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Sampled-data extended state observer for uncertain nonlinear systems

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Abstract

In this paper, we present a sampled-data nonlinear extended state observer (NLESO) design method for a class of nonlinear systems with uncertainties and discrete time output measurement. To accommodate the inter-sample dynamics, an inter-sample output predictor is employed in the structure of the NLESO to estimate the system output in the sampling intervals, where the prediction is used in the proposed observer instead of the system output. The exponential convergence of the sampled-data NLESO is also discussed and a sufficient condition is given by the Lyapunov method. A numerical example is provided to illustrate the performance of the proposed observer.

Keywords: Sampled-data, extended state observer, nonlinear systems

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1 Introduction

The existence of various disturbances and model uncertainties poses major challenges in the design of control systems, where the situation is further complicated in sophisticated control applications with demanding performance requirements such as aerospace industries and modern precision industries. There are abundant research results addressing challenging problems on anti-disturbance control of systems with parametric variations, unmodeled dynamics and external disturbances. The readers can be referred to [1] and references therein for recent advances in control techniques for disturbance/uncertainty estimation and attenuation.

One major category in anti-disturbance control is disturbance observer based control (DOBC) approach,

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where disturbance observers are introduced to estimate and compensate the uncertainties and disturbances. Based on the observation mechanism, disturbance observers in both frequency domain [2,3] and time domain [4, 5] are proposed in the literature. Alternatively, the method of active disturbance rejection control (ADRC) ([6, 7]) is also well discussed, where successful industry applications have been achieved such as MEMS gyroscopes [8], robotics [9] and high precision motion control [10]. As a key part of the ADRC control structure, the extended state observer (ESO) is developed to estimate uncertainties and disturbances simultaneously by lumping them into an an extended state as the "total disturbance" for disturbance elimination purposes.

In recent years, theoretical analysis on the linear ESO (LESO) and the nonlinear ESO (NLESO) has attracted more and more research efforts as well. In [8] and [11], the convergence of LESO is given analytically. Based on time-varying PD-eigenvalues assignment and Kalman filter algorithms respectively, adaptive extended state observers (AESO) in the form of LESO are presented to improve the performance of ESO and cancel the peaking phenomenon [12, 13]. To have more design flexibility for complicated systems, [12], nonlinear extended state observer (NLESO) design and analysis are also discussed for both single-input-single-output (SISO) systems [14], and multiple-input-multiple-output (MIMO) cases [15], as well as uncertain lower triangular nonlinear systems [16]. Furthermore, the NLESO method is improved by replacing constant observer gains with time-varying gains in [17].

Although most of the results stated above, especially for NLESO, are based on continuous time, the ESO typically needs to be implemented in discrete-time in various computer-based control applications. The digital implementations of ESO have also received considerable attention, e.g., different discrete approximation methods [18], the relationship between sampling periods and control bandwidth for LESO [19] and incremental algorithm design [20]. In a similar fashion, the discrete form of NLESO is discussed in [7]. Note that most of the existing works stated above are designed based on a direct discretization of the plant models. However, many complications in sampled-data systems can not be fully addressed by the design methods discussed above, e.g., the existence of perturbations of sampling schedule [21], or control systems with multiple sampling rates [22].

It is noticed that a continuous-discrete observer method was discussed in [21], where an inter-sample output predictor was used to predict the inter-sample dynamics in sampled-data nonlinear observer design. This design has a hybrid structure because the states are estimated in continuous time and the predictor is updated discontinuously only at the sampling time to correct the estimated state trajectory. Such sampled-data observer design has been widely investigated recently, e.g., sampled-data high gain observers for networked control systems [23] where sampling periods are nonuniform, and sampled-data extended high gain observers with multi-rate control applications in electro-hydraulic actuator systems [22].

Inspired by this line of research on continuous-discrete observer design, we propose a sampled-data extended state observer design with nonlinear gain function, where the convergence of the proposed observer is analyzed by a Lyapunov function based method. Meanwhile, the relationship between the observer error bound and the observer gain parameters is also derived. The present work is an extension of [14], which is capable of generating continuous state estimation based on sampled-data system measurement. For applications requiring multiple sampling rates, the proposed design offers the opportunity to compensating high frequency disturbances using an up-sampling compensator, while using the original sampling rate for the rest of control system. For example, when the proposed ESO is employed in an multi-rate ADRC control framework as illustrated in Fig.1, the inter-sample information (by prediction) can be used to handle high frequency disturbances by up-sampling the observer output, while the feedback controller is still running in the original sampling rate to handle low frequency disturbances. Multi-rate control systems have been successfully implemented in various industry applications such as Hard Disk Drive servo systems [24].

The rest of the paper is organized as follows: In Section 2, some definitions and notations which will be used in this paper are given. The system and continuous time ESO formulation are described in Section 3. The sampled-data nonlinear extended state observer for nonlinear uncertain systems with sampled measurements is proposed in Section 4, where the exponential convergence of the observer is also given by using Lyapunov approach. In Section 5, simulation results are presented to show the effectiveness of the design, followed by conclusions in Section 6.

