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An output-based distributed observer and its application to the cooperative linear output regulation problem

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Abstract

In this paper, we first extend an existing stability result for a class of linear switched systems. This extended result will relax the existence conditions of the output-based distributed observer for a leader system subject to jointly connected switching communication networks in the literature. As an application of this output-based distributed observer, we solve the cooperative output regulation problem of a linear multi-agent system subject to jointly connected switching communication networks by composing a purely decentralized control law and the output-based distributed observer based on the certainty equivalence principle.

Keywords: Output-based distributed observer, switched systems, cooperative output regulation

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1 Introduction

Nowadays, more and more complex engineering tasks are performed by coordinating the function of a group of individual subsystems. Typical tasks include formation of mobile robots [2, 3], attitude alignment of multiple spacecrafts [4, 5], mobile sensor area coverage [6, 7], and distributed control of electric power grids [8, 9]. These applications have led to a surge of interest in cooperative control of networked multi-agent systems. A specific feature for the control of a networked multi-agent system is that the control law to be designed has to satisfy the communication constraints imposed by the communication network. We call a control law that satisfies the communication constraints as a distributed control law.

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The distributed observer approach is one of the effective approaches to synthesizing a distributed control law. It was first proposed in [10] to solve the cooperative output regulation problem of a linear multi-agent system subject to static communication constraints, and then in [11] for handling the same problem subject to switching communication constraints. In contrast with the controller design for a single system, information sharing, or, what is the same, cooperation among different subsystems is essential in the design of the distributed control law for a networked multi-agent system. Thus, the core of the approach in [10] and [11] is the employment of the so-called distributed observer, which can provide, through cooperation, an estimation of the leader's state to each follower so that a distributed control law can be synthesized on the basis of a purely decentralized control law and the distributed observer. Such an approach to designing a distributed control law is known as the certainty equivalence principle. A distinguished feature of the distributed observer approach is that the approach reduces a seemingly intractable task of controlling a complex multi-agent system to a leader-following consensus problem of a special class of linear systems. As a result, it offers a systematic way for handling various cooperative control problems of leader-follower multi-agent systems over various communication networks. Other publications making use of the distributed observer can be found in, for example, [8], [9], [12], and [13].

The distributed observer proposed in [10] and [11] is state-based in the sense that the observer relies on the state of the leader system. In practice, it may happen that only the output rather than the state of the leader system is available. Thus, in this paper, we will further consider the design of a distributed observer subject to jointly connected switching communication networks that only uses the output information of the leader system. We call such a distributed observer an output-based distributed observer. For this purpose, we will first extend the stability result for a class of linear switched systems studied in [14] by relaxing the symmetry assumption of a switching matrix to Assumption 1, which includes some interesting non-symmetric cases. This result lends itself to a synthesis of an output-based distributed observer, subject to jointly connected switching communication networks which may not necessarily be undirected. As a direct application of this observer, we will further synthesize a distributed control law to solve the cooperative output regulation problem of a linear multi-agent system subject to jointly connected switching communication networks. It is noted that the cooperative output regulation problem of a linear multi-agent system subject to jointly connected and undirected switching communication networks was also studied in [15]. However, the system in this paper is somehow different from the one in [15] and the proof here is more straightforward and self-contained.

The rest of this paper is organized as follows. In Section 2, we establish a stability result for a class of linear switched systems. In Section 3, we present an output-based distributed observer for a leader system subject to jointly connected switching communication networks. In Section 4, we solve the cooperative output regulation problem of a linear multi-agent system subject to jointly connected switching communication networks via the output-based distributed observer approach. A numerical example is used to illustrate our design in Section 5, and the paper is closed in Section 6 with some concluding remarks.

Notation $\otimes$ denotes the Kronecker product of matrices. $\|x\|$ denotes the Euclidean norm of a vector x and $\|A\|$ denotes the induced norm of a matrix A by the Euclidean norm. $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ denote the maximum and the minimum eigenvalues of a symmetric matrix A, respectively. For $x_i \in \mathbb{R}^{n_i}, i = 1, \dots, m$, $\mathrm{col}(x_1, \dots, x_m) = [x_1^{\mathrm{T}} \cdots x_m^{\mathrm{T}}]^{\mathrm{T}}$. We call a time function $\sigma : [0, \infty) \mapsto \mathcal{P} = \{1, 2, \dots, \rho\}$, where ρ is some positive integer, a piecewise constant switching signal, if there exists a sequence $\{t_j, j = 0, 1, 2, \dots\}$ satisfying $t_0 = 0, t_{j+1} - t_j \geqslant \tau$ for some positive constant τ such that, for all $t \in [t_j, t_{j+1})$, $\sigma(t) = p$ for some $p \in \mathcal{P}$. $\mathcal{P}$ is called the switching index set, t_j is called the switching instant, and τ is called the dwell time.

2 An extended stability result

In this section, we study the stability property of the following class of linear switched systems:

$$\dot{x}(t) = \left(I_N \otimes S - H_{\sigma(t)} \otimes (LC)\right) x(t), \quad \sigma(t) \in \mathcal{P}, \tag{1}$$

where $x(t) \in \mathbb{R}^{Nn}$ is the state, $S \in \mathbb{R}^{n\times n}$ and $C \in \mathbb{R}^{m\times n}$ are constant matrices, and the pair (C, S) is observable; $L \in \mathbb{R}^{n\times m}$ is a gain matrix to be designed; $\sigma : [0, \infty) \mapsto \mathcal{P} = \{1, 2, \dots, \rho\}$ for some positive integer ρ, is a piecewise constant switching signal with switching instants $\{t_j : j = 0, 1, 2, \dots\}$ and dwell time $\tau > 0$; $H_{\sigma(t)} \in \mathbb{R}^{N\times N}$ is a switching matrix dictated by the switching signal

$\sigma(t)$.

The stability property of a system similar to system (1) was first studied in [14] assuming that $H_{\sigma(t)}$ is symmetric and positive semi-definite for all $t \geqslant 0$. Later, the symmetry assumption on $H_{\sigma(t)}$ was relaxed to the assumption [16, Assumption 3.2] that, for all $p = 1, \dots, \rho$, the matrices $H_p^{\mathrm{T}} + H_p - \hat{\mu} H_p^{\mathrm{T}} H_p$ are positive semi-definite for some $\hat{\mu} > 0$. It is clear that Assumption 3.2 of [16] is satisfied automatically if, for all $p = 1, \dots, \rho$, the matrices H_p are symmetric and positive semi-definite. In what follows, we will further relax Assumption 3.2 of [16] to the following one.

Assumption 1 There exist a $\hat{\mu} > 0$ and a symmetric and positive definite matrix Q such that, for all $p = 1, \dots, \rho$, the matrix $H_p^{\mathrm{T}} Q + Q H_p - \hat{\mu} H_p^{\mathrm{T}} Q H_p$ is positive semi-definite.

Also, like in [14], we assume the following one.

Assumption 2 The matrix S is neutrally stable, i.e., all the eigenvalues of S are semi-simple with zero real parts.

