense of  $\lambda$ -norm. However, we can observe some huge overshoot in the sensor of sup-norm even though the monotone convergence is guaranteed in the sense of  $\lambda$ -norm. Such an undesirable phenomenon of  $\lambda$ -norm was first observed by Lee and Bien<sup>[46]</sup>, and it was reported that the pure error term of a PD-type ILC algorithm plays an important role in a bound of the interval where the monotone convergence is guaranteed in the sense of sup-norm. However, we have to obtain an accurate model of the plant in order to get a desired error convergence behavior in a wider range of the interval, since the interval depends on the plant parameters.

To resolve this limitation, Park and Bien proposed a new ILC algorithm with adjustment of learning interval, which is found to be more robust against parameter uncertainty, and achieved monotone convergence of the output error in the sensor of sup-norm<sup>[47]</sup>. It is shown that the monotone convergence of the output error in the sense of sup-norm can be guaranteed when the ILC algorithm (2) is applied to the system (1) for the time interval  $[0, t_k^\lambda]$ , where  $t_k^\lambda$  is the maximum value among the time when  $e^{-\lambda t} \|\mathbf{e}_k(t)\|_\infty$  takes its maximum value over the given time interval  $[0, T]$  and  $\lambda$  is a real number satisfying the inequality

$$\lambda > \|A\|_\infty + \frac{\|C\|_\infty \|AB\Gamma - B\Gamma R\|_\infty}{1 - \rho},$$

where  $\|I - CB\Gamma\|_\infty \leq \rho < 1$ .

Park and Bien also proposed an intervalized learning scheme to achieve monotone convergence of the output error in the sense of sup-norm for an arbitrarily given long time interval<sup>[48]</sup>. In the proposed algorithm, we first choose a real number  $h$  satisfying the following inequality:

$$0 < h < \frac{1}{\|A\|_\infty} \ln \left( 1 + \frac{\|A\|_\infty (1 - \rho)}{2\|C\|_\infty \|AB\Gamma - B\Gamma R\|_\infty} \right),$$

and divide the given time interval  $[0, T]$  into  $N$  subintervals of length  $h$  and a remainder of length  $T - Nh$ , where  $N$  is a maximum integer less than  $T/h$ . It is shown that the monotone convergence of the output error in the sense of sup-norm can be guaranteed when the ILC algorithm (2) is applied to the system (1) for the time interval  $[0, t_{i_k}]$  where

$$\begin{aligned} i_0 &= N + 1, \quad \mathbf{e}[t_{-1}, t_0] = 0, \\ i_{k+1} &= \max \{ i \mid \|\mathbf{e}_k[t_{j-1}, t_j]\|_{\text{sup}} \leq \frac{1}{2} e^{-ah(l_j^k+1)} \times \\ &\quad \|\mathbf{e}_k[t_{j+l_j^k}, t_{j+l_j^k+1}]\|_{\text{sup}}, \quad 0 \leq j \leq i \} + 1, \end{aligned}$$

$$\begin{aligned} l_j^k &= \begin{cases} \min L_j^k, & L_j^k \neq \emptyset, \\ 0, & L_j^k = \emptyset, \end{cases} \\ L_j^k &= \{ l \mid \|\mathbf{e}_k[t_{j+l}, t_{j+l+1}]\|_{\text{sup}} \neq 0, \quad 0 \leq l \leq N - j \}. \end{aligned}$$

Recently, Ruan et al. addressed the monotone con-

vergence of typical PD-type ILC scheme in the sense of Lebesgue- $p$  norm<sup>[10–11]</sup>. It is shown that the output error is monotone convergent on whole operation time interval and the sufficient convergence condition is dominated not only by the system input-output matrices and derivative learning gain but also the system state matrix and the proportional gain. The result is objectively reveals the impact of the inherent system dynamics and the constructive mode of the ILC algorithm on the convergence without any requirement of the system/algorithm-irrelevant parameter.

### 3 Higher-order ILC algorithm

Regarding to the higher-order ILC algorithm, the pioneer scheme<sup>[49]</sup> has been constructed in late 1980's for the following linear time-invariant systems:

$$\begin{cases} \dot{\mathbf{x}}(t) = A\mathbf{x}(t) + Bu(t), \\ y(t) = C\mathbf{x}(t) + Du(t), \\ \mathbf{x}(0) = \xi^0, t \in [0, T], \end{cases} \quad (10)$$

where  $[0, T]$  stands for an operation duration and  $\mathbf{x}(t) \in \mathbb{R}^n$ ,  $u(t) \in \mathbb{R}$  and  $y(t) \in \mathbb{R}$  are  $n$ -dimensional state variable, scalar input and output, respectively.  $A, B, C$  and  $D$  are matrices with appropriate dimensions. We assume that the control task executes repetitively over the finite time interval  $[0, T]$  and the initial state is resettable, for simplicity, is zeroed at each repetition. Given that  $y_d(t)$ ,  $t \in [0, T]$  is a desired trajectory to be tracked. Provided that  $u_1(t)$  is an initial control input to substitute the control input  $u(t)$  of the system (10).