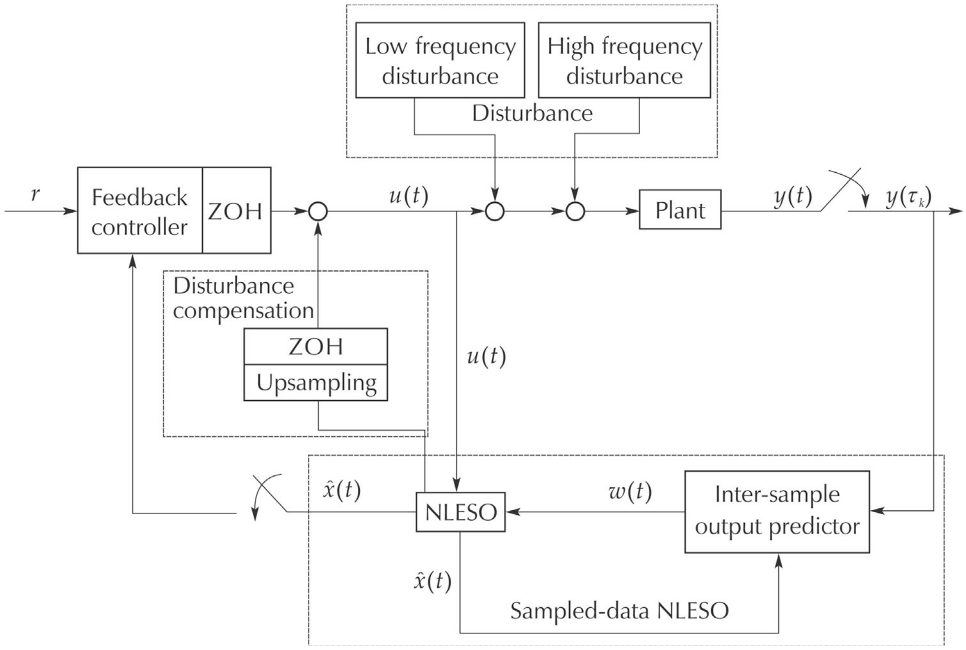


Fig. 1 The diagram of sampled-data control system with NLESO.

2 Notations and definitions

In this section, some mathematical notations used in this paper are introduced. $\mathbb{R} = (-\infty, +\infty)$ is the set of real numbers and $\mathbb{R}^n$ denotes the set of real vectors of n-dimension. C is the continuous function. $\|\cdot\|$ presents the euclidian norm on $\mathbb{R}^n$. Some countable set of time instants is denoted by $\pi = \{\tau_k\}_{k=0}^{\infty}$ and satisfies $0 < r = (\tau_{k+1} - \tau_k) < r_{\max}$ where r is the sampling period and $r_{\max}$ is the upper diameter of the sampling partition.

3 Problem formulation

Consider an n-dimensional SISO nonlinear system

$$y^{(n)} = f(t, y, \dot{y}, \dots, y^{(n-1)}) + bu + d, \tag{1}$$

where $y \in \mathbb{R}$ is the system output, $f \in C(\mathbb{R}^n, \mathbb{R})$ represents a possibly unknown nonlinear dynamics of the system, $u \in C(\mathbb{R}, \mathbb{R})$ is the input, b is a given constant, and $d \in C(\mathbb{R}, \mathbb{R})$ is the external disturbance. Then system (1) can be presented in the following form:

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \quad\vdots \\ \dot{x}_n(t) = x_{n+1}(t) + bu(t), \\ \dot{x}_{n+1}(t) = h, \\ y(t) = x_1(t), \end{cases} \tag{2}$$

where $f + d$, the total disturbance, is set as an extended state of the system, and let $h = \dot{f} + \dot{d}$, $x = [x_1 \ x_2 \ \cdots \ x_n]^{\mathrm{T}} \in \mathbb{R}^n$ is the state of the system, whose initial values can be set as $x_i(t_0)$ for $i = 1, 2, \dots, n$. Inspired by the work of [14], a nonlinear extended state observer can be designed for system (2),

$$\begin{cases} \dot{\hat{x}}_1(t) = \hat{x}_2(t) + \varepsilon^{n-1} g_2\left(\dfrac{y(t) - \hat{x}_1(t)}{\varepsilon^n}\right), \\ \dot{\hat{x}}_2(t) = \hat{x}_3(t) + \varepsilon^{n-1} g_3\left(\dfrac{y(t) - \hat{x}_1(t)}{\varepsilon^n}\right), \\ \quad\vdots \\ \dot{\hat{x}}_n(t) = \hat{x}_{n+1}(t) + bu(t) + g_n\left(\dfrac{y(t) - \hat{x}_1(t)}{\varepsilon^n}\right), \\ \dot{\hat{x}}_{n+1}(t) = \dfrac{1}{\varepsilon} g_{n+1}\left(\dfrac{y(t) - \hat{x}_1(t)}{\varepsilon^n}\right). \end{cases} \tag{3}$$

The above observer is a special form of ESO proposed in [6], where $\hat{x} = [\hat{x}_1 \ \cdots \ \hat{x}_{n+1}] \in \mathbb{R}^{n+1}$ is the estimated state of ESO, the initial condition can be set as $\hat{x}_i(t_0)$, the gain ε describes a small positive constant, $g_i, i = 1, 2, \dots, n+1$ denote chosen nonlinear functions. According to [14], the error dynamics of the ESO are expected to exponentially converge to 0, namely $\hat{x} - x \to 0$, as $\varepsilon \to 0$ and $t \to \infty$.