Remark 1 Assumption 1 is motivated from the recent result in [17], which studied the consensus problem of a discrete-time linear multi-agent system subject to directed and jointly connected switching communication networks. As pointed out in [17], what makes Assumption 1 interesting is that it can be satisfied in some cases where Assumption 3.2 of [16] is not satisfied. This point will be further illustrated with an example in Section 5.

Under Assumption 2, there exists a symmetric and positive definite matrix P such that

$$PS + S^{\mathrm{T}} P = 0. \tag{2}$$

Before establishing the stability property of system (1), we introduce the following result.

Lemma 1 (Lemma 4 of [14]) Given a strictly increasing sequence $\{t_j : j = 0, 1, 2, \dots\}$ with $t_0 = 0$ and $\lim\limits_{j \to \infty} t_j = \infty$. Consider the following system:

$$\dot{\psi}(t) = (M - N(t))\psi(t), \quad t \geqslant 0, \tag{3}$$

where M is a constant matrix, $N(t)$ is bounded over $[0, \infty)$ and is constant over each time interval $[t_j, t_{j+1})$, $j = 0, 1, 2, \dots$. Suppose the solution $\psi(t)$ of system (3) is bounded over $[0, \infty)$ and satisfies

$$\lim_{t \to \infty} N(t)\psi(t) = 0, \quad \lim_{t \to \infty} G\psi(t) = 0$$

for some constant matrix G. Then

$$\lim_{t \to \infty} G M^k \psi(t) = 0, \quad k = 0, 1, 2, \dots.$$

The following result can be viewed as an extension of the partial result of Theorem 1 in [14].

Lemma 2 Under Assumptions 1 and 2, suppose there exists a subsequence $\{j_k : k = 0, 1, 2, \dots\}$ of $\{j : j = 0, 1, 2, \dots\}$ with $t_{j_{k+1}} - t_{j_k} < T$ for some $T > 0$, such that the matrix $(\sum\limits_{r=j_k}^{j_{k+1}-1} H_{\sigma(t_r)})$ is nonsingular. Then, with $L = P^{-1} C^{\mathrm{T}}$, the origin of system (1) is asymptotically stable.

Proof First note that (2) implies that

$$P^{1/2} S P^{-1/2} + (P^{1/2} S P^{-1/2})^{\mathrm{T}} = 0.$$

Then, with $L = P^{-1} C^{\mathrm{T}}$ and $\xi(t) = (I_N \otimes P^{1/2}) x(t)$, system (1) is equivalent to

$$\dot{\xi}(t) = \Big(I_N \otimes \bar{S} - H_{\sigma(t)} \otimes (\bar{C}^{\mathrm{T}} \bar{C})\Big)\xi(t), \quad \sigma(t) \in \mathcal{P}, \tag{4}$$

where $\bar{S} = P^{1/2} S P^{-1/2}$ and $\bar{C} = C P^{-1/2}$.

Let

$$V(\xi(t)) = \xi^{\mathrm{T}}(t)(Q \otimes I_n)\xi(t).$$

The time derivative of $V(\xi(t))$ along the trajectory of system (4) exists on every time interval $[t_j, t_{j+1})$, $j = 0, 1, 2, \dots$, and is given by

$$\begin{aligned}
&\dot{V}(\xi(t))|_{(4)} \\
&= \xi^{\mathrm{T}}(t)\Big(Q \otimes \bar{S}^{\mathrm{T}} - (H_{\sigma(t)}^{\mathrm{T}} Q) \otimes (\bar{C}^{\mathrm{T}} \bar{C}) \\
&\quad + Q \otimes \bar{S} - (Q H_{\sigma(t)}) \otimes (\bar{C}^{\mathrm{T}} \bar{C})\Big)\xi(t) \\
&= -\xi^{\mathrm{T}}(t)((H_{\sigma(t)}^{\mathrm{T}} Q + Q H_{\sigma(t)}) \otimes (\bar{C}^{\mathrm{T}} \bar{C}))\xi(t) \\
&\leqslant -\hat{\mu}\xi^{\mathrm{T}}(t)((H_{\sigma(t)}^{\mathrm{T}} Q H_{\sigma(t)}) \otimes (\bar{C}^{\mathrm{T}} \bar{C}))\xi(t) \\
&= -\hat{\mu}\|(Q^{1/2} H_{\sigma(t)} \otimes \bar{C})\xi(t)\|^2 \leqslant 0.
\end{aligned} \tag{5}$$

Thus, we have $V(\xi(t)) \leqslant V(\xi(0))$ for all $t \geqslant 0$, and hence $\|\xi(t)\| \leqslant \dfrac{\lambda_{\max}(Q)}{\lambda_{\min}(Q)}\|\xi(0)\|$ for all $t \geqslant 0$. Moreover, $\lim\limits_{t \to \infty} V(\xi(t))$ exists.

Since $\sigma(t) \in \mathcal{P}$ and $\mathcal{P}$ is a finite set, $H_{\sigma(t)}$ is bounded over $[0, \infty)$. Then, by (4), $\dot{\xi}(t)$ is bounded over $[0, \infty)$ and so is $\ddot{V}(\xi(t))$. Therefore, by Corollary 1 of [14], $\lim\limits_{t \to \infty} \dot{V}(\xi(t)) = 0$, and it follows from (5) that

$$\lim_{t \to \infty} (H_{\sigma(t)} \otimes \bar{C})\xi(t) = 0. \tag{6}$$

Similar to the proof of Theorem 1 in [14], we show that the origin of system (1) with $L = P^{-1}C$ is asymptotically stable by the following three steps.

Step 1 Let $\eta(t) = (H_{\sigma(t)} \otimes I_n)\xi(t)$. We first show that (6) implies

$$\lim_{t\to\infty} \eta(t) = 0. \tag{7}$$

For this purpose, we note that (6) implies that

$$\lim_{t\to\infty}(I_N \otimes \bar{C})(H_{\sigma(t)} \otimes I_n)\xi(t) = \lim_{t\to\infty}(I_N \otimes \bar{C})\eta(t) = 0. \tag{8}$$

Since $\xi(t)$ is bounded over $[0,\infty)$, $\eta(t)$ is also bounded over $[0,\infty)$. Moreover, the derivative of $\eta(t)$ exists on every time interval $[t_j, t_{j+1})$, $j = 0,1,2,\dots$, and is given by

$$\dot{\eta}(t) = (I_N \otimes \bar{S} - H_{\sigma(t)} \otimes (\bar{C}^{\rm T}\bar{C}))\eta(t),$$

which is in the form of (3) with $\psi(t) = \eta(t)$ and

$$M = I_N \otimes \bar{S},\ \ N(t) = H_{\sigma(t)} \otimes (\bar{C}^{\rm T}\bar{C}).$$

Let $G = I_N \otimes \bar{C}$. Then, (6) and (8) imply

$$\lim_{t\to\infty} N(t)\eta(t) = 0,\ \ \lim_{t\to\infty} G\eta(t) = 0.$$

By Lemma 1, we have

$$\lim_{t\to\infty}(I_N \otimes \bar{C})(I_N \otimes \bar{S})^k\eta(t) = 0,\ \ k = 0,1,2,\dots. \tag{9}$$

Note that the observability of the pair (C, S) implies the observability of the pair $(\bar{C}, \bar{S})$, and hence the observability of the pair $(I_N \otimes \bar{C}, I_N \otimes \bar{S})$. Therefore, from (9), we have $\lim\limits_{t\to\infty} \eta(t) = 0$, i.e., equation (7) holds.