The second-order ILC algorithm is formulated as<sup>[49]</sup>

$$\begin{aligned} u_1(t) &: \text{arbitrarily given.} \\ u_{k+1}(t) &= P_1 u_k(t) + P_2 u_{k-1}(t) + Q_1 e_k(t) + \\ & \quad Q_2 e_{k-1}(t), k = 1, 2, \dots, \end{aligned} \quad (11)$$

where  $P_1$  and  $P_2$  are weighting coefficients satisfying  $P_1 + P_2 = I$ ,  $Q_1$  and  $Q_2$  are respective the first order and second order proportional learning gains. Besides, for general inherent nonlinear systems taking a form of (12), the pioneer work has further generalized the second-order P-type ILC law to  $N$ th-order P-type ILC law (13) exhibited as follows:

$$\begin{cases} \dot{\mathbf{x}}(t) = f(\mathbf{x}, t) + B(t)u(t), \\ y(t) = g(\mathbf{x}, t) + D(t)u(t), \\ \mathbf{x}(0) = \xi^0, t \in [0, T], \end{cases} \quad (12)$$

where  $f(\mathbf{x}, t)$  and  $g(\mathbf{x}, t)$  are nonlinear continuous functions with respect to the state variable  $\mathbf{x}$  and time variable  $t$  satisfying Lipschitz continuity condition. The  $N$ th order P-type ILC scheme is constructed as

$$\begin{aligned} u_{k+1}(t) &= P_1 u_k(t) + \dots + P_N u_{k-N+1}(t) + \\ & \quad Q_1 e_k(t) + \dots + Q_N e_{k-N+1}(t). \end{aligned} \quad (13)$$

In the case when the tracking error is measured in the sense of  $\lambda$ -norm, the convergence results are shown

as follows:

**Theorem 1** Assume that the control signal  $u(t)$  of the system (10) is undertaken by  $u_{k+1}(t)$  of the algorithm (11) successively. If the initial state of the system (10) is resettable and the weighting coefficients and the learning gains of the algorithm (11) together with the direct feed-through gain  $D$  of the system (10) satisfy  $P_1 + P_2 = I$  and  $\|P_1 - Q_1 D\|_\infty + \|P_2 - Q_2 D\|_\infty < 1$ , then  $\lim_{k \rightarrow \infty} y_k(t) = y_d(t)$ <sup>[49]</sup>.

**Theorem 2** Suppose that the matrices  $P_i$  and  $Q_i$  ( $i = 1, 2, \dots, N$ ) of the algorithm (13) satisfy  $\sum_{i=1}^N P_i = I$  and the polynomial has

$$P_l(z) = z^N - l_1 z^{N-1} - \dots - l_N,$$

all its roots inside the unit circle, where

$$l_i = \sup_{t \in [0, T]} \|P_i - Q_i D(t)\|_\infty.$$

If the initial state of the system (12) is resettable, then  $\lim_{k \rightarrow \infty} y_k(t) = y_d(t)$ <sup>[49]</sup>.

As both the algorithm (11) and (13) weight the control inputs and tracking errors more than one iteration, they are regarded to possess more robustness to noise. In addition, it is illustrated that they may operate with better tracking performance than the lower-order algorithm. Though the better performance feature needs to be argued in a rigorous manner, the higher-order ILC has attracted a great deal attention. Its further development has been shown in [29, 50–51] and the results has approved the opinion. The opinion, however, became controversial after the  $Q$  factor-based derivation has been made that the convergence speed of a higher-order P-type ILC scheme, of which the learning gains are chosen to ensure its  $Q$  factor minimal, for a nonlinear system with direct feed-through term is not faster than the lower-order P-type ILC scheme in virtue of the well-defined  $Q$  factor in the sense of  $\lambda$ -norm<sup>[52]</sup>. The derivation has developed a mathematical technique to assess the learning performance regardless of some mathematical mistakes to be mended as commented in [53]. In order to clarify the opinion in [49] and affirm the result in [52] in a rigorous manner, the paper [10] refined the concept of  $Q$  factor raised in [52] and reformulated the typical second-order P-type ILC to a more understandable form. The idea of the reformation is to separate the second-order P-type ILC command into two iteration-wise parts, one is the first-order learning component and another is the second-order one, and then to describe the second-order P-type ILC rule in a weighting form of these two components as follows:  $u_1(t)$ ,  $t \in [0, T]$ , an arbitrarily-chosen initial input signal,