Although the above plant system, as well as the ESO design, can be discretized directly for computer-based digital implementations, some complications (e.g., nonuniform sampling systems, or multi-rate sampling systems) cannot be well addressed. Recall the multi-rate ADRC control architecture depicted in Fig. 1, we would like to investigate the sampled-date NLESO design based on discrete time system output and generate continuous observer output, such that the ESO output can be up-sampled to handle high frequency disturbances out of the control bandwidth using original sample rate. For such purposes, we would like to investigate the sampled-data case of NLESO (3) by applying the continuous-discrete observer design technique similar to [21].

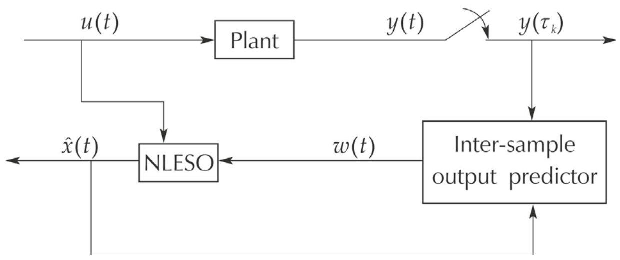


Fig. 2 The diagram of sampled-data system with NLESO.

4 Sampled-data NLESO design

In this section, we consider the sampled-data NLESO design problem. As depicted in the block diagram in Fig. 1, the sampled-data NLESO is composed of an inter-sample output predictor and a NLESO, where the measurement of the system output is only available at each sampling time. Besides the control input u, the prediction of output w is the other input of the NLESO instead of the real system output y, and some information of state estimations is used in the inter-sample output predictor. The observer is designed in continuous time and the states of the observer can be potentially sampled digitally, even with a sampling time different from that of the output measurement when discretization, thus facilitates multi-rate control system design.

First, we consider system (1) with the sampled-data output measurement as

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \quad \vdots \\ \dot{x}_n(t) = x_{n+1}(t) + bu(t), \\ \dot{x}_{n+1}(t) = h, \\ y(\tau_k) = x_1(\tau_k), \end{cases} \tag{4}$$

where the output data can be measured at each sampling time τ_k. Then a sampled-data NLESO with output predictor can be designed as

$$\begin{cases} \dot{\hat{x}}_1(t) = \hat{x}_2(t) + \varepsilon^{n-1} g_1\big(\dfrac{w(t) - \hat{x}_1(t)}{\varepsilon^n}\big), \\ \dot{\hat{x}}_2(t) = \hat{x}_3(t) + \varepsilon^{n-2} g_2\big(\dfrac{w(t) - \hat{x}_1(t)}{\varepsilon^n}\big), \\ \quad \vdots \\ \dot{\hat{x}}_n(t) = \hat{x}_{n+1}(t) + bu(t) + g_n\big(\dfrac{w(t) - \hat{x}_1(t)}{\varepsilon^n}\big), \\ \dot{\hat{x}}_{n+1}(t) = \dfrac{1}{\varepsilon} g_{n+1}\big(\dfrac{w(t) - \hat{x}_1(t)}{\varepsilon^n}\big), \end{cases} \tag{5}$$

where $\hat{x}$ denotes contious-time estimate state of x. $w(t)$ is the prediction of output y between two consecutive sampling instants, which is updated at the start of each sampling interval. Moreover, the output predictor for the time interval between two consecutive measurements can be shown as

$$\begin{cases} \dot{w}(t) = \hat{x}_2(t), \quad t \in [\tau_k, \tau_{k+1}), \\ w(\tau_{k+1}) = y(\tau_{k+1}), \end{cases} \tag{6}$$

Then, according to systems (4) and (5) we set

$$\begin{cases} e_i(t) = x_i(t) - \hat{x}_i(t), \quad i = 1, 2, \dots, n+1, \\ \eta_i(t) = \dfrac{e_i(\varepsilon t)}{\varepsilon^{n+1-i}}, \\ e_w(t) = w(t) - y(t), \\ \varphi(t) = \dfrac{e_w(\varepsilon t)}{\varepsilon^n}, \quad t \in [\tau_k, \tau_{k+1}). \end{cases} \tag{7}$$

Note that $w(t) - \hat{x}(t) = e_w(t) + e_1(t)$ in each sampling interval. Thus the error equation of system (4) can be obtained as

$$\begin{cases} \dot{\eta}_1(t) = \eta_2(t) - g_1(\eta_1(t) + \varphi(t)), \\ \dot{\eta}_2(t) = \eta_3(t) - g_2(\eta_1(t) + \varphi(t)), \\ \quad \vdots \\ \dot{\eta}_n(t) = \eta_{n+1}(t) - g_n(\eta_1(t) + \varphi(t)), \\ \dot{\eta}_{n+1}(t) = \varepsilon \vartheta(t) - g_{n+1}(\eta_1(t) + \varphi(t)), \\ \dot{\varphi}(t) = \dfrac{\hat{x}_2 - x_2}{\varepsilon^n}, \quad t \in (\tau_k, \tau_{k+1}), \end{cases} \tag{8}$$

where

$$\vartheta(t) = \frac{\mathrm{d}}{\mathrm{d}t}(f(\varepsilon t, x_1(\varepsilon t), \dots, x_n(\varepsilon t)) + d(\varepsilon t)). \tag{9}$$

In what follows, we will give the main results of this paper. First of all, some assumptions are made for the sampled-data NLESO.