Step 2 Next, we further show that (7) implies

$$\lim_{t\to\infty}(H_{\sigma(t+T_0)} \otimes I_n)\xi(t) = 0 \tag{10}$$

for any finite $T_0 \geqslant 0$.

From (4), we have, for any $t \geqslant 0$,

$$\xi(t + T_0) = {\rm e}^{(I_N\otimes\bar{S})T_0}\xi(t) + \Delta(t, T_0), \tag{11}$$

where

$$\Delta(t, T_0) = \int_t^{t+T_0} {\rm e}^{(I_N\otimes\bar{S})(t+T_0-s)}(-H_{\sigma(s)} \otimes \bar{C}^{\rm T}\bar{C})\xi(s){\rm d}s.$$

Let $\Upsilon = \max\limits_{t\in[0,T_0]} \|{\rm e}^{(I_N\otimes\bar{S})t}\|$. Since $\|{\rm e}^{(I_N\otimes\bar{S})t}\|$ is continuous on $[0, T_0]$, Υ is finite. Then, we have

$$\begin{aligned}\|\Delta(t, T_0)\| &\leqslant \Upsilon\|I_N \otimes \bar{C}^{\rm T}\| \int_t^{t+T_0} \|(H_{\sigma(s)} \otimes \bar{C})\xi(s)\|{\rm d}s\\ &\leqslant \Upsilon\|I_N \otimes \bar{C}^{\rm T}\|T_0 \sup_{t\leqslant s\leqslant t+T_0} \|(H_{\sigma(s)} \otimes \bar{C})\xi(s)\|.\end{aligned}$$

According to (6), we obtain

$$\lim_{t\to\infty} \Delta(t, T_0) = 0. \tag{12}$$

Thus, by (7), (11), (12), and the fact that $H_{\sigma(t+T_0)} \otimes I_n$ is bounded over $[0,\infty)$, we have

$$\begin{aligned}&\lim_{t\to\infty} {\rm e}^{(I_N\otimes\bar{S})T_0}(H_{\sigma(t+T_0)} \otimes I_n)\xi(t)\\ &= \lim_{t\to\infty}(H_{\sigma(t+T_0)} \otimes I_n){\rm e}^{(I_N\otimes\bar{S})T_0}\xi(t)\\ &= \lim_{t\to\infty}(H_{\sigma(t+T_0)} \otimes I_n)\left(\xi(t + T_0) - \Delta(t, T_0)\right)\\ &= 0,\end{aligned}$$

which gives (10) by noting that the matrix ${\rm e}^{(I_N\otimes\bar{S})T_0}$ is nonsingular for any $T_0 \geqslant 0$.

Step 3 Finally, we show that the origin of system (1) is asymptotically stable with $L = P^{-1}C$.

Letting $t = t_{j_k}$ and $t+T_0 = t_r$ for $r = j_k, j_k+1,\dots, j_{k+1}-1$, in (10) gives

$$\lim_{k\to\infty}(H_{\sigma(t_r)} \otimes I_n)\xi(t_{j_k}) = 0,\ \ r = j_k, j_k + 1,\dots, j_{k+1} - 1.$$

The summation of the above equations is

$$\lim_{k\to\infty}\Big(\sum_{r=j_k}^{j_{k+1}-1} H_{\sigma(t_r)} \otimes I_n\Big)\xi(t_{j_k}) = 0.$$

Since the matrix $\Big(\sum\limits_{r=j_k}^{j_{k+1}-1} H_{\sigma(t_r)}\Big)$ is nonsingular, we have

$$\lim_{k\to\infty} \xi(t_{j_k}) = 0. \tag{13}$$

According to (5), $\|(Q^{1/2} \otimes I_n)\xi(t)\|$ is non-increasing over $[0,\infty)$. This fact together with (13) concludes that $\lim\limits_{t\to\infty} \xi(t) = 0$, and therefore

$$\lim_{t\to\infty} x(t) = \lim_{t\to\infty}(I_N \otimes P^{-1/2})\xi(t) = 0.$$

Thus, the proof is completed. □

Remark 2 Lemma 2 can be viewed as a continuous-time counterpart of Lemma 2 of [17], which gave a stability result for a class of linear discrete-time switched systems.

3 An output-based distributed observer

In this section, we propose an output-based distributed observer for the leader system described by the following linear system:

$$\dot{v}_0 = Sv_0, \ \ y_{m_0} = Cv_0, \tag{14}$$

where $v_0 \in \mathbb{R}^n, y_{m_0} \in \mathbb{R}^m$ are the state and the output, $S \in \mathbb{R}^{n\times n}$ and $C \in \mathbb{R}^{m\times n}$ are constant matrices, and the pair (C, S) is observable.

Let $\sigma : [0,\infty) \mapsto \mathcal{P} = \{1, 2, \dots, \rho\}$ be a piecewise constant switching signal with switching instants $\{t_j : j = 0, 1, 2, \dots\}$ and dwell time $\tau > 0$, and let $\bar{\mathcal{G}}_{\sigma(t)} = (\bar{\mathcal{V}}, \bar{\mathcal{E}}_{\sigma(t)})$ denote a switching digraph [1], where $\bar{\mathcal{V}} = \{0, 1, \dots, N\}$ and, for all $t \geqslant 0$, $\bar{\mathcal{E}}_{\sigma(t)} \subseteq \bar{\mathcal{V}} \times \bar{\mathcal{V}}$ satisfies the following assumption.

Assumption 3 There exists a subsequence $\{j_k : k = 0, 1, 2, \dots\}$ of $\{j : j = 0, 1, 2, \dots\}$ with $t_{j_{k+1}} - t_{j_k} < T$ for some $T > 0$, such that the union digraph $\bigcup_{r=j_k}^{j_{k+1}-1} \bar{\mathcal{G}}_{\sigma(t_r)}$ has the property that there is a directed path from node 0 to every other node.

Remark 3 Assumption 3 is called the jointly connected condition [18]. It is perhaps the mildest condition on the switching digraph $\bar{\mathcal{G}}_{\sigma(t)}$ as it allows the digraph to be disconnected at every time instant.