$$\begin{aligned} u_2(t) &= u_1(t) + \Gamma_1 e_1(t), \\ u_{k+1}(t) &= w_1 [u_k(t) + \Gamma_1 e_k(t)] + \\ & \quad w_2 [u_{k-1}(t) + \Gamma_2 e_{k-1}(t)], \end{aligned}$$

$$t \in [0, T], k = 2, 3, 4, \dots \quad (14)$$

Here,  $w_1$  and  $w_2$  are weighting coefficients satisfying  $0 \leq w_1 < 1$ ,  $0 \leq w_2 \leq 1$  and  $w_1 + w_2 = 1$ .

For comparison, the following algorithm (15) is termed as the first-order P-type ILC rule.  $u_1(t)$ ,  $t \in [0, T]$ , an arbitrarily-chosen initial input signal,

$$u_{k+1}(t) = u_k(t) + \Gamma_1 e_k(t), \quad t \in [0, T], k = 1, 2, 3, \dots, \quad (15)$$

where the term  $u_k(t) + \Gamma_1 e_k(t)$  is called as the first-order learning component and the term  $u_{k-1}(t) + \Gamma_2 e_{k-1}(t)$  is assigned as the second-order learning component, respectively, whilst  $\Gamma_1$  is assigned as the first order proportional learning gain and  $\Gamma_2$  is named as the second order proportional learning gain, respectively.

By analyzing the property of the characteristics polynomial of the second-order ILC (14), the convergence speed in terms of the  $Q_p$  factor in the sense that the tracking error is measured in Lebesgue- $p$  norm is as follows:

**Theorem 3**<sup>[10]</sup> Assume that the second-order P-type ILC law (14) is applied to the system (10) and the system matrices  $A, B, C, D$  and the learning gains  $\Gamma_1$  and  $\Gamma_2$  satisfy the conditions in the following:

- i)  $\rho_1 = |1 - D\Gamma_1| + \|C \exp(A \cdot (\cdot)) B \Gamma_1\|_1 < 1$ ;
- ii)  $\rho_2 = |1 - D\Gamma_2| + \|C \exp(A \cdot (\cdot)) B \Gamma_2\|_1 < 1$ ;
- iii)  $\lim_{k \rightarrow \infty} \frac{\|e_{k+1}(\cdot)\|_p}{\|e_k(\cdot)\|_p}$  exist.

Then, if  $\rho_1^2 > \rho_2$ , the second-order ILC law (14) is  $Q_p$ -faster than the first-order law (15); If  $\rho_1^2 = \rho_2$ , the second-order ILC law (14) is  $Q_p$ -equivalent to the first-order law (15); If  $\rho_1^2 < \rho_2$ , the second-order ILC law (14) is  $Q_p$ -slower than the first-order law (15).

It is worth remarking that the newly-developed result not only theoretically supports the pioneer opinion in [49] but also rigorously approves the result in [52] while tracking error is measured in the form of Lebesgue- $p$  norm. Further, the article [11] exploited the second-order PD-type ILC rule to the LTI system with no direct feed-through term and derived the similar result with the reference [10].

#### 4 ILC schemes for large-scale systems

Large-scale system is a system that consists of a number of interdependent/interconnected constituents which serve particular functions/responsibilities, share resources/information, and are governed by a set of interrelated goals and constraints<sup>[53]</sup>. Because large-scale systems are structurally complex, multidimensional and highly interacting and so on, its controller profile is usually designed in a decentralized form to reduce the complexity, that is, each subsystem has its local independent controller with no exchange information with other sub-

systems. The dynamics of such kind of (linear time-invariant) LTI large-scale systems is usually described as follows:

$$\begin{cases} \dot{\mathbf{x}}^{(i)}(t) = A^{(i)}(t)\mathbf{x}^{(i)}(t) + B^{(i)}(t)\mathbf{u}^{(i)}(t) + \sum_{j=1, j \neq i}^N A^{(ij)}(t)\mathbf{x}^{(j)}(t), \\ \mathbf{y}^{(i)}(t) = C^{(i)}(t)\mathbf{x}^{(i)}(t) + D^{(i)}(t)\mathbf{u}^{(i)}(t), \\ \mathbf{x}^{(i)}(0) = \xi_0^{(i)}, t \in [0, T]. \end{cases} \quad (16)$$