Assumption 1 The unknown functions f, d are continuously differentiable with respect to their variables, for some positive constants $c_j, j = 0, 1, \dots, n$ and positive integer q, such that

$$|u| + |f| + |\dot{d}| + |\frac{\partial f}{\partial t}| + |\frac{\partial f}{\partial x}| \leqslant c_0 + \sum_{j=1}^{n} c_j |x_j|^q. \quad (10)$$

Assumption 2 The solutions x_i to system (2) and disturbance d satisfy $|d| + |x_i(t)| \leqslant m_1$ for some constant $m_1 > 0; i = 1, 2, \dots, n$ and $t \geqslant 0$.

Assumption 3 For $\forall \eta = [\eta_1 \ \ \eta_2 \ \cdots \ \eta_{n+1}]^{\mathrm{T}} \in \mathbb{R}^{n+1}$, there exist constants λ_i, for $i = 1, \dots, 4$ and positive definite radially unbounded and continuous differentiable functions $V_1, W_1: \mathbb{R}^{n+1} \to \mathbb{R}$ such that

i) $\lambda_1 \|\eta\|^2 \leqslant V_1(\eta) \leqslant \lambda_2 \|\eta\|^2$,
$\lambda_3 \|\eta\|^2 \leqslant W_1(\eta) \leqslant \lambda_4 \|\eta\|^2$,

ii) $\sum\limits_{i=1}^{n} \dfrac{\partial V_1}{\partial \eta_i}(\eta_{i+1} - g_i(\eta_1)) - \dfrac{\partial V_1}{\partial \eta_{n+1}} g_{n+1}(\eta_1) \leqslant -W_1(\eta)$,

iii) $|\dfrac{\partial V_1}{\partial \eta_{n+1}}| \leqslant \beta \|\eta\|$, $|\sum\limits_{i=1}^{n+1} \dfrac{\partial V_1}{\partial \eta_{n+1}}| \leqslant \alpha \|\eta\|$.

Assumption 4 Functions $g_i(\cdot) \in C(\mathbb{R}, \mathbb{R})$ are globally Lipschitz on a compact set ζ of z, namely for $(z_1, z_2) \in \mathbb{R} \times \mathbb{R}$, there exists $\gamma > 0$ such that

$$|g_i(z_1) - g_i(z_2)| \leqslant \gamma |z_1 - z_2|. \quad (11)$$

Theorem 1 Consider the sampled-data system (4). If Assumptions 1–4 hold, then the states of the sampled-data NLESO (5) exponentially converge to the states and extended state of system (4), namely for $\sigma > 0$, there exists a sufficiently small ε, a ε-dependent T and a positive bounded $r_{\max}$ such that

$$\begin{aligned} &|\hat{x}_i(t) - x_i(t)| \leqslant \sigma, \quad i = 1, 2, \dots, n+1, \\ &t > T, t \in [\tau_k, \tau_k + r), \quad r \in (0, r_{\max}]. \end{aligned} \quad (12)$$

Proof Under Assumptions 1 and 2 and the dynamic of extended state (9), there exists a constant $M > 0$, such that $|\vartheta(t)| \leqslant M$.

Inspired by [23], we consider the following candidate Lyapunov function:

$$\begin{aligned} U(t) &= V_1(\eta(t)) + V_2(t) \\ &= V_1(\eta(t)) + \theta \kappa(t) |\varphi(t)|^2, \end{aligned} \quad (13)$$

where we introduce an additional term $V_2(t)$ with respect to the output predictor, θ is a positive constant which can be computed as follows and $\kappa(t)$ is a positive and bounded function. This function satisfies the following conditions:

$$\begin{cases} \kappa(\tau_k) = \mu, & k \in \mathbb{N}^+, \ \ \mu > 1, \\ \dot{\kappa}(t) < 0, & t \in [\tau_k, \tau_{k+1}), \\ \kappa(\tau_k + r) = \mu^{-1}, & r \in (0, r_{\max}]. \end{cases} \quad (14)$$

First, under Assumptions 3 and 4, we consider the time derivative of $V_1(\eta(t))$ along the solution $\eta(t)$ to system (4), and obtain