Given system (14) and the switching digraph $\bar{\mathcal{G}}_{\sigma(t)}$, as in [15], we denote by $\bar{\mathcal{A}}_{\sigma(t)} = [a_{ij}(t)]_{i,j=0}^N \in \mathbb{R}^{(N+1)\times(N+1)}$ the weighted adjacency matrix of $\bar{\mathcal{G}}_{\sigma(t)}$ and define a distributed dynamic compensator as follows:

$$\dot{v}_i = Sv_i + L\sum_{j=0}^{N} a_{ij}(t)C(v_j - v_i), \ \ i = 1, \dots, N, \tag{15}$$

where $L \in \mathbb{R}^{n\times m}$ is a gain matrix to be designed, and, for $i = 1, \dots, N$, $v_i \in \mathbb{R}^n$. It can be seen that (15) depends on $Cv_0 = y_{m_0}$ instead of v_0. If there exists L such that, for any initial condition $v_i(0)$, $i = 0, 1, \dots, N$, the solutions of systems (14) and (15) exist for all $t \geqslant 0$ and satisfy

$$\lim_{t\to\infty}(v_i(t) - v_0(t)) = 0, \ \ i = 1, \dots, N$$

asymptotically, then system (15) is called an output-based distributed observer for the leader system (14).

In the special case where $C = I_n$, we can take $L = \mu I_n$ with μ a positive constant. Then, (15) reduces to

$$\dot{v}_i = Sv_i + \mu\sum_{j=0}^{N} a_{ij}(t)(v_j - v_i), \ \ i = 1, \dots, N, \tag{16}$$

which was proposed in [11]. By Lemma 2 and Remark 4 of [11], under Assumption 3 and the assumption that the matrix S has no eigenvalues with positive real parts, for any $\mu > 0$ and any initial condition $v_i(0)$, $i = 0, 1, \dots, N$, the solutions of systems (14) and (16) exist for all $t \geqslant 0$ and satisfy

$$\lim_{t\to\infty}(v_i(t) - v_0(t)) = 0, \ \ i = 1, \dots, N$$

exponentially. For this reason, system (16) is called a (state-based) distributed observer for the leader system (14).

Now, let $\tilde{v}_i = v_i - v_0, i = 1, \dots, N$, and $\tilde{v} = \mathrm{col}(\tilde{v}_1, \dots, \tilde{v}_N)$. Then, system (15) can be compactly written as

$$\dot{\tilde{v}} = (I_N \otimes S - \mathcal{H}_{\sigma(t)} \otimes (LC))\tilde{v}, \ \ \sigma(t) \in \mathcal{P}, \tag{17}$$

where, with $\bar{\mathcal{L}}_{\sigma(t)} \in \mathbb{R}^{(N+1)\times(N+1)}$ denoting the Laplacian of $\bar{\mathcal{G}}_{\sigma(t)}$, the matrix $\mathcal{H}_{\sigma(t)} \in \mathbb{R}^{N\times N}$ is obtained from $\bar{\mathcal{L}}_{\sigma(t)}$ by removing the first row and the first column.

In order to apply Lemma 2 to show that (15) is indeed an output-based distributed observer for the leader system (14), we need to make one more assumption as follows.

Assumption 4 There exist a $\hat{\mu} > 0$ and a symmetric and positive definite matrix Q such that, for all $p = 1, \dots, \rho$, the matrix $\mathcal{H}_p^{\mathrm{T}}Q + Q\mathcal{H}_p - \hat{\mu}\mathcal{H}_p^{\mathrm{T}}Q\mathcal{H}_p$ is positive semi-definite.

We are now ready to present the following result.

Lemma 3 Given systems (14) and (15), under Assumptions 2–4, for any initial conditions $v_i(0)$, $i = 0, 1, \dots, N$, the solutions of systems (14) and (15) with $L = P^{-1}C^{\mathrm{T}}$, where P is the symmetric and positive definite solution to (2), exist for all $t \geqslant 0$ and satisfy

$$\lim_{t\to\infty}(v_i(t) - v_0(t)) = 0, \ \ i = 1, \dots, N.$$

Proof By the same argument as used in Remark 14 of [14], under Assumption 3, the matrix $(\sum_{r=j_k}^{j_{k+1}-1} \mathcal{H}_{\sigma(t_r)})$ is

[1] See Appendix for a summary of notation on digraph.

nonsingular. By Lemma 2, system (17) is asymptotically stable with $L = P^{-1}C^{\mathrm{T}}$. Thus, the proof is completed. □

4 An application

In this section, we apply the output-based distributed observer to solve the cooperative output regulation problem of the following class of linear multi-agent systems:

$$\begin{cases} \dot{x}_i = A_i x_i + B_i u_i + E_{iv} v_0 + E_{iw} w_i, \\ y_{m_i} = C_{m_i} x_i + D_{m_i} u_i + F_{m_i v} v_0 + F_{m_i w} w_i, \\ e_i = C_i x_i + D_i u_i + F_{iv} v_0 + F_{iw} w_i, \quad i = 1, \dots, N, \end{cases} \tag{18}$$

where, for $i = 1, \dots, N$, $x_i \in \mathbb{R}^{n_i}, y_{m_i} \in \mathbb{R}^{p_{m_i}}, e_i \in \mathbb{R}^{p_i}$, and $u_i \in \mathbb{R}^{m_i}$ are the state, measurement output, error output, and control input of the ith subsystem, respectively; $v_0 \in \mathbb{R}^n$ is the measurable exogenous signal such as the reference input to be tracked, and it is assumed to be generated by (14); $w_i \in \mathbb{R}^{q_i}$ is the unmeasurable exogenous signal such as local external disturbances to each subsystem, and it is assumed to be generated by

$$\dot{w}_i = S_i w_i, \quad i = 1, \dots, N. \tag{19}$$

In practice, the communication among system (14) and different subsystems of (18) is subject to some constraints due to, say, the physical distance among them. In particular, the output signal y_{m_0} of system (14) may not be available for the control u_i of all subsystems of (18). To describe the communication constraints among various subsystems, as in [11], we view the system composed of (14) and (18) as a multi-agent system with (14) as the leader and the N subsystems of (18) as followers. Then, given a piecewise constant switching signal $\sigma(t)$, we can define a switching digraph $\bar{\mathcal{G}}_{\sigma(t)} = (\bar{\mathcal{V}}, \bar{\mathcal{E}}_{\sigma(t)})$ where $\bar{\mathcal{V}} = \{0, 1, \dots, N\}$. We associate node 0 with the leader system (14) and node $i, i = 1, \dots, N$, with the ith follower subsystem of (18). The edge set $\bar{\mathcal{E}}_{\sigma(t)} \subseteq \bar{\mathcal{V}} \times \bar{\mathcal{V}}$ describes the communication constraints: for $i = 1, \dots, N, j = 0, 1, \dots, N, (j, i) \in \bar{\mathcal{E}}_{\sigma(t)}$ if and only if u_i can use the information of agent j for control at time instant t.