Here, the superscript  $(i)$  is employed to mark number of the subsystem  $i$ .  $\mathbf{x}^{(i)}(t) \in \mathbb{R}^{n^{(i)}}$ ,  $\mathbf{u}^{(i)}(t) \in \mathbb{R}^{l^{(i)}}$  and  $\mathbf{y}^{(i)}(t) \in \mathbb{R}^{q^{(i)}}$  are  $n^{(i)}$ -dimensional state vector,  $l^{(i)}$ -dimensional control input and  $q^{(i)}$ -dimensional output, respectively. Specifically,  $\sum_{j=1, j \neq i}^N A^{(ij)}(t)\mathbf{x}^{(j)}(t)$  presents the state interactions from other subsystems, and  $A^{(i)}(t)$ ,  $B^{(i)}(t)$ ,  $C^{(i)}(t)$ ,  $D^{(i)}(t)$  and  $A^{(ij)}(t)$  are matrices with appropriate dimensions.

In practical engineering, batch industrial processes, such as acrylonitrile-butadiene-styrene polymerization reactor and cement manufacturing process, are multi-operation repetitive systems. For the sake of pursuing an expectable transient response, ILC scheme is adoptable. The basic ILC scheme is decentralized PID-type as follows:  $\mathbf{u}_1^{(i)}(t)$ ,  $t \in [0, T]$ , an arbitrarily-chosen initial input signal.

$$\mathbf{u}_{k+1}^{(i)}(t) = \mathbf{u}_k^{(i)}(t) + \Gamma_p(t)e_k^{(i)}(t) + \Gamma_i(t) \int_0^t e_k^{(i)}(\tau) d\tau + \Gamma_d(t)\dot{e}_k^{(i)}(t), \quad t \in [0, T], k = 1, 2, 3, \dots \quad (17)$$

In reference [17], the authors firstly proposed a decentralized state-space PD-type ILC scheme (17) for the large-scale system (16) with  $D^{(i)}(t) = 0$  which uses an inverse model of each subsystems but no information of other subsystems. Under the assumption that the error is measured in the form of  $\lambda$ -norm and the initial state is resettable together with the desired control input is reachable, it is proved that the input error  $\delta \mathbf{u}_k^{(i)} = \mathbf{u}_d^{(i)}(t) - \mathbf{u}_k^{(i)}(t)$  between the desired input  $\mathbf{u}_d^{(i)}(t)$  and the  $k$ th iteration-wise input  $\mathbf{u}_k^{(i)}(t)$  converges to zero as the iteration number  $k$  tends to infinity. Besides, the reference proposed decentralized input-output-based P-type ILC rule for the system (16) and derived that the transmission term  $D^{(i)}(t)$  plays a crucial role in guaranteeing the convergence property. Similar to the early convergence analysis in the sense of  $\lambda$ -norm, the sufficient convergence condition does not rely on the system state parameters, neither the interactions among the subsystems. The further work including the ILC structure and the derivation technique of the convergence analysis is analogous to the reference [17] except the tracking error measure is a relaxed  $\lambda$ -norm-like mode<sup>[18]</sup>. Because the decentralized large-scale systems description of (16) can be rewritten as a

general LTI system model in a whole point of view, the result is no other than the obtained one for general LTI system.

For many large-scale industrial processes, such as petrochemical process, electrical power plants and metallurgical process, the whole process is usually designed to operate continuously under some optimal operating conditions. The operation may, however, deviate away from the normal operating set-point due to changing of the properties of raw materials or environmental conditions including aging of catalyst and wearing of some equipment. This implies that the steady-state optimization for control decision, that is, set-point, should be updated so as to minimize the energy cost, save raw materials and thus enhance the efficiency. A typical steady-state optimization scheme is based on a two-layer hierarchical structure, whose supreme layer is a coordinator whilst the lower layer is a set of local decision-making units.