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}t} V_1(\eta(t)) \\ &= \sum_{i=1}^{n} \frac{\partial V_1}{\partial \eta_i} [\eta_{i+1}(t) - g_i(\frac{w(\varepsilon t) - \hat{x}_1(\varepsilon t)}{\varepsilon^n})] \\ &\quad - \frac{\partial V_1}{\partial \eta_{n+1}} g_{n+1}(\frac{w(\varepsilon t) - \hat{x}_1(\varepsilon t)}{\varepsilon^n}) + \frac{\partial V_1}{\partial \eta_{n+1}} \varepsilon \vartheta(t) \\ &= \sum_{i=1}^{n} \frac{\partial V_1}{\partial \eta_i} [\eta_{i+1} - g_i(\eta_1 + \varphi)] - \frac{\partial V_1}{\partial \eta_{n+1}} g_{n+1}(\eta_1 + \varphi) \\ &\quad + \frac{\partial V_1}{\partial \eta_{n+1}} \varepsilon \vartheta \\ &\leqslant \sum_{i=1}^{n} \frac{\partial V_1}{\partial \eta_i} (\eta_{i+1} - g_i(\eta_1)) - \frac{\partial V_1}{\partial \eta_{n+1}} g_{n+1}(\eta_1) \\ &\quad + \sum_{i=1}^{n+1} \frac{\partial V_1}{\partial \eta_i} \gamma |\varphi| + \frac{\partial V_1}{\partial \eta_{n+1}} \varepsilon \vartheta \\ &\leqslant -W(\eta) + \alpha \gamma |\varphi| \, \|\eta\| + \varepsilon M \beta \, \|\eta\| \\ &\leqslant -\lambda_3 \|\eta\|^2 + \alpha \gamma |\varphi| \, \|\eta\| + \varepsilon M \beta \, \|\eta\|. \end{aligned} \quad (15)$$

The following bound regarded to (15) can be derived by recalling the Young inequality:

$$\left| \gamma \|\eta\| \, |\varphi| \alpha \right| \leqslant \frac{1}{2}(\alpha^2 \|\eta\|^2 + \gamma^2 |\varphi|^2). \quad (16)$$

Hence (15) can be written as

$$\frac{\mathrm{d}}{\mathrm{d}t} V_1(\eta(t)) \leqslant (-\lambda_3 + \frac{1}{2}\alpha^2) \|\eta\|^2 + \varepsilon M \beta \, \|\eta\| + \frac{1}{2} \gamma^2 |\varphi|^2. \quad (17)$$

Then we obtain the time derivative of $V_2(t)$ as

$$\begin{aligned} \dot{V}_2 &= \theta \dot{\kappa} |\varphi|^2 + 2\theta \kappa |\varphi| |\dot{\varphi}| \\ &\leqslant \theta \dot{\kappa} |\varphi|^2 + 2\theta \kappa \frac{1}{\varepsilon} |\varphi| \|\eta\| \\ &\leqslant \theta \dot{\kappa} |\varphi|^2 + \theta^2 \kappa^2 |\varphi|^2 + \frac{1}{\varepsilon^2} \|\eta\|^2. \end{aligned} \quad (18)$$

Note that

$$|\dot{\varphi}| = |\frac{\dot{e}_w}{\varepsilon^n}| = \frac{1}{\varepsilon} |\eta_2| \leqslant \frac{1}{\varepsilon} \|\eta\|. \quad (19)$$

Combining (17) with (18), we can obtain

$$\begin{aligned}\dot{U} &= \dot{V}_1 + \dot{V}_2 \\ &\leqslant -(\lambda_3 - \frac{1}{2}\alpha^2 - \frac{1}{\varepsilon^2})\|\eta\|^2 + \varepsilon M\beta\,\|\eta\| \\ &\quad + (\frac{1}{2}\gamma^2 + \theta\dot{\kappa} + \theta^2\kappa^2)|\varphi|^2. \end{aligned} \tag{20}$$

We can choose

$$\dot{\kappa}(t) = -\theta(\kappa^2(t) + 1),\ \ t \in [\tau_k, \tau_k + r),\ r \in (0, r_{\max}]. \tag{21}$$

Then we have

$$\dot{U} \leqslant -(\lambda_3 - \frac{1}{2}\alpha^2 - \frac{1}{\varepsilon^2})\|\eta\|^2 + \varepsilon M\beta\,\|\eta\| + (\frac{1}{2}\gamma^2 - \theta)|\varphi|^2. \tag{22}$$

Choosing

$$\begin{cases}\theta = \sqrt{\dfrac{1}{2}\gamma^2}, \\ \lambda_3 = \dfrac{\alpha^2/2 + 1/\varepsilon^2}{1-\delta},\ \ \delta \in (0,1).\end{cases} \tag{23}$$

Thus, using Assumption 3, we derive

$$\begin{aligned}\dot{U} &\leqslant -\lambda_3\delta\|\eta\|^2 + \varepsilon M\beta\,\|\eta\| \\ &\leqslant -\frac{\lambda_3\delta}{\lambda_2}V_1 + \frac{\varepsilon M\beta}{\sqrt{\lambda_1}}\sqrt{V_1} \\ &\leqslant -\frac{\lambda_3\delta}{\lambda_2}U + \frac{\varepsilon M\beta}{\sqrt{\lambda_1}}\sqrt{U}. \end{aligned} \tag{24}$$