We describe our control law as follows:

$$\begin{cases} u_i = f_i(\zeta_i), \quad i = 1, \dots, N, \\ \dot{\zeta}_i = g_i(\zeta_i, y_{m_i}, \zeta_j, y_{mj}, j \in \bar{\mathcal{N}}_i(t)), \end{cases} \tag{20}$$

where, for $i = 1, \dots, N$, $\bar{\mathcal{N}}_i(t) = \{j \in \bar{\mathcal{V}} \,|\, (j, i) \in \bar{\mathcal{E}}_{\sigma(t)}\}$ is the neighbor set of node i at time t, both $f_i(\cdot)$ and $g_i(\cdot)$ are linear functions in their arguments, and ζ_i is the state of the dynamic compensator. In particular, it can be seen that, at each time instant $t \geqslant 0$, u_i can make use of y_{m_0} if and only if the leader is a neighbor of the ith follower, or mathematically, $0 \in \bar{\mathcal{N}}_i(t)$. Such a control law is called a distributed control law.

We are now ready to describe the cooperative linear output regulation problem.

Problem description Given systems (14), (18), (19), and a switching digraph $\bar{\mathcal{G}}_{\sigma(t)}$, design a distributed control law of the form (20) such that, for any initial conditions $v_0(0)$, $x_i(0)$, $w_i(0)$, and $\zeta_i(0)$, $i = 1, \dots, N$, the solution of the closed-loop system exists for all $t \geqslant 0$ and satisfies

$$\lim_{t \to \infty} e_i(t) = 0, \quad i = 1, \dots, N.$$

Some standard assumptions for solving the above problem are listed below.

Assumption 5 For $i = 1, \dots, N$, the pairs (A_i, B_i) are stabilizable.

Assumption 6 For $i = 1, \dots, N$, the pairs

$$\left([\, C_{m_i} \;\; F_{m_i w} \,], \begin{bmatrix} A_i & E_{iw} \\ 0 & S_i \end{bmatrix} \right)$$

are detectable.

To introduce the next assumption, let $\bar{S}_i = \begin{bmatrix} S & 0 \\ 0 & S_i \end{bmatrix}$, $E_i = [\, E_{iv} \;\; E_{iw} \,]$, and $F_i = [\, F_{iv} \;\; F_{iw} \,]$.

Assumption 7 For $i = 1, \dots, N$, the linear matrix equations

$$\begin{cases} X_i \bar{S}_i = A_i X_i + B_i U_i + E_i, \\ 0 = C_i X_i + D_i U_i + F_i \end{cases} \tag{21}$$

have solution pairs (X_i, U_i).

Remark 4 Assumptions 5–7 are standard as they have been used in both the classical linear output regulation problem [19–21] and the cooperative linear output regulation problem [11, 15] by a measurement output feedback control law. In particular, equations in (21) are called the regulator equations, whose solvability imposes a necessary condition for the solvability of the (cooperative) linear output regulation problem.

To motivate our approach, we assume, tentatively, that every follower subsystem of (18) can access the state v_0 of the leader system (14). In this case, it is possible to find a so-called purely decentralized control law to solve our problem.

For $i = 1, \dots, N$, under Assumption 5, there exists $K_{1i} \in \mathbb{R}^{m_i \times n_i}$ such that $A_i + B_i K_{1i}$ is Hurwitz. Under Assumption 7, let (X_i, U_i) be a solution pair to the regulator equations in (21), define $K_{2i} = U_i - K_{1i} X_i \in \mathbb{R}^{m_i \times (n+q_i)}$, and further partition K_{2i} as $K_{2i} = [K_{2iv} \ K_{2iw}]$ with $K_{2iv} \in \mathbb{R}^{m_i \times n}$ and $K_{2iw} \in \mathbb{R}^{m_i \times q_i}$. Moreover, under Assumption 6, there exists $L_i \in \mathbb{R}^{(n_i+q_i) \times p_{m_i}}$ such that

$$\begin{bmatrix} A_i & E_{iw} \\ 0 & S_i \end{bmatrix} + L_i [C_{m_i} \ F_{m_i w}]$$

is Hurwitz. Then, it can be shown, as in the classical linear output regulation theory [19–21], that the following dynamic measurement output feedback control law solves our problem:

$$\begin{cases} u_i = [K_{1i} \ K_{2iw}] \begin{bmatrix} \hat{x}_i \\ \hat{w}_i \end{bmatrix} + K_{2iv} v_0 \\ \begin{bmatrix} \dot{\hat{x}}_i \\ \dot{\hat{w}}_i \end{bmatrix} = \begin{bmatrix} A_i & E_{iw} \\ 0 & S_i \end{bmatrix} \begin{bmatrix} \hat{x}_i \\ \hat{w}_i \end{bmatrix} + \begin{bmatrix} B_i \\ 0 \end{bmatrix} u_i + \begin{bmatrix} E_{iv} \\ 0 \end{bmatrix} v_0 \\ \qquad - L_i \Big(y_{m_i} - [C_{m_i} \ F_{m_i w}] \begin{bmatrix} \hat{x}_i \\ \hat{w}_i \end{bmatrix} \\ \qquad - D_{m_i} u_i - F_{m_i v} v_0\Big), \quad i = 1, \dots, N. \end{cases} \tag{22}$$

Nevertheless, the control law (22) is not in the form of (20) since, for each $i = 1, \dots, N$, the control u_i has to rely on the state v_0 of the leader system (14). We call such a control law a purely decentralized control law. To obtain a distributed control of the form (20), composing the purely decentralized control law (22) and the output-based distributed observer (15) yields the following control law:

$$\begin{cases} u_i = [K_{1i} \ K_{2iw}] \begin{bmatrix} \hat{x}_i \\ \hat{w}_i \end{bmatrix} + K_{2iv} v_i, \\ \begin{bmatrix} \dot{\hat{x}}_i \\ \dot{\hat{w}}_i \end{bmatrix} = \begin{bmatrix} A_i & E_{iw} \\ 0 & S_i \end{bmatrix} \begin{bmatrix} \hat{x}_i \\ \hat{w}_i \end{bmatrix} + \begin{bmatrix} B_i \\ 0 \end{bmatrix} u_i + \begin{bmatrix} E_{iv} \\ 0 \end{bmatrix} v_i \\ \qquad - L_i \Big(y_{m_i} - [C_{m_i} \ F_{m_i w}] \begin{bmatrix} \hat{x}_i \\ \hat{w}_i \end{bmatrix} \\ \qquad - D_{m_i} u_i - F_{m_i v} v_i\Big), \\ \dot{v}_i = S v_i + L \sum\limits_{j=0}^{N} a_{ij}(t) C (v_j - v_i), \quad i = 1, \dots, N. \end{cases} \tag{23}$$

Remark 5 It can be seen that the first two equations of (23) are obtained from (22) by replacing v_0 with $v_i, i = 1, \dots, N$, which are provided by the third equation of (23). This way of synthesizing a control law is known as the certainty equivalence principle. Moreover, it is noted that (23) is in the form of (20) with $\zeta_i = \mathrm{col}(\hat{x}_i, \hat{w}_i, v_i)$, $i = 1, \dots, N$.

Theorem 1 Under Assumptions 2–7, the distributed dynamic measurement output feedback control law (23) solves the cooperative output regulation problem of the system composed of (14), (18), and (19).