Due to the unavoidable discrepancy between the mathematical model and the reality, the realistic steady-state information is required to be fed back to the coordinator for model modification so as to improve the model-based optimum. It is interesting to notice that conventional PID-type controlled process usually operates in a slow response mode with a long settling time or in a rapid response with an oscillatory overshoot. To improve the dynamic transient performance such as decreasing the overshoot, accelerating the transient response and shortening the settling time of such kind of magnitude-iteration-varying industrial processes, a decentralized magnitude-varying PD-type iterative learning control strategy is developed in reference [19], in which the distinct magnitudes of the step set-point changes sequence of each subsystem are considered by introducing some appropriate amplified coefficients both in the updating law and in the selection of the desired reference trajectories. In particular, the convergence analysis involving the proposed iterative learning control algorithm is conducted in the sense of Lebesgue- $p$  norm by adopting generalized Young inequality of convolution integral. The conclusion shows that not only the system input-output matrices and the derivative learning gain but also the system state matrix and the proportional learning gain dominate the convergence. In addition, owing to the magnitude-varying property and the decentralized ILC mode, the asymptotical tracking error exists unless the large-scale system is decoupling or the whole set-point vector is proportional. The investigation adopts Lebesgue- $p$  norm and thus the result covers the existing result in the sense of  $\lambda$ -norm for repetitive unique desired trajectory tracking.

## 5 Prospect for long-term learning control

Human is a very complex large-scale system, whose characteristics are high-dimensionality, com-

plexity, subjectivity, ambiguity, inconsistency and non-stationary, etc. For a human-friendly welfare intelligent machine such as a service robot for aged or disabled user, the robot must identify and understand the user's commands with emotional information such as facial expressions, voice tone or hand gesture, or by physical information such as electromyography, electrocardiogram or electroencephalography, or by behavioral information such as posture, gait patterns and gestures. It is of no doubt that a mathematical model of the human system is quite difficult to use for realistic control of human-in-the-loop systems such as a smart home. This means that some forms of intelligence are needed for the counterpart of human such as a robot to have proper interaction with human. It would be desirable to fuse the above-mentioned information by some logical intent. Heuristically, it is observed that life-long learning capability for a service robot is essential to co-exist, cooperate to serve well for human in the long run. The life-long learning/adaptive capability is noted essential for the intelligent service robotic system to be human-friendly. The life-long learning, also termed as continuous learning, emphasizes learning through the entire lifespan of a system<sup>[54]</sup>. Here, we say 'life-long learning' as the repetitively inductive learning process by monitoring plus the deductive learning process by control and feedback. Besides, by virtue of the science and technology in incessant progress, robotic clone is possible in our future life. As a matter of fact that human and his/her human (animal) clone are the same at the beginning but may become ever-different as the time elapses, whilst human and his/her robotic clone are different at the beginning but must become more and more similar as the time passes. It is hoped that, as time goes by, a robot can become a mechanical clone that resembles human-self by life-long learning/teaching and thus can serve human better. The learning mode will be an incessant interaction between the target and the learning agent. The learning process will be multi-layer, that is, learning would take place from physical layer to mental layer then emotional layer and so on. Life-long learning is also useful for knowledge discovery in database, which is the nontrivial process of identifying valid, novel, potentially useful and ultimately understandable patterns in data<sup>[55]</sup>. Up to date, the gap between the people's expectation and the current level technology is too big. It is believable that the gap can be shorten by advancing multidisciplinary technologies such as material technology to renew battery and sensor, biological technology to update the understanding of implementable brain as well as information technology to synthesize the complex and time-varying information along with life-long learning control techniques. In short, learning will be a key notion for future robotic systems and the notion of the existing learning algo-

rithms such as ILC must go through a paradigm change.

## 6 Conclusions

This paper reviews some ILC techniques retrospectively, including convergence analysis, initial state shift issue, higher-order scheme as well as the structure for large-scale systems. The paper also gives a brief prospective investigation trend for long-term learning control. The review mainly shows the investigation stages and focuses of those techniques. It is observed that the investigative progress has been achieved in a long-term time consuming manner. It of course needs a great deal effort and devotion as well as passion. Sometimes frustration or mistakes are unavoidable. However, some topics need to be considered in a further deep level. For instance, in terms of a realistic control system, its dynamics can be mathematically modeled as either a continuous-time system by an ordinary differential equation or a discrete-time system by an ordinary difference equation. This implies that those two forms of system description must be equivalent in the sense that the increment of the state variable with respect to time variable at two adjacent time points is equivalent to the differentiation of the state variable over the subinterval confined by the two adjacent time points. But, in the authors' knowledge, the convergence results with respect to the two dynamical forms seem not to be in accordance with each other obviously. This requires a rigorous elaboration.

Further, with the development of computer science and internet technology, it is inevitable that the prominently-existing ILC techniques will be implemented by means of network to share the resource and minimize the cost and so on. In this circumstance, the networked ILC strategy must seek the manners to deal with the internet induced time delay and data dropout problem. This would be one of challenging issues for future research.

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