By Assumption 3 again, integrating (24) on the interval $[\tau_k, t]$ yields

$$U(t) \leqslant U(\tau_k)\mathrm{e}^{-\frac{\lambda_3\delta}{\lambda_2}(t-\tau_k)} + (\frac{\varepsilon M\beta\lambda_2}{\sqrt{\lambda_1}\lambda_3\delta})^2. \tag{25}$$

Thus, we have

$$\sqrt{U(t)} \leqslant \sqrt{U(\tau_k)}\mathrm{e}^{-\frac{\lambda_3\delta}{2\lambda_2}(t-\tau_k)} + \frac{\varepsilon M\beta\lambda_2}{\sqrt{\lambda_1}\lambda_3\delta}. \tag{26}$$

According to Assumption 3, we obtain

$$\begin{aligned}\|\eta(t)\| &\leqslant \sqrt{\frac{V(\eta(t))}{\lambda_1}} \\ &\leqslant \sqrt{\frac{U(\tau_k)}{\lambda_1}}\mathrm{e}^{-\frac{\lambda_3\delta}{2\lambda_2}(t-\tau_k)} + \frac{\varepsilon M\beta\lambda_2}{\lambda_1\lambda_3\delta},\ \ t \in [\tau_k, \tau_k + r). \end{aligned} \tag{27}$$

Consider (12)–(14) and the fact that $\varphi(\tau_k) = 0$, $\eta(\tau_k) = \eta(\tau_k^-)$ at the time instant $t = \tau_k$, then we will have

$$\begin{aligned}U(\tau_k) &= V_1(\eta(\tau_k)) + \theta\kappa(\tau_k)|\varphi(\tau_k)|^2 \\ &= V_1(\eta(\tau_k)) + \theta\mu|\nu(\tau_k)|^2 \\ &\leqslant V_1(\eta(\tau_k^-)) + \theta\kappa(\tau_k^-)|\varphi(\tau_k^-)|^2 \leqslant U(\tau_k^-). \end{aligned} \tag{28}$$

Then, we have

$$U(\tau_k) \leqslant U(\tau_{k-1})\mathrm{e}^{-\frac{\lambda_3\delta}{\lambda_2}(\tau_k^- - \tau_{k-1})} + (\frac{\varepsilon M\beta\lambda_2}{\sqrt{\lambda_1}\lambda_3\delta})^2. \tag{29}$$

$$U(t) \leqslant U(\tau_{k-1})\mathrm{e}^{-\frac{\lambda_3\delta}{\lambda_2}(t-\tau_{k-1})} + (\frac{\varepsilon M\beta\lambda_2}{\sqrt{\lambda_1}\lambda_3\delta})^2(1 + \mathrm{e}^{-\frac{\lambda_3\delta}{\lambda_2}(t-\tau_k)}). \tag{30}$$

Thus, we obtain

$$\sqrt{U(t)} \leqslant \sqrt{U(t_0)}\mathrm{e}^{-\frac{\lambda_3\delta}{2\lambda_2}(t-t_0)} + \Delta, \tag{31}$$

where $\Delta = \dfrac{\varepsilon M\beta\lambda_2}{\sqrt{\lambda_1}\lambda_3\delta}\sqrt{1 + \sum\limits_{i=1}^{k}\mathrm{e}^{-\frac{\lambda_3\delta}{\lambda_2}(t-\tau_i)}}$ is bounded.

Thus, we can write (27) as

$$\|\eta(t)\| \leqslant \sqrt{\frac{U(t_0)}{\lambda_1}}\mathrm{e}^{-\frac{\lambda_3\delta}{2\lambda_2}(t-t_0)} + \frac{\Delta}{\sqrt{\lambda_1}},\ \ t \in [\tau_k, \tau_k + r), \tag{32}$$

which indicates, for $t \in [\tau_k, \tau_k + r)$

$$\begin{aligned}|e_i(t)| &= \varepsilon^{n+1-i}|\eta_i(\frac{t}{\varepsilon})| \\ &\leqslant \varepsilon^{n+1-i}(\sqrt{\frac{U(t_0)}{\lambda_1}}\mathrm{e}^{-\frac{\lambda_3\delta}{2\lambda_2}(t/\varepsilon - t_0)} + \frac{\Delta}{\sqrt{\lambda_1}}). \end{aligned} \tag{33}$$

It means that the observer error is ultimately bounded and we can choose ε small enough to reduce the bound of error. Moreover, the right hand side of (30) converges exponentially to 0, as $\varepsilon \to 0$. In addition, we can compute the value of $r_{\max}$ as

$$r_{\max} = \frac{1}{\theta}(\arctan(\mu) - \arctan(\mu^{-1})). \tag{34}$$

5 Numerical simulations

In this section, a numerical example is given to illustrate the effectiveness of the proposed observer. Inspired by [14] and [17], consider the following nonlinear system:

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = f(t,x) + u(t) + d(t), \\ y(\tau_k) = x_1(\tau_k). \end{cases} \tag{35}$$