Proof For $i = 1, \dots, N$, let

$$\tilde{x}_i = x_i - X_i \begin{bmatrix} v_0 \\ w_i \end{bmatrix}, \quad \tilde{u}_i = u_i - U_i \begin{bmatrix} v_0 \\ w_i \end{bmatrix},$$

$$\begin{bmatrix} \bar{x}_i \\ \bar{w}_i \end{bmatrix} = \begin{bmatrix} \hat{x}_i \\ \hat{w}_i \end{bmatrix} - \begin{bmatrix} x_i \\ w_i \end{bmatrix}, \quad \tilde{v}_i = v_i - v_0.$$

Under Assumption 7, by making use of the solution to the regulator equations in (21), we have, for $i = 1, \dots, N$,

$$\begin{aligned} \dot{\tilde{x}}_i &= A_i x_i + B_i u_i + E_i \begin{bmatrix} v_0 \\ w_i \end{bmatrix} - X_i \bar{S}_i \begin{bmatrix} v_0 \\ w_i \end{bmatrix} \\ &= A_i \tilde{x}_i + B_i \tilde{u}_i \end{aligned} \tag{24}$$

and

$$\begin{aligned} e_i &= C_i x_i + D_i u_i + F_i \begin{bmatrix} v_0 \\ w_i \end{bmatrix} \\ &= C_i \tilde{x}_i + D_i \tilde{u}_i. \end{aligned} \tag{25}$$

According to our control law (23), we have, for $i = 1, \dots, N$,

$$\begin{aligned} \tilde{u}_i &= [K_{1i} \ K_{2iw}] \begin{bmatrix} \hat{x}_i \\ \hat{w}_i \end{bmatrix} + K_{2iv} v_i - U_i \begin{bmatrix} v_0 \\ w_i \end{bmatrix} \\ &= [K_{1i} \ K_{2iw}] \begin{bmatrix} \bar{x}_i \\ \bar{w}_i \end{bmatrix} + K_{1i} x_i + K_{2iv} \tilde{v}_i \\ &\quad + (K_{2i} - U_i) \begin{bmatrix} v_0 \\ w_i \end{bmatrix} \\ &= [K_{1i} \ K_{2iw}] \begin{bmatrix} \bar{x}_i \\ \bar{w}_i \end{bmatrix} + K_{1i} \tilde{x}_i + K_{2iv} \tilde{v}_i \end{aligned} \tag{26}$$

and

$$\begin{bmatrix} \dot{\bar{x}}_i \\ \dot{\bar{w}}_i \end{bmatrix} = \left(\begin{bmatrix} A_i & E_{iw} \\ 0 & S_i \end{bmatrix} + L_i [C_{m_i} \ F_{m_i w}] \right) \begin{bmatrix} \bar{x}_i \\ \bar{w}_i \end{bmatrix}$$

$$+\Big(\begin{bmatrix} E_{iv} \\ 0 \end{bmatrix} + L_i F_{m_i v}\Big)\tilde{v}_i. \tag{27}$$

We first note that, by Lemma 3, under Assumptions 2–4, $\lim_{t\to\infty} \tilde{v}_i(t) = 0$, $i = 1,\dots,N$. Furthermore, under Assumption 6, system (27) is a strictly stable linear time-invariant system subject to an asymptotically decaying input. Thus, we have

$$\lim_{t\to\infty} \begin{bmatrix} \bar{x}_i(t) \\ \bar{w}_i(t) \end{bmatrix} = 0, \quad i = 1,\dots,N. \tag{28}$$

Next, substituting (26) into (24) yields

$$\begin{aligned} \dot{\tilde{x}}_i &= (A_i + B_i K_{1i})\,\tilde{x}_i \\ &\quad + B_i([\,K_{1i} \;\; K_{2iw}\,]\begin{bmatrix} \bar{x}_i \\ \bar{w}_i \end{bmatrix} + K_{2iv}\tilde{v}_i), \end{aligned}$$

which again, under Assumption 5, can be viewed as a strictly stable linear time-invariant system subject to an asymptotically decaying input. Thus, we have

$$\lim_{t\to\infty} \tilde{x}_i(t) = 0, \quad i = 1,\dots,N \tag{29}$$

and therefore by (25), (26), (28), and (29),

$$\lim_{t\to\infty} e_i(t) = 0, \quad i = 1,\dots,N.$$

□

5 An example

In this section, we present an example to illustrate our design. Consider the cooperative output regulation problem for a group of double integrator systems subject to local sinusoidal disturbances:

$$\begin{aligned} \dot{x}_{1i} &= x_{2i}, \\ \dot{x}_{2i} &= u_i + i * w_{2i}, \\ y_{m_i} &= x_{1i} - i * w_{1i}, \\ e_i &= x_{1i} - y_{m_0}, \quad i = 1,2,3,4, \end{aligned}$$

where the local disturbance $w_i = \mathrm{col}(w_{1i}, w_{2i})$ to each subsystem is generated by

$$\dot{w}_i = \begin{bmatrix} 0 & 2i \\ -2i & 0 \end{bmatrix} w_i, \quad i = 1,2,3,4$$

and the reference signal y_{m_0} is generated by the following leader system:

$$\dot{v}_0 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} v_0, \quad y_{m_0} = [1 \;\; 0]v_0.$$

For this example, $N = 4$ and

$$S = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad C = [1 \;\; 0], \quad S_i = \begin{bmatrix} 0 & 2i \\ -2i & 0 \end{bmatrix},$$

$$A_i = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad E_{iv} = 0,$$

$$E_{iw} = \begin{bmatrix} 0 & 0 \\ 0 & i \end{bmatrix}, \quad C_{m_i} = [1 \;\; 0], \quad D_{m_i} = 0,$$

$$F_{m_i v} = 0, \quad F_{m_i w} = [-i \;\; 0], \quad C_i = [1 \;\; 0],$$

$$D_i = 0, \quad F_{iv} = [-1 \;\; 0], \quad F_{iw} = 0.$$

It is easy to check that Assumptions 2 and 5–7 hold. In particular, the regulator equations in (21) admit the following solutions:

$$X_i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad U_i = [-1 \;\; 0 \;\; 0 \;\; -i], \quad i = 1,2,3,4.$$

Assume that the switching digraph $\bar{\mathcal{G}}_{\sigma(t)}$ is dictated by the following switching signal:

$$\sigma(t) = \begin{cases} 1, & \text{if } sT \leqslant t < (s + \frac{1}{4})T, \\ 2, & \text{if } (s + \frac{1}{4})T \leqslant t < (s + \frac{1}{2})T, \\ 3, & \text{if } (s + \frac{1}{2})T \leqslant t < (s + \frac{3}{4})T, \\ 4, & \text{if } (s + \frac{3}{4})T \leqslant t < (s + 1)T, \end{cases}$$

where $T = 2$ and $s = 0,1,2,\dots$. The four digraphs $\bar{\mathcal{G}}_i, i = 1,2,3,4$, are described in Fig. 1, where node 0 is associated with the leader system and the other four nodes are associated with the four follower subsystems. It can be seen that Assumption 3 is satisfied even though $\bar{\mathcal{G}}_{\sigma(t)}$ is disconnected for all $t \geqslant 0$.