We take the system input $u(t)$, external disturbance $d(t)$, and nonlinear function $f(t,x)$ in the above system respectively as

$$\begin{aligned} &u(t) = 1 + \sin t, \\ &d(t) = -0.6\cos(2t) + \sin(0.4\pi t + 1) - 0.8, \\ &f(t,x) = -x_1(t) - 2x_2(t) + \sin(x_1(t) + 2x_2(t)). \end{aligned}$$

By following the design procedure in the above section, We can design the following sampled-data nonlinear extended state observer

$$\begin{cases} \dot{\hat{x}}_1(t) = \hat{x}_2(t) + \dfrac{3}{\varepsilon}(w(t) - \hat{x}_1(t)) + \varepsilon\varrho(\dfrac{w(t) - \hat{x}_1}{\varepsilon^2}), \\ \dot{\hat{x}}_2(t) = \hat{x}_3(t) + u(t) + \dfrac{3}{\varepsilon^2}(w(t) - \hat{x}_1(t)), \\ \dot{\hat{x}}_3(t) = \dfrac{1}{\varepsilon^3}(w(t) - \hat{x}_1(t)); \\ w(t) = \hat{x}_2(t), \quad t \in [\tau_k, \tau_{k+1}), \\ w(\tau_{k+1}) = y(\tau_{k+1}), \end{cases} \tag{36}$$

where the nonlinear function $\varrho : \mathbb{R} \to \mathbb{R}$ is defined as

$$\varrho(\rho) = \begin{cases} -\dfrac{1}{5}, & \rho \in (-\infty, -\dfrac{\pi}{2}], \\ \dfrac{1}{5}\sin\rho, & \rho \in (-\dfrac{\pi}{2}, \dfrac{\pi}{2}], \\ \dfrac{1}{5}, & \rho \in (\dfrac{\pi}{2}, +\infty). \end{cases} \tag{37}$$

In this case, the global Lipschitz nonlinear functions g_i in the ESO in (5) can be specified as

$$g_1(z) = 3z + \varrho(z), \quad g_2(z) = 3z, \quad g_3(z) = z.$$

It is straightforward that

$$|g_i(z_1) - g_i(z_2)| \leqslant \frac{16}{5}|z_1 - z_2|. \tag{38}$$

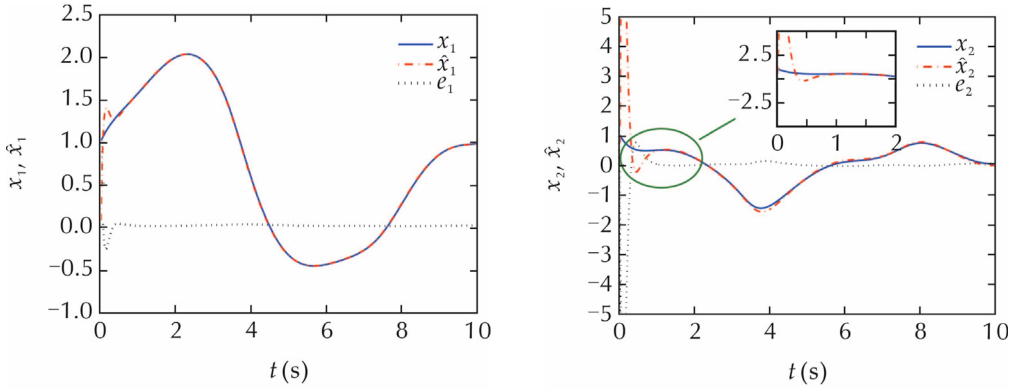
Note that Assumptions 1–4 are all satisfied. Thus, (36) is a well-defined sampled-data NLESO for system (35) according to Theorem 1. Now we can define the Lyapunov function as

$$\begin{aligned} U(t) &= V_1 + V_2 \\ &= (\eta^{\mathrm{T}} P \eta + \int_0^{\eta_1} \varrho(s)\mathrm{d}s) + \theta\kappa(t)|\varphi(t)|^2. \end{aligned} \tag{39}$$

The positive definite matrix P can be chosen according to [14], and the convergence can be guaranteed by the method presented in Section 4.

The initial states of the plant (30) and the ESO (31) are set as $(1,1)^{\mathrm{T}}$ and $(0,0,0)^{\mathrm{T}}$, respectively. The time step for calculation is 0.005 s, and the sampling time of the output measurement is $r = 0.01$ s.

We now take the gain parameter as $\varepsilon = 0.1$ and $\varepsilon = \dfrac{1}{30}$. As depicted in Fig. 3 and Fig. 4, it can be clearly seen that the states $\hat{x}_1, \hat{x}_2, \hat{x}_3$ of the designed sampled-data NLESO track the states of the plant and the extended state very well with high observer gain. Moreover, the tracking performance of the observer is further improved when the parameter ε decreases. In addition, the relationship between prediction w and the output y is also illustrated in Fig. 3 (d) and Fig. 4 (d).



(a) (b)

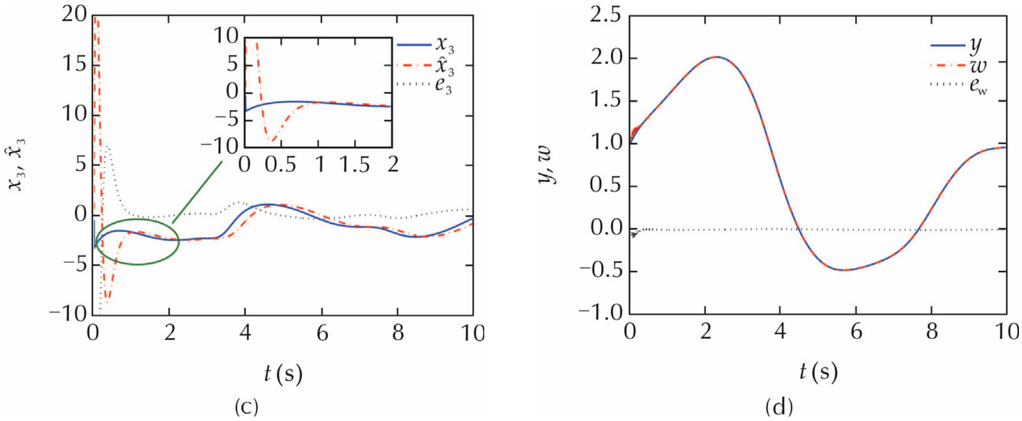


Fig. 3 Numerical simulations for system (32) by sampled-data NLESO with $\varepsilon = 0.1$. (a) x_1 and $\hat{x}_1$. (b) x_2 and $\hat{x}_2$. (c) x_3 and $\hat{x}_3$. (d) y and w.

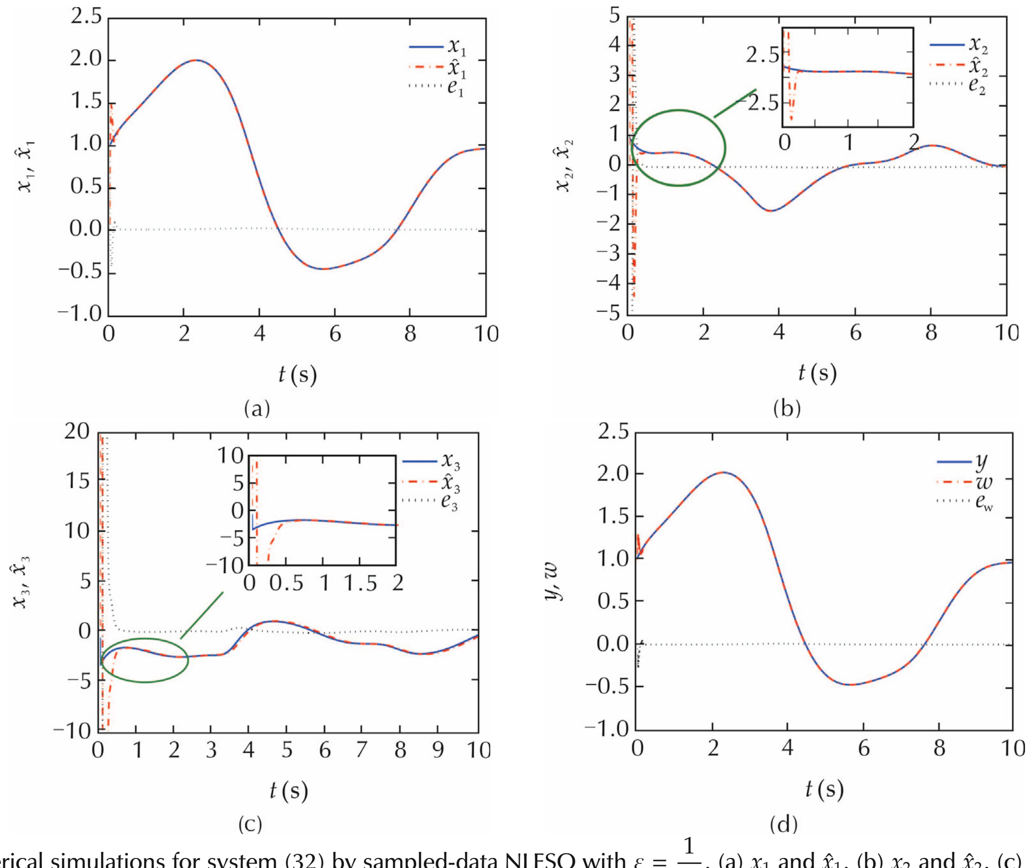


Fig. 4 Numerical simulations for system (32) by sampled-data NLESO with $\varepsilon = \frac{1}{30}$. (a) x_1 and $\hat{x}_1$. (b) x_2 and $\hat{x}_2$. (c) x_3 and $\hat{x}_3$. (d) y and w.

6 Conclusions

In this paper, a sampled-data nonlinear extended state observer for uncertain nonlinear systems subject to discrete time measurement was developed, where the inter-sample dynamics and sampling schedule were considered. The exponential convergence of the observer was analyzed by introducing a Lyapunov function chosen for hybrid systems. The relations between the observer error bound and the observer parameters were explicitly given. The numerical simulation results demonstrated the convergence of the proposed observer and inter-sample output predictor. Future works, along this line of research, include sampled-data ADRC or sampled-data output feedback control based on the proposed observer, as well as their industrial applications.

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