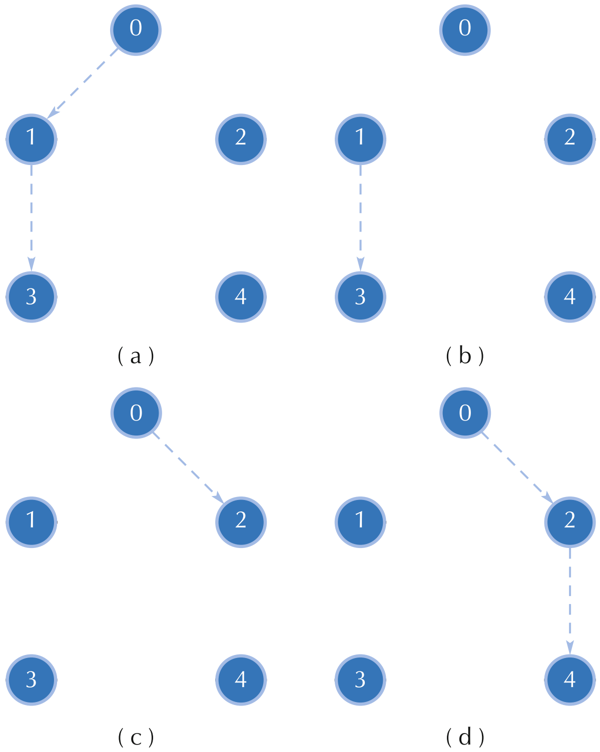


Fig. 1 Switching topology $\bar{\mathcal{G}}_{\sigma(t)}$ with $\mathcal{P} = \{1,2,3,4\}$. (a) $\bar{\mathcal{G}}_1$. (b) $\bar{\mathcal{G}}_2$. (c) $\bar{\mathcal{G}}_3$. (d) $\bar{\mathcal{G}}_4$.

Let $a_{ij}(t) = 1$, $i, j = 0,1,2,3,4$, whenever $(j,i) \in \bar{\mathcal{E}}_{\sigma(t)}$. Then, the four matrices associated with the four digraphs $\bar{\mathcal{G}}_i, i = 1,2,3,4$, are given as follows:

$$\mathcal{H}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathcal{H}_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$
$$\mathcal{H}_3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathcal{H}_4 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}.$$

Clearly, none of the matrices $\mathcal{H}_i, i = 1,2,3,4$, are symmetric. Moreover, there exists no $\hat{\mu} > 0$ such that the matrix $\mathcal{H}_2^{\mathrm{T}} + \mathcal{H}_2 - \hat{\mu}\mathcal{H}_2^{\mathrm{T}}\mathcal{H}_2$ is positive semi-definite. Thus, for this example, Assumption 3.2 of [16] cannot be satisfied. However, let

$$Q = \begin{bmatrix} 2 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Then, it can be verified that, for $i = 1,2,3,4$, $\mathcal{H}_i^{\mathrm{T}} Q + Q\mathcal{H}_i - \hat{\mu}\mathcal{H}_i^{\mathrm{T}} Q \mathcal{H}_i$ are positive semi-definite for $\hat{\mu} \leqslant 1$. Thus, Assumption 4 is satisfied.

By Theorem 1, we can design a distributed dynamic measurement output feedback control law of the form (23) with the following design parameters: $K_{1i} = [-8\ \ -4]$, $K_{2i} = [7\ \ 4\ \ 0\ \ -i]$, $i = 1,2,3,4$, $L_1 = [-51\ \ -68\ \ -37\ \ 3]^{\mathrm{T}}$, $L_2 = [-13\ \ -12\ \ 0\ \ 6]^{\mathrm{T}}$, $L_3 = [-6\ \ -4\ \ 3\ \ 2]^{\mathrm{T}}$, $L_4 = [-3\ \ -2\ \ 3\ \ 0]^{\mathrm{T}}$, and $L = C^{\mathrm{T}}$.

Simulation of the closed-loop system is performed with randomly generated initial conditions. Fig. 2 shows the evolution of the estimation errors of the output-based distributed observer and Fig. 3 shows the evolution of the error outputs of the followers.

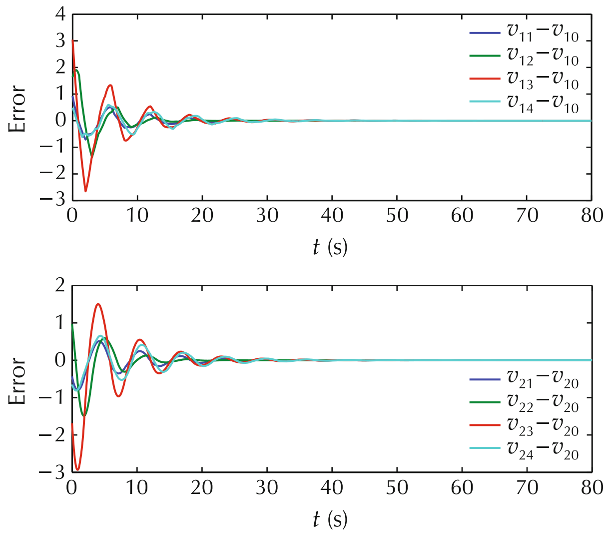


Fig. 2 Profile on the estimation errors on the leader's state v_0.

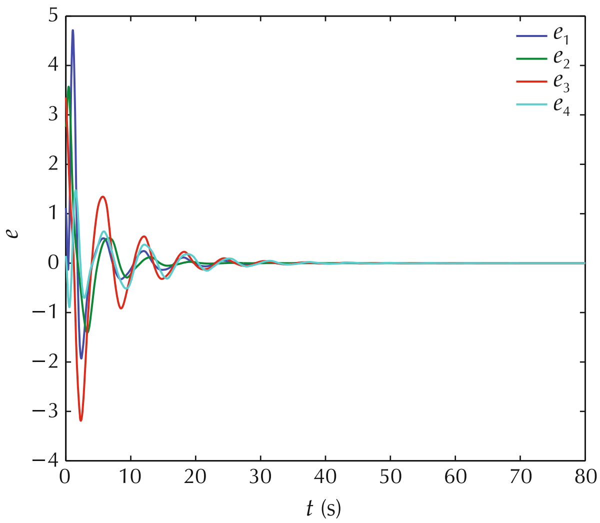


Fig. 3 Profile on the error outputs of the followers.

6 Conclusions

In this paper, we have extended a stability result for a class of linear switched systems, which, in turn, has led to an output-based distributed observer subject to directed and jointly connected switching communication networks. As an application of this output-based distributed observer, we have solved the cooperative output regulation problem of a linear multi-agent system subject to jointly connected switching communication networks. It is also possible to apply the output-based distributed observer to the leader-following attitude consensus problem of multiple rigid spacecraft systems studied in [4] and the leader-following consensus problem of multiple Euler-Lagrange systems studied in [12].

References

[1] T. Liu, J. Huang. An output-based distributed observer and its application. *Proceedings of the American Control Conference*, New York: IEEE, 2018: 2491 – 2496.

[2] Z. Lin, B. Francis, M. Maggiore. Necessary and sufficient graphical conditions for formation control of unicycles. *IEEE Transactions on Automatic Control*, 2005, 50(1): 121 – 127.

[3] R. Olfati-Saber, J. A. Fax, R. M. Murray. Consensus and cooperation in networked multi-agent systems. *Proceedings of the IEEE*, 2007, 95(1): 215 – 233.

[4] H. Cai, J. Huang. The leader-following attitude control of multiple rigid spacecraft systems. *Automatica*, 2014, 50(4): 1109 – 1115.

[5] J. R. Lawton, R. W. Beard. Synchronized multiple spacecraft rotations. *Automatica*, 2002, 38(8): 1359 – 1364.

[6] J. Cortés, S. Martínez, T. Karatas, et al. Coverage control for mobile sensing networks. *IEEE Transactions on Robotics and Automation*, 2004, 20(2): 243 – 255.

[7] P. Ögren, E. Fiorelli, N. E. Leonard. Cooperative control of mobile sensor networks: Adaptive gradient climbing in a distributed environment. *IEEE Transactions on Automatic Control*, 2004, 49(8): 1292 – 1302.

[8] H. Cai, G. Hu, F. L. Lewis, et al. A distributed feedforward approach to cooperative control of AC microgrids. *IEEE Transactions on Power Systems*, 2016, 31(5): 4057 – 4067.

[9] H. Cai, G. Hu. Distributed nonlinear hierarchical control of AC microgrid via unreliable communication. *IEEE Transactions on Smart Grid*, 2017, 9(4): 2429 – 2441.

[10] Y. Su, J. Huang. Cooperative output regulation of linear multi-agent systems. *IEEE Transactions on Automatic Control*, 2012, 57(4): 1062 – 1066.

[11] Y. Su, J. Huang. Cooperative output regulation with application to multi-agent consensus under switching network. *IEEE Transactions on Systems, Man, and Cybernetics – Part B: Cybernetics*, 2012, 42(3): 864 – 875.

[12] H. Cai, J. Huang. The leader-following consensus for multiple uncertain Euler-Lagrange systems with an adaptive distributed observer. *IEEE Transactions on Automatic Control*, 2016, 61(10): 3152 – 3157.

[13] G. S. Seyboth, W. Ren, F. Allgöwer. Cooperative control of linear multi-agent systems via distributed output regulation and transient synchronization. *Automatica*, 2016, 68: 132 – 139.

[14] Y. Su, J. Huang. Stability of a class of linear switching systems with applications to two consensus problems. *IEEE Transactions on Automatic Control*, 2012, 57(6): 1420 – 1430.

[15] J. Huang. Certainty equivalence, separation principle, and cooperative output regulation of multi-agent systems by the distributed observer approach. *Control of Complex Systems: Theory and Applications*. Elsevier, 2016: 421 – 449.

[16] J. Huang. The consensus for discrete-time linear multi-agent systems under directed switching networks. *IEEE Transactions on Automatic Control*, 2017, 62(8): 4086 – 4092.

[17] J. Liu, J. Huang. Leader-following consensus of linear discrete-time multi-agent systems subject to jointly connected switching digraphs. *Science China Information Sciences*, 2018, 61: 112208:1 – 112208:8.

[18] A. Jadbabaie, J. Lin, A. S. Morse. Coordination of groups of mobile autonomous agents using nearest neighbor rules. *IEEE Transactions on Automatic Control*, 2003, 48(6): 988 – 1001.

[19] B. A. Francis. The linear multivariable regulator problem. *SIAM Journal on Control and Optimization*, 1977, 15(3): 486 – 505.

[20] J. Huang. *Nonlinear Output Regulation: Theory and Applications*. Philadelphia: SIAM, 2004.

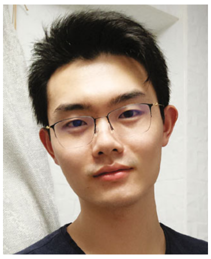
[21] A. Isidori, C. I. Byrnes. Output regulation of nonlinear systems. *IEEE Transactions on Automatic Control*, 1990, 35(2): 131 – 140.

Appendix

A digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ consists of a finite set of nodes $\mathcal{V} = \{1, \dots, N\}$ and an edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. An edge of $\mathcal{E}$ from node i to node j is denoted by (i, j) and node i is called a neighbor of node j. Let $\mathcal{N}_i = \{j \in \mathcal{V} \mid (j, i) \in \mathcal{E}\}$. Then, $\mathcal{N}_i$ is called the neighbor set of node i. The edge (i, j) is called undirected if $(i, j) \in \mathcal{E}$ implies $(j, i) \in \mathcal{E}$. The digraph $\mathcal{G}$ is undirected if every edge in $\mathcal{E}$ is undirected. If the digraph contains a set of edges of the form $\{(i_1, i_2), (i_2, i_3), \dots, (i_{k-1}, i_k)\}$, then this set is called a directed path of $\mathcal{G}$ from node i_1 to node i_k. Given a set of ρ digraphs $\{\mathcal{G}_i = (\mathcal{V}, \mathcal{E}_i), i = 1, \dots, \rho\}$, the digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with $\mathcal{E} = \bigcup_{i=1}^{\rho} \mathcal{E}_i$, is called the union of the digraphs $\mathcal{G}_i$ and is denoted by $\mathcal{G} = \bigcup_{i=1}^{\rho} \mathcal{G}_i$.

The weighted adjacency matrix of a digraph $\mathcal{G}$ is a nonnegative matrix $\mathcal{A} = [a_{ij}]_{i,j=1}^{N} \in \mathbb{R}^{N\times N}$, where $a_{ii} = 0$ and $a_{ij} > 0$ if and only if $(j, i) \in \mathcal{E}$. The Laplacian of $\mathcal{G}$ is then defined as $\mathcal{L} = [l_{ij}]_{i,j=1}^{N} \in \mathbb{R}^{N\times N}$, where $l_{ii} = \sum_{j=1}^{N} a_{ij}$ and $l_{ij} = -a_{ij}$ for $i \neq j$.

Given a piecewise constant switching signal $\sigma(t)$, and a set of digraphs $\mathcal{G}_i = (\mathcal{V}, \mathcal{E}_i)$, $i = 1, \dots, \rho$, with the corresponding weighted adjacency matrices denoted by $\mathcal{A}_i$, $i = 1, \dots, \rho$, we call the time-varying digraph $\mathcal{G}_{\sigma(t)} = (\mathcal{V}, \mathcal{E}_{\sigma(t)})$ a switching digraph and denote the weighted adjacency matrix and the Laplacian of $\mathcal{G}_{\sigma(t)}$ by $\mathcal{A}_{\sigma(t)}$ and $\mathcal{L}_{\sigma(t)}$, respectively.



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